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Performance and Robustness Metrics for the Compensation of Processes with Time Delay

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Performance and Robustness Metrics for the Compensation of Processes with Time Delay

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Abstract:

Proportional-integral (PI) or proportional-integral-derivative (PID) controllers are still by far the most common form of automatic feedback control employed in the process industries at present. It is estimated that more than 90% of control loops in the process control industry are of the PID type. Unfortunately, it is a common occurrence, in the majority of industries where this type of control is employed, for the performance of PI/PID controlled processes to be poor. There are a number of factors influencing this poor performance, namely controller tuning problems, controller equipment design restrictions, deficiencies in control strategy design, etc.

The research was conducted with the intention of developing an effective strategy for assessing the performance and robustness of closed loop systems controlled by PI and PID controllers. Subsequent to an initial investigation into the most effective performance assessment measures, a Matlab based software tool to automate the evaluation process was developed. This software tool incorporated an identification stage followed by an evaluation stage and was developed using Matlab™ 6.5. A comparison of two distinctive identification techniques was conducted with the intention of identifying the most efficient means of extracting the most constructive information with minimal disturbance to the system under test. The identification techniques investigated included a relay based techniques and a Pseudorandom binary sequence testing scheme. A thorough investigation was carried out into identifying the most efficient means of applying these testing schemes. With respect to the Pseudorandom binary sequence based identification technique, a comparison of a number of technique parameter selection methods was carried out and a guideline for applying a pseudorandom binary test signal was developed. Also, a PI(D) tuning rule database was created. The effectiveness of the overall evaluation strategy, as well as the capabilities of the identification techniques was investigated in order to validate the merit of the strategy developed.

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1 Introduction:

1.1 Problem Investigated:

Monitoring of process variables is useful, not only for assessing the status of the process, but also for controlling product quality [1]. In the testing of thousands of control loops in hundreds of operating plants, one of the leading companies in integrated process control system analysis and loop tuning, Techmation Inc., found that more than 30% of the automatic control loops actually increase variability over manual control due to poor controller tuning [2, 3]. According to a survey carried out by Entech [4], only about 20% of all control loops surveyed were found to actually reduce process variability in automatic mode. Of the remaining 80% of control loops, 30% were found to oscillate due to control valve nonlinearities, 30% were found to oscillate due to controller tuning and controller equipment design limitations, 15% were found to oscillate due to deficiencies in control strategy design and 5% were found to oscillate due to process design. Another reason for poor control system performance is that often there are numerous (more than a thousand) loops in a large process plant and not enough control engineers to maintain every loop.

Jamsa-Jounela *et al.* make the point that in order to ensure highest product quality it is essential to maintain the control system in an adequate manner [5]. Vishnubhotla *et al.* discuss how the current standard practice for industrial process control is to install DCS (Distributed Control Systems) and PLC (Programmable Logic Controller) control system platforms [6]. These system platforms accumulate large volumes of process data, but there are very few data mining tools.

It is apparent, therefore, that there is a strong need for automatic assessment and monitoring of control loop performance. The goal of monitoring should be to provide information that can be used to assess the current status of the existing control system and to assist control engineers in deciding whether redesign is necessary [7]. When the controller performance is determined to be inadequate, it is important to ascertain whether an acceptable level of performance can be achieved with the existing control structure [8]. With this objective in mind, the motivation of the following research is aimed at developing techniques to analytically determine performance and robustness criteria for proportional-integral (PI) and proportional-integral-derivative (PID) controlled processes. A brief explanation of P, PI and PID control is presented in Appendix G. Through the course of this research, a software tool will be developed, in a Matlab^{TM} environment, to automate the evaluation and performance assessment procedure.

1.2 Thesis Layout:

The thesis is structured as follows. In Chapter 2 the results of a comprehensive literature review of current control system performance assessment techniques is presented. The chief objectives of an effective evaluation technique are discussed along with a classification of a number of useful assessment metrics. The key objective of this Chapter is to familiarise the reader with a number of possible means of system evaluation, identifying the benefits associated with each technique while also providing a means of comparing the identified categories to help evaluate their suitability in performance assessment. An investigation into the direction of current industrial research, with respect to performance evaluation procedures, is discussed and an overview of currently available assessment software packages is presented.

In Chapter 3, the results of the literature review of Chapter 2 are utilised in the development of an assessment and evaluation strategy that can be applied to closed loop systems with the objective of evaluating their performance. A graphical user interface (GUI) is developed using Matlab 6.5, incorporating Humusofts' real time library, to facilitate real time system testing. An explanation of the review and selection process of a variety of identification techniques incorporated in the assessment tool, ultimately leading to the selection of the pseudo random binary sequence (PRBS) and relay based approaches to system identification, along with an explanation of the stages involved in the development of a PI(D) database and information regarding the approach taken in selecting an appropriate process modelling technique is also presented.

Chapter 4 presents a review of the results obtained from a variety of simulations carried out using the GUI's developed in Chapter 3. The simulations were carried out with the objective of developing a guideline for applying the identification techniques in the most effective manner. Both the PRBS and relay based identification techniques require user inputs before the system-testing phase can begin. This Chapter concentrates on the development of a guideline for the choice of each of these technique parameters along with an investigation into the limitations and capabilities of each of the identification and evaluation methods developed. Also, a review of the results obtained from a variety of simulations carried out with the objective of validating the overall evaluation strategy developed in Chapter 2 is presented.

Finally, Chapter 5 presents the conclusions obtained during the course of the research. A section containing avenues of possible future work is also presented. Appendices A to G will be referred through the course of the thesis and, in most cases, contain detailed explanations of issues and results encountered during the research.

2 System Evaluation:

2.1 Introduction:

The following Chapter presents the results of a comprehensive literature review of current control system performance assessment techniques. Subsequent to a brief introduction outlining the key aims of a good evaluation technique, a number of assessment metrics are categorised and defined. The key objective of this Chapter is to familiarise the reader with a number of possible means of system evaluation, identifying the benefits associated with each technique while also providing a means of comparing the identified categories to help evaluate their suitability in performance assessment. Section 2.2 presents a comprehensive overview of a number of assessment techniques and also provides references to papers that have evaluated the usefulness of each of the identified techniques. Also mentioned in this Chapter (section 2.3) is the importance of identifying a control loop's objective and incorporating this objective in the evaluation process. An investigation into the direction of current industrial research, with respect to performance evaluation procedures, is discussed in section 2.4. In section 2.5 an overview of currently available assessment software packages is presented. At the end of this Chapter the reader should not only be familiar with the methods discussed but should also have enough information to further investigate any of the evaluation techniques identified should they see fit.

2.2 Performance Assessment Techniques:

The goal of monitoring should be to provide information that can be used to assess the current status of the existing controller and assist control engineers in deciding whether redesign is necessary [7]. If a controller's performance is determined to be inadequate, it is important to ascertain whether an acceptable level of performance can be achieved using the existing control structure [8]. Following a preliminary literature review, it was decided to divide the assessment techniques investigated into the following categories:

1. Time domain assessment techniques,
2. Frequency domain assessment techniques,
3. Minimum variance control (MVC) as a benchmark,

4. Statistical analysis techniques
5. Problem specific assessment techniques.

The subsequent sections will present a comprehensive and in-depth analysis of each of the categories identified above.

2.2.1 Time Domain Assessment Techniques:

The dynamic response characteristics of a closed loop system may be accurately assessed using a number of useful time domain measures. These measures include rise time, settling time and integral error measures (see Figure 2-1).

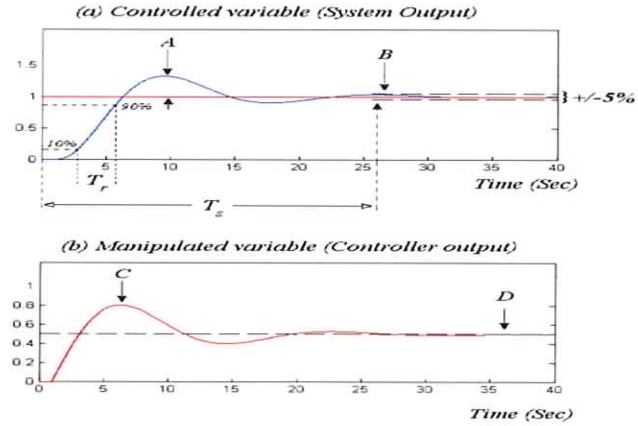

Figure 2-1 - Typical transient response of a feedback control system to a step set point change

The rise time (T_r) has a number of definitions, depending on which author is consulted. It has been defined by one author as the time taken for the step response of a closed loop system to change from 10% to 90% of its final steady state value [9]. Rise time is less frequently measured between the 5% and 95% points or the 1% and 99% points [10]. However, according to the authors of [11], the rise time may be defined as the time from when the step change in the set point is applied until the time when the controlled variable (system output) first reaches the new set point value. In some cases the rise time is even defined as the time taken for the system closed loop step response to reach 63% of its final steady state value. For the purpose of this thesis, the rise time will be defined as the time taken for the closed loop step response of a system to change from 10% to 90% of its final steady state value, as this appears to be the most widely accepted definition. A short rise time is usually desired. The settling time (T_s) is defined as the time the system takes to attain a 'nearly constant' value, usually ± 5 or ± 2 percent of its final value [11]. Again, a short settling time is usually desired.

An alternative set of dynamic response characteristics are the 'integral error' measures. The integral error measures indicate the cumulative deviation of the controlled variable from its set point during the transient response. The Integrated Error (IE) criterion is simply the integral of the deviation of the controlled variable with respect to time. However, this measure is not normally used because positive and negative errors cancel in the integral, resulting in the possibility of large positive and negative errors giving a small IE [11].The Integral of Absolute Error (IAE) criterion is determined from the sum of the area of the controlled variable above and below the set point. It is accepted that IE can be related to economic performance, thus minimising IAE will achieve the same goal as minimising IE while ensuring stable return to the set point [11]. The Integral of Squared Error (ISE) criterion is appropriate when large deviations cause greater performance degradation than small deviations. The Integral of Time multiplied by Absolute Error (ITAE) criterion penalises deviations that endure for a long period of time. The formulae for calculating the integral error measures are given below:

$$IAE = \int_0^\infty |SP(t) - CV(t)|.dt \qquad (2.2.1)$$

$$ISE = \int_0^\infty [SP(t) - CV(t)]^2.dt \qquad (2.2.2)$$

$$ITAE = \int_0^\infty t.|SP(t) - CV(t)|.dt \qquad (2.2.3)$$

$$IE = \int_0^\infty [SP(t) - CV(t)].dt \qquad (2.2.4)$$

For the above equations, SP is the set point and CV is the controlled variable (closed loop system output).

The following paragraphs present a number of papers in which these time domain assessment measures, or slight variations of these measures, are evaluated. Jamasa-Jounela *et al.* present a set of performance indices appropriate to process monitoring and assessment [5]. These indices include IAE, ITAE, rise time and settling time. Swanda and Seborg have developed a methodology to assess the performance of PI controllers from closed loop response data for a set point step change [12]. This method is based on two new dimensionless performance indices, the dimensionless settling time and the dimensionless IAE. This methodology is also applicable to PID controllers. Horch and Stattin extend this method to analyse the settling time of a set point step response, normalised by the apparent process time delay [13]. Ruel discusses a number of metrics used to assess loop performance [14]. These include IAE, set point crossing, and average error. Huang and Jeng assess a simple feedback system by analysing IAE and rise time observed from the response of the system to a step set point change [15]. Optimal IAE's and associated rise times are computed. Comparing its current IAE to the optimal IAE allows an assessment of the performance of the system.

There are a variety of alternative time domain measures that may be used to assess a closed loop system's performance. These include offset, decay ratio, manipulated variable overshoot, maximum deviation of the controlled variable, and magnitude of the controlled variable in response to a sine disturbance. Offset can be defined as the difference between the final steady state value of the set point and that of the controlled variable. In most cases, a zero steady state offset is desired [11]. The decay ratio (B/A), see Figure 2-1, can be defined as the ratio of neighbouring peaks in an underdamped controlled variable response. Usually, periodic behaviour with large amplitude variation is avoided; therefore, a small decay ratio is usually desired, and an overdamped response is sometimes desired [11]. The manipulated variable overshoot (C/D), see Figure 2-1, is of concern because the manipulated variable is also a process variable that influences performance. Large variations in the manipulated variable can, in some situations, cause long-term degradation in equipment performance. The overshoot is the maximum amount that the manipulated variable exceeds its final steady state value and is usually expressed as a percentage. Some overshoot is acceptable in some cases, depending on the loop objective [11]. The maximum deviation of the controlled variable from the set point is an important measure of the process degradation experienced due to system disturbances. Usually a small value is desirable so that the process variable remains close to its set point [11]. In many cases the disturbance is composed predominantly of one or a few sine waves [16]. Therefore, the behaviour of the control system in response to sinusoidal inputs is of great practical importance. Through this analysis the relationship between the frequency of the disturbances and the control performance may be deduced. Control performance is assessed by measuring the amplitude of the output sine wave; the metric is often expressed as the ratio of the output to input sine wave amplitudes [11].

The following papers discuss some further time domain evaluation techniques. Stanfelji *et al.* present a method for monitoring and diagnosing the performance of single loop-control systems based primarily on normal operating data [17]. This method involves analysing the autocorrelation and cross correlations of a time series of control loop variables. Hagglund describes a procedure for the automatic detection of sluggish control loops obtained from conservatively tuned controllers [18]. The 'idle index' was developed, by Hagglund, as a measure of the sluggishness of the control loop.

2.2.2 Frequency Domain Assessment Techniques:

This section will review some of the more common frequency domain assessment metrics.

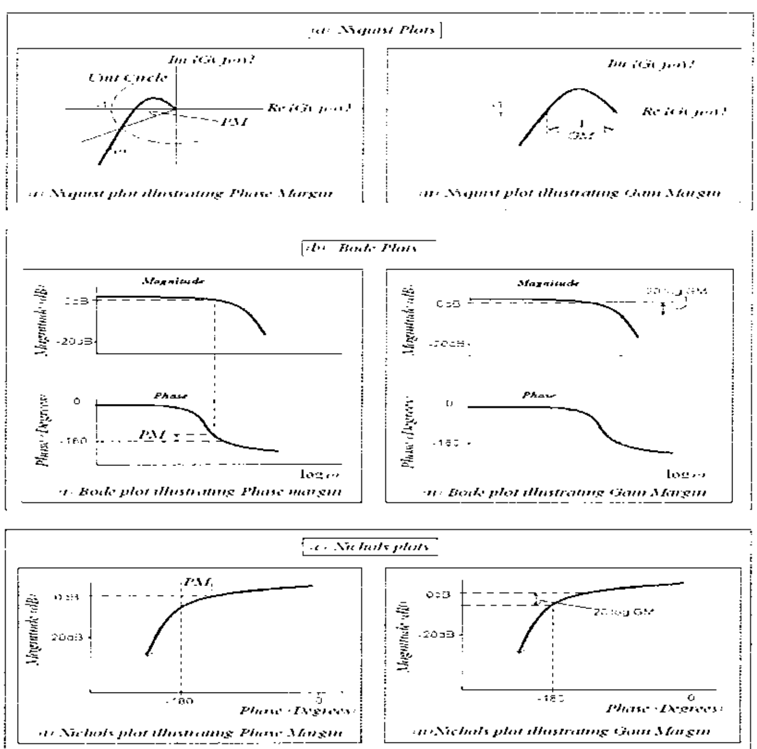

Figure 2-2 - Typical Nyquist, Bode and Nichols plots illustrating Gain and Phase margins.

Three different types of plots are commonly used to graphically illustrate the frequency response of a controlled system, see Figure 2-2. These three plots are the Nyquist, Bode and Nichols plots. Nyquist plots, also called polar plots, may be obtained by either plotting the real versus the imaginary part of the open loop frequency domain transfer function, G(jω) (using rectangular coordinates), or by plotting the magnitude versus phase angle of G(jω) (using polar coordinates). Bode plots require two curves to be plotted; these plots show how the magnitude ratio and phase angle of G(jω) vary with frequency. The Nichols plot is a single curve in a coordinate system with phase angle as the abscissa and magnitude ratio as the ordinate. Frequency is a parameter along the curve [19].

Phase margin and gain margin are two commonly used frequency domain assessment measures. The following definitions will be given with respect to the Nyquist plots of Figure 2-2a. Phase margin (PM) is defined as the angle between the negative real axis and a radial line drawn from the origin to the point where the open loop frequency domain transfer function polar plot intersects the unit circle (see Figure 2-2a(i)). The bigger the phase margin, the more stable the closed loop system tends to be. Phase margins of 45^{o} are often considered appropriate [19]. The gain margin (GM) is defined as the reciprocal of the intersection of the open loop frequency domain transfer function polar plot on the negative real axis (Figure 2-2a(ii)). Similarly, the larger the gain margin, the more stable the system. Typically gain margin values of about 2 (i.e. 6dBs) are recommended [19]. Both PM and GM are a direct measure of how much the phase and gain of the open loop system may vary before the closed loop system becomes unstable. Astrom and Hagglund discuss a simple method for estimating the critical gain of a controlled system, from which the gain margin may be deduced [20].

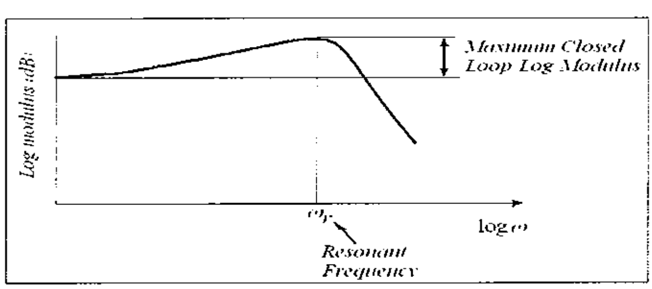

Figure 2-3 - Plot illustrating a typical maximum closed loop log modulus, L_{cmax}

The maximum closed loop log modulus (or magnitude), L_{cmax}, is another quantity used to assess performance in the frequency domain, see Figure 2-3. While the phase and gain margin specifications can sometimes give poor results when the shape of the frequency response curve is unusual, the maximum closed loop log modulus does not have this problem. It is a direct measure of the closeness of the open loop frequency domain transfer function to the (-1,0) point at all frequencies [19]. Chiou and Yu evaluate a monitoring procedure that identifies the maximum closed loop log modulus in two to three relay feedback experiments [21]. Ju and Chiu assess a monitoring procedure, incorporating the FFT (Fast Fourier Transform) technique, to identify L_{cmax} on line (i.e. when the control system is up and running) [7]. This method addresses some of the problems associated with the technique discussed by Chiou and Yu [21] i.e. too many relay tests are required, the frequency search range is confined to the third quadrant, and the identified value of L_{cmax} cannot be used on-line to redesign the controller. Belanger and Luyben propose a new test to locate the peak regulator log modulus [22]. The test involves the insertion of a relay between the controlled variable and a specified load disturbance model, with the feedback controller on automatic. This causes the plant to exhibit a sustained oscillation at the frequency where the L_{cmax} curve exhibits a peak.

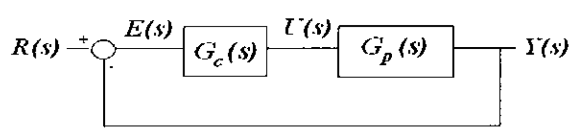

Figure 2-4 - Simple Feedback system

The sensitivity function is another important quantity that may be used to judge controller performance, see Figure 2-5. It can be used to describe the effects disturbances of different frequency have on the controlled variable. For a controller to achieve good disturbance rejection the sensitivity function should be made as small as possible.

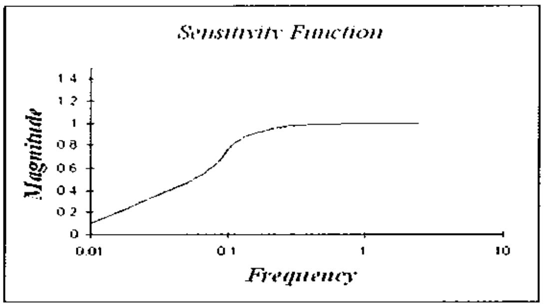

Figure 2-5 - Sensitivity function of a closed loop system containing a first order system

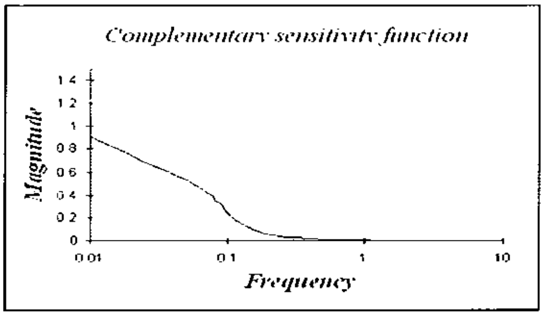

Figure 2-6 - Complementary Sensitivity function of a closed loop system containing a first order system

The sensitivity function of the control system illustrated in Figure 2-4 may be determined using the following formula:

$$Sensitivity\ function = \frac{1}{\left[1 + G_c(s)G_p(s)\right]} \tag{2.2.5}$$

where $G_c(s)$ is the controller transfer function and $G_p(s)$ is the process transfer function. The complementary sensitivity function is, as its name suggests, is simply defined as (1 - sensitivity function). This function simply relates the controlled variable to the desired output (i.e. the set point) and, for the set-up as illustrated in Figure 2-4, may be calculated using the following formula:

$$\begin{matrix} Complementary \\ sensitivity\ function \end{matrix} = \frac{G_c(s)G_p(s)}{\left[1 + G_c(s)G_p(s)\right]} = \frac{Y(s)}{R(s)} \tag{2.2.6}$$

The ideal objective (perfect tracking) is to have the controlled variable equal to the set point, regardless of measurement noise and disturbances affecting the system. Of course, the ideal objective is unattainable [23]. For simplicity, we often interpret the objective in terms of magnitudes only. That is, we would like to have the magnitude of the controlled variable equal to the magnitude of the set point for all frequency ω.

The typical situation is that the magnitude of the set point, and the magnitude of disturbances affecting the system are small for large ω, and the magnitude of the measurement noise is small for small ω. Thus for good tracking performance, we require that [23]:

- The magnitude of the sensitivity function be small for small ω, so that the effect of the disturbance input is attenuated,
- The magnitude of the complementary function be small for large ω, so that the effect of the sensor noise is attenuated,
- The magnitude of the complementary function is unity (0 db) for small ω so that the (low-frequency) characteristics of the reference input are unaffected.

For the case of the control loop illustrated in Figure 2-4, the complementary sensitivity function is the same as the closed loop frequency response of the system. Therefore, it is desirable that the closed loop frequency response is 0dBs at low frequencies, drops off at higher frequencies and the peak in the closed loop frequency response should be kept as small as possible.

The capacity based method for quantifying controllability is a method used to quantitatively incorporate the economics of control into conventional steady-state design methods [19]. Elliot and Luyben outline a generic methodology called the capacity based economic approach that can be used to compare or screen preliminary plant designs by quantifying both steady-state economics and dynamic controllability [24]. Elliot *et al.* demonstrate that the capacity based economic approach can be successfully applied to a large industrial scale process [25]. Elliot and Luyben analyse the effectiveness of the capacity based economic approach when controlling a complex recycle system consisting of a reactor and two distillation columns [26].

Kendra and Cinar discuss a method used to estimate the closed loop transfer function of a system by exciting the reference input with a zero mean, pseudo random binary sequence and observing the process output and error response [27]. Performance assessment is based on the comparison between the observed frequency response characteristics and the design specifications.

Another useful frequency domain measure is the closed loop bandwidth of a control system. The closed loop bandwidth of a system can be defined as the frequency at which the closed loop frequency response has declined by 3dB from its low frequency value [28]. The bandwidth of a system can be related to the transient response of said system. For example, as the bandwidth of the system increases, the rise time of the step response of the system will decrease.

2.2.3 Minimum Variance Benchmarks:

Minimum variance control (MVC) is considered the optimal feedback control provided that the process can be described by a linear transfer function with additive disturbance [29],[30]. Spring states that minimum variance is a better benchmark than zero variance for evaluating controller performance [31]. Control systems cannot reduce the variance in product quality (due to controlled variable variance) below the variance inherent in the process (variance due the physical structure of the plant). On the basis of minimum variance, an investment in controller maintenance can be evaluated realistically.

This minimum variance benchmark control may or may not be achievable in practice depending on process invertibilty and other process physical constraints [6]. Also, it is worth noting that this technique requires knowledge of the process time delay, which may not always be available. However, as a benchmark, it provides useful information such as how much 'potential' there is to improve controller performance. Thornhill *et al.* make the point that minimum variance control may require excessively vigorous action of the manipulated variable and, as a result, can lead to maintenance problems for the actuators [32].

The following papers present an overview of the MVC method. Harris discusses how an estimate of the best possible control can be obtained by fitting a univariate time series to process data collected under routine control [30]. Harris *et al.* discuss some of the concepts associated with assessing the effectiveness of a control system [33]. Also discussed in this paper, is how these concepts were initially developed using a performance benchmark of minimum variance control for single input single output (SISO) systems. Thornhill *et al.* examine some of the factors that influence the minimum variance performance measure of a SISO control loop [32]. The authors show that, for an arbitrary controller, the calculated minimum variance benchmark is different for servo and regulator operation. Grimble discusses the use of the generalised minimum variance control law for control loop performance assessment and benchmarking [34]. Huang and Shah discuss, in detail, some of the theory behind the MVC method [35].

Based on MVC theory, a performance index (the Harris index) was first introduced by Harris [30]. This index compares the actual variance in the controlled variable of the closed loop system under test, to the controlled variable of a minimum variance controller. Desborough and Harris present a normalised performance index used to characterise the performance of control systems [29], [36]. This index provides a measure of the proximity of the current control to minimum variance control. Time domain and spectral interpretations of the index are discussed and a fast, simple on-line method for estimating the index is given. Bezergianni and Georgakis introduce a modified version of the Harris index in which the closed loop performance is compared both with that obtained with the best theoretical control action (minimum variance control), and no control action [37]. Vishnubhotla *et al.* discuss a method of performance assessment based on the Harris index [6]. The resulting index gives an indication of the level of performance of the control loop, and an indication of the action required to improve performance. Spring discusses a performance index based on minimum variance control [31]. Ko and Edgar outline a scheme for the estimation of achievable PI control performance, measured by controlled variable variance, in linear processes with dead time when stochastic load disturbances are affecting the process [38].

A number of papers have been written discussing modifications to the MVC benchmark. Eriksson and Isaksson discuss how this technique provides an inadequate measure of performance if the aim is not control of statistically random disturbances [39]. Some modifications to the Harris index are suggested. Horch and Isaksson discuss a modification to the index introduced by Harris [40]. The modified index and the original index are then evaluated and compared using data from industrial processes. Isaksson discusses the MVC benchmarking technique and suggests a set of alternative indices [41]. Huang discusses some of the aspects associated with the minimum variance control law for linear time variant processes [42], [43]. Alternative benchmarks that are more suitable for time variant processes are suggested. Venkatesan introduces a minimum variance feedback control algorithm (MVFCA) that can be used to calculate a series of adjustments required at the input that minimises the variance of the output variable [44]. Kucera presents a tutorial paper emphasising the contribution of V. Peterka to the steady state minimum variance control problem [45]. Qin presents an overview of the current status of control performance monitoring using minimum variance principles [46].

Minimum variance control (MVC) as a benchmark (as discussed in [29]) or variants of it, is used in virtually all industrial controller assessment packages due to its theoretical and practical advantages [47]. Hugo lists some of these software packages as follows: Performance assessment tool-kit [48]; loop scout [49]; Process Doc [50]; and Aspen Watch [51]. Software packages such as Probe [52] and Plant Triage [53] also offer a number of useful routines and algorithms related to the MVC benchmark [47]. The main advantage that MVC as a benchmark has over the other four categories of assessment techniques is that it not only gives an indication as to the current level of performance of the controlled system under investigation, but it can also determine whether or not current performance can be improved by retuning the controller. Vishnubhotla *et al.* highlight this point by stating that 'as a benchmark (MVC) ... provides useful information such as how well the current controller was tuned compared to the minimum variance controller and how much 'potential' there is to improve controller performance' [6]. For example, an index (ratio of minimum achievable controlled variable variance to actual controlled variable variance) value of 1 indicates that current performance cannot be improved by retuning the existing controller. However, an index value below 1 indicates retuning the controller will have an impact on improving system performance.

2.2.4 Statistical Analysis Techniques:

The goal of statistical process monitoring (SPM) is to detect the existence, magnitude and time of occurrence of changes that cause a process to deviate from its desired operation [1]. Cinar and Undey discuss a number of useful techniques for the monitoring of process variables in [1]. These methods include Shewhart control charts, moving average control charts, cumulative sum charts and partial least squares methods.

The likelihood method is a useful technique for assessing performance [1]. This method may be used to determine if the error response characteristics are acceptable based on specified dynamic performance bounds. Dynamic response characteristics such as overshoot or settling time can be extracted from the pulse response of a fitted time series model of the controlled variable error (difference between the controlled variable and the set point). The pulse response of the estimated controlled variable error can be compared to the pulse response of the desired specification to determine if the output error characteristics are acceptable. Tyler and Morari propose a framework in which acceptable performance is expressed by constraints on the closed loop transfer function impulse response coefficients [54]. Using likelihood methods, a hypothesis test is outlined to determine if control deterioration has occurred. Zhang and Ho propose the use of the likelihood ratio method as a means of sensitivity analysis of stochastic system performance [55].

Li *et al.* developed a monitor to automatically detect poor control performance [56]. The monitor provides a measure (Relative Performance Index – RPI) of control loop performance relative to an acceptable reference model. Zhong demonstrates how to improve the effectiveness of equipment monitoring and process induced defect control through properly selecting, validating and using the hypothetical distribution models [57]. Mosca and Agnoloni study the early detection problem of stability losses or close-to-instability conditions in feedback control systems, where the plant dynamics are uncertain and possibly time-varying [58].

2.2.5 Problem Specific Analysis:

This section is devoted to a number of more 'problem specific' assessment techniques, as opposed to the more general methods discussed in the previous sections. The focus of this section is on methods to both detect and diagnose oscillations in control loops. The techniques discussed here may well be considered special cases of the methods discussed in previous sections.

The first step in dealing with an under performing control loop with suspected oscillation disturbances is the detection stage. Hagglund presents a closed loop performance monitor (CLPM) to detect oscillations in the control loop [59]. The procedure presented is automatic in the sense that no additional parameters, other than the normal controller parameters, have to be specified. Huang *et al.* discuss a method of determining the presence of oscillations in selected frequency ranges, based on the regularity of the zero crossings of filtered auto-covariance functions [60]. Chang *et al.* present a system-wide dynamic performance monitoring system (DPMS), which includes special features such as oscillation detection [61]. Stenman *et al.* propose a model-based method for detecting static friction (stiction) in control valves [62]. In contrast to existing methods, only limited process knowledge is needed and it is not required that the loop has oscillating behaviour. Wallen proposes an integrated system for valve diagnostics and automatic PID tuning [63]. The purpose of the method is to detect non-linearities such as friction and hysteresis since these may drastically decrease the control performance.

Once an oscillation has been detected, the next step is to determine its cause. Thornhill and Hagglund present a set of 'operational signatures' that indicate the cause of an oscillation [64]. This method involves the offline analysis of ensembles of data from control loops. Horch proposes a simple method for the diagnosis of oscillations in process control loops based on the cross correlation between manipulated variable and controlled variable [65]. This method is shown to correctly identify the two most important reasons for oscillations in control loops in the process industry, namely, external oscillating disturbances and stiction in control valves. Taha *et al.* present an on line automatic procedure for the diagnosis of oscillations in control loops [66]. This method works without disturbing normal plant operation.

2.3 Objective Based Performance Assessment:

It is important to mention that the process of system performance evaluation should be based not only on the metrics discussed in previous sections, but should also be dependant on the control loop objective. Different loop objectives will require different control constraints. For example, flow loops are typically dominated by low order processes and tight control is the assumed objective. On the other hand, pressure loops are often moderately fast processes and moderately tight control is the assumed objective while temperature processes range from simple low order processes to high order processes with complex dynamics and interactions. Level loops, on the other hand, can have either a surge attenuation or a regulatory objective. Sluggish control is the assumed objective in well performing level loops in order to attenuate variability in the feed stream. Simply put, the threshold value defining 'excellent' performance for one loop type may be completely different to the threshold value defining 'excellent' performance for a different loop type. Therefore, it is important, when evaluating performance, to thoroughly understand the current loop objective and not overemphasise the minimisation of metric values [67].

2.4 Direction of Evaluation Concepts:

In the current industrial climate, control performance assessment and monitoring applications have become mainstream, particularly in the refining, petrochemical, pulp and paper and mining industries [68]. These applications have been deemed necessary because of the diversity of each control loop; there is no single optimised controller design that may be used to ensure optimal system performance. Also, most large-scale industrial plants will contain a disproportionately large number of control loops compared to maintenance personnel thus making the practice of monitoring and diagnosis a particularly challenging prospect.

The basic idea of process control involves diverting process variability from variables where large variations may be costly to variables or tanks where this variability can be more cost effectively absorbed [68]. Controller performance assessment involves three key stages:

- Collect suitable data for analysis
- Benchmark current performance against a suitable standard
- Present advice and reports that control engineers can use to improve control

At present the first two of these stages are considered relatively straightforward and amply catered for. The current focus tends to be towards the third point, developing a means of translating the information gathered into 'useful' information that relates to business objectives. The need to prioritise economically critical loops has been identified as an extremely significant step in achieving improved overall system performance. This practice of prioritising loops should depend on the extent of a loops performance degradation and its 'criticality' in the overall plant operation. A critical loop may be defined as a loop whose output has the potential to cause uncontrollable disturbances on other loops [68].

One of the main problems associated with providing meaningful advice is that in most industrial scale process plants there are a large number of processes running with, in some cases, hundreds of loops. Therefore, the key issue becomes resolving a means of providing an overview of the overall plant operation while still presenting important details associated with every control loop within each unit of the overall process. Professor Sirish Shah provides a solution to this problem in the form of a procedure known as tree mapping [69]. Tree mapping graphically illustrates the overall plant operation performances while still providing the freedom to examine individual under-performing loops (see Figure 2-7).

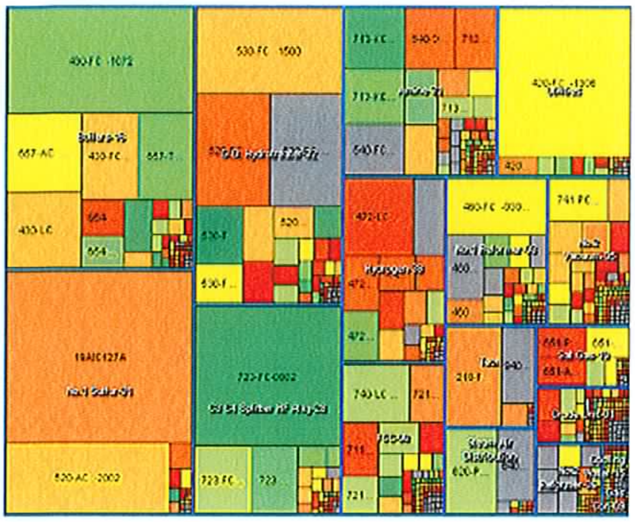

Figure 2-7 - Tree map [69]

The colours in the tree map vary from green to red. Green areas on the map represent 'well behaved loops', i.e. loops that are performing to specification. Red areas on the other hand highlight loops in need of attention. It is also possible to click on any area of the map in order to 'drill down' and obtain more information regarding that particular loop.

Honeywell Hi-spec solutions, manufacturers of the Loop scout assessment software package, have adopted this tree mapping approach in their product [70]. Based on their own industrial survey, Honeywell have decided to prioritise their research resources on valve diagnosis. Their other research directions include 'behaviour clustering', which is a means of identifying groups of measurements with similar trends, identified through analysis of their power spectra.

Work is also currently being untaken in Oxford University in developing an active closed loop analysis and diagnosis (CLAD) monitor. This monitoring system involves injecting a test signal into a loop to obtain important information. The signal used in this testing scheme has a low signal-to-noise ratio in order to keep system disruption to a minimum [71].

2.5 Currently Available Software Packages:

There are a number of reliable assessment software packages currently available on the market. These include 'Loop performance manager' from ABB, 'Loop Scout' from Honeywell, 'Plant Triage' by Expertune, 'PROBEwatch' by ISC, 'Process Doctor' from Matrikon, 'Control Wizard' by PAS, 'Control Monitor' from Control Arts Inc and 'PCT Loop Optimiser' from ProControl Technology [68].

Of the assessment packages listed, a summary investigation by the author into three of these tools, namely 'Plant Triage', 'Loop Scout' and 'Control Monitor', provided some important information into the types of benchmarks or assessment measures considered significant by these companies. The Harris index is standard in all available software packages reviewed. Process modelling and subsequent estimation of achievable time domain characteristics, such as closed loop rise time and settling time, based on this model and a suggested 'optimised controller' were also features highlighted in each of the packages. Statistical analysis and oscillation detection and diagnosis also featured in Honeywells' Loop Scout and Expertunes' Plant Triage software packages. Thus, the metrics or evaluation measures presented in this Chapter provide a reasonably comprehensive overview of the majority of accepted evaluation measures currently being used.

2.6 Summary and Conclusion:

The Chapter provided a review of a variety of common, effective evaluation techniques. The techniques discussed were divided into five categories namely, time domain, frequency domain, MVC benchmarking, statistical analysis and problem specific assessment measures. In the time domain category, measures such as closed loop system rise and settling time were discussed. These metrics provide a direct measure of a system's response time to an applied input. Also discussed in this category were the integral error measures, which could provide a direct link between a system's control performance and its economic performance. Also, measures such as offset, decay ratio, overshoot and maximum deviation were examined.

The second category of evaluation techniques examined was the frequency domain measures. These included gain margin, phase margin and the maximum closed loop log modulus. Using these techniques it was possible to discern information regarding a control system's robustness. It is therefore possible to estimate how much the characteristics of a process can vary before the closed loop system becomes unstable. Also discussed were the sensitivity and complementary sensitivity functions. These functions can be use to evaluate a system's disturbance rejection capabilities.

The next assessment technique discussed was the MVC benchmarking method. This metric provided a means of assessing how much a controlled variables variance could be reduced, thus reducing off-specification product. This benchmark is used in virtually all-current assessment software packages. Statistical analysis techniques were evaluated in the subsequent section. These methods can be used to detect possibly indiscernible patterns in a processes' output that may be attributable to an under performing control system. The final category discussed was the problem specific evaluation schemes. These schemes involved both the detection and diagnosis of oscillations in a control loop.

Finally, a section was devoted to highlighting the importance of objective oriented performance assessment. This section discussed the importance of incorporating the control loop objective into the performance evaluation procedure. When evaluating a system's performance, it is paramount that the loop objective is fully understood and integrated into the assessment process.

After investigating the currently available assessment software packages and evaluating the standard practices in performance monitoring it was decided to develop an evaluation tool along the lines of the tools discussed in section 2.5. The assessment strategy to be developed would consist of two main stages, namely an identification stage followed by a subsequent evaluation step. The identification stage would involve the application of a test signal and the subsequent manipulation of the system's response in an effort to characterise its dynamic behaviour. The evaluation stage would involve processing of data and the application of a number of the metrics; it was decided to direct the main focus on the time domain and frequency assessment measures, as these quantities are the most widely accepted and intuitive of all the metrics discussed in this Chapter.

The time domain measures employed were rise time, settling time and percentage overshoot. From close examination of the dates of the articles researched with respect to the time domain characteristics investigated, it can be seen that time domain assessment metrics do not appear to be an active topic of research. However, metrics such as rise time, settling time and percentage overshoot have become standard assessment measures and are widely accepted as good indicators of a system's time domain performance. Therefore, it was felt that it was necessary to include these assessment measures in the evaluation tool to be developed.

The frequency domain metrics included gain and phase margin, maximum closed loop log modulus along with plots of the sensitivity function. Traditionally, these frequency domain assessment measures tend to be more widely used in an academic environment than in an industrial setting. Therefore, it was decided that it would be beneficial to evaluate the usefulness of incorporating these frequency domain measures in the evaluation tool to be developed.

A simple graphical user interface (GUI) was developed through Matlab 6.5 in order to facilitate a user-friendly approach to presenting both the identification stage requirements and the subsequent illustration of the evaluation measures. The software tool to be developed would differ from currently available assessment tools in two key areas. Firstly, the assessment tool would focus on the application of what could be considered traditionally academic frequency domain evaluation measures (i.e. phase and gain margins) in order to evaluate system performance. Secondly, the tool to be developed would incorporate a tuning rule database to benchmark the current system's performance against possible alternatives. While it has been identified that the majority of currently available assessment software packages incorporate MVC as a benchmark, it was decided that a significant amount of research has already been conducted in this area and no significant benefits would be obtained in investigating this area further. Therefore, it was decided to focus primarily on the time and frequency domain assessment measures and avoid the inclusion of MVC as a benchmark.

3 Development of Assessment Strategy:

3.1 Introduction:

This Chapter is intended to develop further the results of the literature review of Chapter 2 and develop an assessment and evaluation tool that can be applied to real time systems. The tool developed is Matlab based and incorporates Humusoft's real time library to facilitate real time testing. Section 3.2 presents and explains the evaluation strategy developed. Section 3.3 explains the review and selection process of the identification techniques incorporated in the assessment tool, along with the steps involved in the development of a PI(D) database. This section also contains information regarding the approach taken in selecting an appropriate process modelling technique as well as information concerning the implementation of the overall evaluation strategy. Finally, section 3.4 provides an explanation of the graphical user interface developed in order to facilitate a user-friendly approach to implementing the evaluation strategy.

3.2 Explanation of Evaluation Strategy:

Based on the literature review and subsequent findings of Chapter 2, the following evaluation strategy was developed. The strategy involves two main stages, namely an identification stage followed by an evaluation stage.

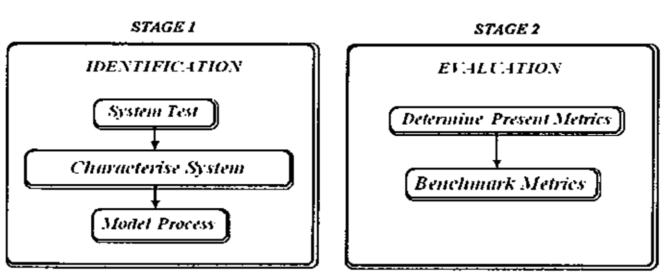

Figure 3-1 - Evaluation Strategy

The figure to the left graphically illustrates the strategy developed. Both the identification and evaluation stages can be seen to consist of a number of steps.

The identification stage consists of three steps. First, a system test is carried out. It is desirable that this testing phase does not cause too much added disturbance to the normal operation of the plant while still providing enough information to evaluate the appropriate metrics needed to characterise the performance of the system. Step two of the identification process involves the off-line processing of the system's response signals. As discussed in Chapter 2, it was decided to concentrate on a number of frequency domain metrics therefore this step involves generating, as accurately as possible, an estimate of the frequency response of the system under test. The final phase of the identification procedure involves estimating an appropriate process model to mathematically characterise the behaviour of the system under test.

The evaluation stage is concerned with generating compelling information from the system test data and presenting it in a manner that will characterise the behaviour of the system in a meaningful way. Firstly the metrics considered significant must be evaluated and presented in an obvious and easily understandable form. Next, it was decided that it would be desirable to generate an array of benchmarks in order to facilitate the comparison of the current closed loop system's metrics with possible alternatives. This was done by developing a *PI(D)* tuning rule database containing a number of tuning rules and their associated formulae. It was therefore possible to compare the current system's performance to a number of possible alternatives to determine whether or not improved control is achievable with an alternative set of controller parameters.

Each step of the two stages discussed has no inherent technique associated with it. Each could be considered 'plug and play' to some extent. In the following sections, the techniques used in the implementation of each of these steps will be presented and justified.

3.3 Development of Assessment Tool:

3.3.1 Review and Selection of Identification Technique:

The dynamics of a system may be determined, and thereafter modelled, by examining the system's response to an input test signal. Over the years, a number of deterministic test signals have been identified as being particularly proficient with respect to yielding valuable system information. These test signals include pulses, steps, ramps and more recently repetitive binary pseudo-noise signals. This section will review a number of methods, based on these test signals, with the intention of identifying an efficient means of evaluating a control system, with the main objective being the extraction of as much information regarding the system's behaviour as possible without driving the system into a region of unstable operation.

One of the first model identification directives was developed by Ziegler and Nichols and involved the application of a step input to the process under test [72]. Step test signals have the advantage of containing all frequencies in one easily reproduced event and therefore have been the most commonly applied test over the years [73]. However, though straightforward, this practice tends to yield process information mostly around the steady state because step changes hardly excite the processes high frequency dynamics. In addition, step changes tend to drive the process to an altered steady state causing problems for the on-going production. Generally, several step changes of different amplitudes are made in both positive and negative directions and the results of these tests are averaged [74]. Quite often, process noise masks the step response and to get round this, the size of the step may have to be increased to a level that, at best, takes the system into a non-linear region and at worst, is too large for safe plant operation [75]. Appendix B.1 contains an example of an implementation of a step testing procedure.

A very common method used in measuring a processes frequency response up to the end of the 1960's was the combination of a slowly swept sine wave with a tracking filter [73]. Because sinusoidal disturbances contain a single frequency, the information produced from a single test – phase and gain – is not very useful by itself. Several different frequencies must be tried over a spectrum where the process phase lag varies from 0 to beyond 180 degrees. These tests tend to be very time consuming, with such a low return on investment that they are rarely used anymore. Appendix B.4 illustrates a typical implementation of a sine wave testing scheme.

It has been well documented that there is a one-to-one relationship between a system's impulse response and its frequency response [76]. If the impulse response can be determined, it is possible to transform this information into the frequency response through the use of the Fast Fourier Transform (FFT). A weakness of pulse testing, however, is the rather low energy content of the test signal. The pulse stimulus is very brief while the response data collection time must be many times longer in order to capture the low frequency information. This allows an opportunity for noise to intrude into the measurement. In traditional impulse response testing schemes, employing repetitive pulse stimuli and averaging the resulting responses is one technique used to deal with the problem of noise [76]. The stimuli must be spaced enough in time so that low frequency information can be obtained and device pulse response can decay sufficiently.

Relay based system identification is becoming more commonplace particularly in autotuning control schemes. There are a number of interesting features associated with relay based system testing. Two main advantages with this form of testing are 1) it is a closed loop test, which keeps the controlled variable under control and is usually preferred to open loop tests, and 2) a linear stable process with relay feedback is likely to automatically reach a sustained stationary oscillation known as a 'limit cycle'. Using the amplitude and period of this oscillation, approximate information about the process critical point (point on the frequency response curve when input/output phase ratio is at -180 degrees) can easily be determined. By employing various techniques it is possible to identify additional process information at frequencies other than the process critical point.

Pseudo random binary sequences (PRBS's), also known as pseudo noise (PN), linear feedback shift register (LFSR) sequences or maximal length binary sequences (m-sequences), are widely used in the field of system identification [77]. A pseudo random binary sequence, as its name suggests, is a semi-random binary sequence in the sense that it appears random within the sequence length, but the entire sequence repeats indefinitely. A PRBS sequence is an ideal test signal, as it simulates the random characteristics of a digital signal and can be easily generated. Pseudorandom binary sequences (PRBS's) are very effective as persistent excitation stimuli in dynamic testing [74]. Because the PRBS testing method is based on cross-correlation techniques, it is highly immune to extraneous noise of all kinds and as a result, its amplitude can be controlled to within safe limits without the risk of driving the plant outside the bounds of linear operation. The PRBS signal can also be easily coupled with the command input signal (set point) for normal plant operation. PRBS signal energy can be controlled over a range of frequencies with low amplitude by appropriate choice of the PRBS test signal parameters.

To summarise, in comparison to traditional testing methods such as step testing, sinusoidal testing and impulse testing, testing using PRBS signals appears to be, not only a viable option, but also in some respects an attractive alternative. Also, due to the relatively simple nature of the relay based approach and its general acceptance in industry with respect to its use in the area of autotuning, it was felt that relay based system identification procedures merited further investigation. By developing two methods of system identification further, a basis for comparison between the two techniques was provided, which proved useful in highlighting the benefits and drawbacks associated with each technique.

3.3.1.1 PRBS Approach:

Pseudo random binary signals have a number of extremely useful properties with respect to their use as a test signal. Appendix A.1 to A.4 provides a detailed discussion of these properties. The following section illustrates how PRBS signals may be used to identify the impulse response and, by application of the FFT, the frequency response of a system under test.

Correlation may be defined as a measure of similarity between two sequences. If the two sequences compared are different, 'crosscorrelation' is the term used and if they are the same, 'autocorrelation' is the term used. Mathematically, the autocorrelation of a sequence x(k) of length L may expressed as follows [78]:

$$R_{xx}(m) = \frac{\sum_{k=1}^{L-m}\left(x(k)-\bar{x}\right)\left(x(k+m)-\bar{x}\right)}{\sum_{k=1}^{L}\left(x(k)-\bar{x}\right)^2} \qquad (3.2.1),$$

$$m = 1,2,3...,L$$

where $\bar{x}$ is the mean of the sequence x(k). For the case of a PRBS sequence, its 'cyclic' autocorrelation function has the following values:

$$R_{xx}(k\Delta t) = \begin{cases} V^2 & k = 0 \\ -\dfrac{V^2}{L} & k \neq 0 \end{cases} \qquad (3.2.2)$$

where V is the bit interval voltage level, k is an integer and Δt is the pulse period (duration of each bit) of the PRBS [77].

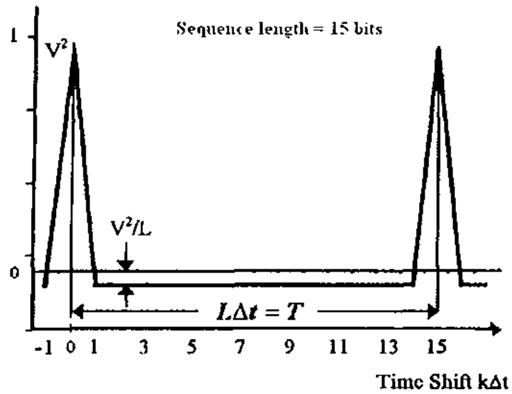

Figure 3-2 – Autocorrelation function for a PRBS of length 15 bits [77]

From (3.2.2), it can be seen that the autocorrelation function is a periodic triangular pulse train. This is illustrated in Figure 3-2 to the left. As the pulse period Δt vanishes and L becomes large the autocorrelation function tends closer to that of a periodic white noise source.

It has been well documented that the cross-correlation between the input x(k) and the output y(k) of a linear system, is related to the auto-correlation of the input by a convolution with the impulse response [79]:

$$\begin{aligned} & y(k) = h(k) \bullet x(k) \\ => \quad & x(k) * y(k) = h(k) \bullet x(k) * x(k) \qquad (3.2.3) \\ => \quad & R_{xy}(k) = h(k) \bullet R_{xx}(k) \end{aligned}$$

where • symbolises convolution and * represents correlation. As already discussed, an important property of any PRBS is that its auto-correlation function is essentially an impulse. This impulse is represented by the Dirac delta function:

$$R_{xx}(k) \approx \delta(k) \quad (3.2.4)$$

The result of convolving a sequence with a Dirac delta function is the sequence itself. Thus the impulse response h(k) can be found by cross-correlating the PRBS input x(k) with the output y(k):

$$R_{xy}(k) = \delta(k) \bullet h(k) = h(k) \qquad (3.2.5)$$

Hence it is possible to measure the impulse response of linear systems by calculating the cross-correlation between the PRBS input signal and the system output signal. The system's frequency response may then be determined by applying the FFT to the system's impulse response [80], [81], [76].

In order to implement the PRBS technique, it is necessary for the user to make a suitable choice for three parameters namely, the sequence length L, the bit interval voltage level or pulse amplitude V and the pulse period Δt. An investigation into an appropriate method for choosing the pulse amplitude V is discussed in Chapter 4 section 4.3. Also methods for choosing the pulse period Δt and sequence length L are discussed in section 4.2 of Chapter 4.

3.3.1.2 Relay Based Approach:

There are a number of different ways in which a relay feedback system (RFS) may be set up in order to obtain information regarding a control system's dynamic response characteristics. An investigation into a number of these different structures can be found in Appendix A.5 to A.11. The most efficient of these relay-based methods appears to be the 'Weighting' method as discussed in [82].

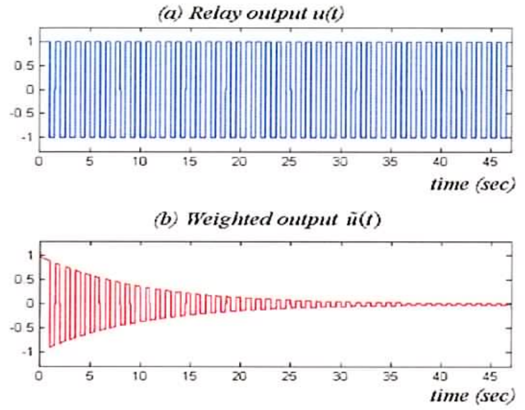

Figure 3-3 - Relay output and weighted version of relay output

This technique involves applying a decay weighting on the signals such that the weighted signals die off as time passes. This technique is also known as windowing. If y(t) and u(t) are the controlled variable and relay output in a relay feedback system, a decaying exponential $e^{-\alpha t}$ ($\alpha>0$) may be introduced to moderate these signals.

Figure 3-3 illustrates how a typical relay output u(t) is affected when the decaying exponential is applied to it, thus producing weighted output ũ(t). The shifted process frequency response $G(j\omega_l + \alpha)$ may be determined using the following formula:

$$G(j\omega_l+\alpha)=\frac{Y(j\omega_l+\alpha)}{U(j\omega_l+\alpha)}=\frac{FFT\{\tilde{y}(kT)\}}{FFT\{\tilde{u}(kT)\}}, \quad l=1,\ 2,\ 3\ldots,\frac{N}{2} \qquad (3.2.6),$$

where $\omega_l = 2\pi l/(NT_{samp})$, N is the total number of samples taken of the output, T_{samp} is the sampling period and $\tilde{y}(t)$ and $\tilde{u}(t)$ are the weighted controlled variable and weighted relay output respectively. The α value may be calculated using the following formula:

$$\alpha \geq -\frac{\ln \varepsilon}{T_f} \qquad (3.2.7),$$

Where ε is a threshold value and usually takes a value of 10^{-4}~10^{-6} and T_f is the total experiment time needed. According to [82], α should be kept less than 0.2 in order to prevent losing too much information when forming $\tilde{y}(t)$ and $\tilde{u}(t)$. The total experiment time T_f needed for this test depends on the period of oscillation of the closed loop system under test and also the number of periods to be taken. The more oscillations captured during the test period the more accurate the final result becomes.

As was the case for the PRBS based approach, there are a number of parameters that must be chosen for a successful implementation of this relay technique. These parameters include the relay output level, hysteresis values and the number of oscillations to be taken. An analysis into an appropriate means for choosing the relay output level and hysteresis width is discussed in section 4.3 of Chapter 4 and an investigation into the number of oscillations to be taken is detailed in section 4.2 of Chapter 4.

3.3.2 Manipulating Frequency Response Information:

Consider the simple feedback system of Figure 3-4.

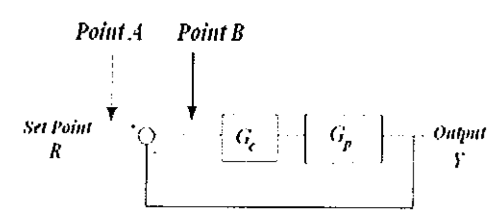

Figure 3-4 – Simple feedback system

For the case of the PRBS system-testing technique the binary testing sequence would be superimposed onto the set point and thus injected into the closed loop system at point A of Figure 3-4. The signals of interest in this testing procedure are therefore the PRBS signal and the controlled variable, Y. After applying the FFT to these signals, as discussed in section 3.3.1.1, the closed loop frequency response, M(jω), of the control system, is determined:

$$M(j\omega) = \frac{G_c(j\omega)G_p(j\omega)}{1 + G_c(j\omega)G_p(j\omega)} \qquad (3.2.8)$$

While this is useful in itself, it is also possible to determine the open loop frequency response, $G_c(j\omega)G_p(j\omega)$, using this closed loop data. The open loop frequency response may be calculated by application of the following formula:

$$G_c(j\omega)G_p(j\omega) = \frac{M(j\omega)}{1 - M(j\omega)} \qquad (3.2.9)$$

Now considering the case of the relay-based approach, the relay is inserted at point B of Figure 3-4 and it is the open loop frequency response of the system that is obtained. In a similar fashion to that of the PRBS case, it is possible to obtain the closed loop frequency response of the system by simply applying equation (3.2.8). Using both the open loop and closed loop frequency response, important metrics of the system may be determined. Furthermore, it was decided that it would be desirable to keep the process under direct control at all times during the testing procedure. It is for this reason that the PRBS test signal was not added directly into the closed loop system at the input to the process (i.e. between G_c and G_p, see Figure 3-4). Similarly, the relay was not inserted into the closed loop at the input to the process for the same reason.

3.3.3 Development of PI(D) Tuning Rule Database:

As discussed in section 3.2, it was considered desirable to create a method of benchmarking in order to contrast the existing system performance with possible alternatives. This benchmarking process was implemented in the form of a PI(D) database. This PI(D) database was based primarily on work previously carried out by Feeney and O'Dwyer [83]. Their work involved the development of a user interface capable of retrieving either a PI or PID tuning rule, based on a users gain and phase margin specifications. In order for the retrieval process to be successful it was necessary to determine the time delay (t_d) to time constant (T_c) ratio of the process to which the rule would be applied. Once the user had selected a particular rule, knowledge of the steady-state gain of the process was necessary in order to calculate the controller parameters associated with that rule. The database developed involved the use of Microsoft Access and Microsoft Excel. Some slight modifications were made to this system in order that it run completely in a Matlab environment. In total, 46 PI tuning rules and 26 PID tuning rules were incorporated into the database. This was done to keep the database manageable and the decision to enlarge the database could be made once its effectiveness has been verified. The following figures should help explain the database operation in more detail.

Rule	td/Tc	Gm	Pm
1	0.05	1.56	21.34
1	0.10	1.60	24.53
1	0.15	1.63	27.83
⋮	⋮	⋮	⋮
1	9.95	11.43	93.47
1	10.00	11.49	93.45
2	0.05	2.41	36.40
2	0.10	2.46	41.01
2	0.15	2.51	45.75
⋮	⋮	⋮	⋮
2	9.95	17.18	92.51
2	10.00	17.26	92.49
⋮	⋮	⋮	⋮
46	0.10	9.39	54.25
46	0.15	7.72	53.02
46	0.20	6.58	52.61
⋮	⋮	⋮	⋮
46	9.95	2.13	65.24
46	10.00	2.13	65.25

Figure 3-5 - PI Database Stage 1

At the heart of the database is the original work of Dr. O'Dwyer in which Gain margin (Gm) and Phase margin (Pm) values were evaluated and plotted for approximately 200 tuning rules applied to a first order lag plus delay model (FOLPD) with a varying time delay to time constant ratio (t_d/T_c) [84]. The t_d/T_c ratio was varied from 0.05 to 10 for each of the tuning rules. A set of Matlab variables was then generated representing the ratio of time delay to time constant (td/Tc), gain margin (Gm) and phase margin (Pm) for each of the 200 or so PI(D) tuning rules. It is these variables that make up stage 1 of the database, see Figure 3-5.

The database consists of two main parts; the first part, as already discussed, consists of an array containing the t_d/T_c, Gm and Pm values for each rule, along with a reference number, labelled *'Rule'* in Figure 3-5. The second part of the database contains a list of the tuning rules names and their associated controller parameter formulae. Each tuning rule in this list also has a reference number that corresponds to the reference numbers in the first part of the database. Figure 3-6 illustrates a typical layout for the second part of the database.

No.	Rule	Kc	Ti
1	*Ziegler & Nichols pg19*	(0.9*Tm)/(Km*Td)	3.33*Td
2	*Chien et al. Regulator 0% overshoot pg20*	0.6*Tm/(Km*Td)	4*Td
3	*Chien et al. servo 0% overshoot pg20*	0.35*Tm/(Km*Td)	1.17*Tm
4	*Chien et al. servo 20% overshoot pg20*	0.6*Tm/(Km*Td)	Tm
5	*Two constraints Murrill pg21*	(0.928/Km)*(Tm/Td)^0.946	(Tm/1.078)*(Td/Tm)^0.583
⋮	⋮	⋮	⋮
42	*Rivera et al. alt. lambda = 1.7TD pg43*	(2*Tm+Td)/(2*1.7*Td*Km)	Tm+0.5*Td
43	*Rivera et al. alt. lambda = TC pg43*	(2*Tm+Td)/(2*Td*Km)	Tm+0.5*Td
44	*Servo minimum ISTSE Zhuang/Atherton pg31*	0.361*Ku	0.083*Tu*(1.935*Km*Ku+1)
45	*GM=3.45, PM=46 deg Astrom/Hagglund pg51*	0.2015*Ku	0.1537*Tu
46	*Calcev and Gorez pg52*	0.3536*Ku	0.1592*Tu

Figure 3-6 - PI Tuning Rule Database Stage 2

The database query works as follows: once the desired Gm and Pm values have been specified and the current t_d/T_c has been determined, Matlab scans through part one of the database and returns the reference number of any tuning rules that will provide the desired results for that particular t_d/T_c ratio. This reference number is then passed to the second part of the database where the corresponding tuning rule names and their associated formulae are extracted. Based on the parameters for a FOLPD model of the process, the required controller parameters can be calculated using these formulae.

The concept behind the benchmarking process is as follows: after the system testing and modelling phase, the current Gm and Pm and a set of FOLPD model parameters are obtained and passed on to the PI(D) database. The database is scanned and any entries found to match the current t_d/T_c ratio and current Gm, Pm results, to a tolerance of $\pm$ 10%, are extracted. The controller parameters associated with these extracted tuning rules are then calculated and an offline simulation is performed to determine the possible achievable performance with these alternative tuning rules. The current metrics, along with the possible alternatives are then presented for comparison. Based on these results, it is possible to evaluate whether improved control is possible with an alternative set of controller parameters.

3.3.4 Approach to Process Modelling:

A vital part of the identification stage is developing a mathematical model of the dynamic system based on the measured data from the system-testing phase. A decision must be made when characterising a system to develop either a parametric or non-parametric model. Parametric identification methods are techniques for estimating parameters for a given model structure. Nonparametric identification methods are techniques to estimate model behaviour without necessarily using a given parameterised model set. Typical nonparametric methods include correlation analysis, which estimates a system's impulse response, and spectral analysis, which estimates a system's frequency response [85]. Both modelling procedures have different advantages and disadvantages and a decision must be made based on the application of the process model. As discussed in section 3.3.3, in order to access the database it is necessary to first determine a t_d/T_c ratio and an estimate of the process gain; thus, a FOLPD model of the process must be developed. Therefore a parametric approach to system modelling is necessary in this instance.

According to [85], the most common models are based on difference-equation descriptions, such as autoregressive with exogenous input (ARX) models, output error (OE) models, autoregressive moving average with exogenous input (ARMAX) models, finite impulse response (FIR) models, and Box-Jenkins (BJ) models. At a basic level, it is sufficient to think of these models as variants of the ARX model that also include a characterisation of the properties of the disturbance. Matlabs system identification toolbox provides algorithms for identifying models based on these techniques. These modelling approaches focus on characterising a system's behaviour based on its time domain responses, i.e. input and output signals. However, as one of the primary objectives is to determine, as accurately as possible, an estimate of the frequency response of the system under test, it seems prudent to use a parametric modelling technique that entails the processing of frequency domain data to develop the system model. One such modelling technique is the 'Gradient approach'. The gradient approach is a least squares based approach to process modelling and requires open loop frequency response information [86].

It should be noted at this stage that there are many other types of modelling procedures that have not been mentioned in this section but which may be considered suitable for the modelling requirements discussed. It should also be noted that particular tuning rules within the database require that the process be modelled in a particular manner using a particular technique in order to fully reach their potential. However, in an attempt to prevent the research from becoming unmanageable, it was decided to accept the gradient approach as the most appropriate modelling method. This area may merit further investigation in future work; however, with the objective of verifying the validity of the assessment strategy in mind, the gradient approach is a satisfactory technique to incorporate into the assessment tool.

3.3.5 Implementation of Strategy:

Subsequent to exploring the necessary techniques involved in each stage of the evaluation strategy, the next step was implementing the strategy and its components in a Matlab based environment. The two identification techniques require user inputs and provide graphical displays at the conclusion of the evaluation process. It was decided, therefore, that the best approach to presenting and facilitating the implementation of these assessment techniques was to develop a graphical user interface (GUI) in Matlab, for each technique. Figure 3-7 presents a flowchart describing the operation of each of the two assessment methodologies. It can be seen that both the relay and PRBS based techniques are broadly similar in their evaluation schemes.

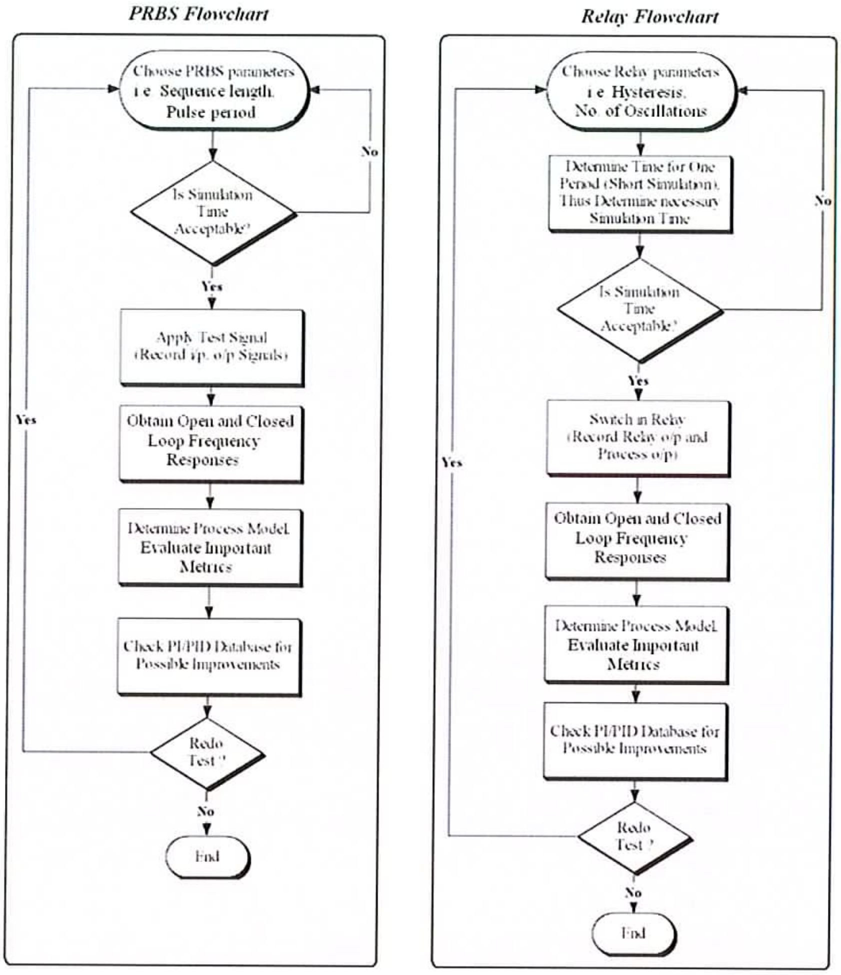

Figure 3-7 - System Evaluation Flowchart

From examining Figure 3-7, it can be seen that the initial stages of both techniques require the user to specify particular parameters before the test can begin. Based on these parameters, an estimate of the necessary simulation time can be made and the user may then choose to accept or reject this simulation time. If the simulation time is accepted the system test can begin, if rejected the technique parameters must be altered until the user is happy with the simulation time necessary. When the test is finished, the response signals recorded are processed in order to determine open and closed loop frequency responses along with a FOLPD model of the system under test. Using this information, it is then possible to evaluate time and frequency domain metrics to characterise the performance of the system. The next step is the consultation of the PI(D) database to determine whether improved control is possible, after which the user must decide whether or not the results obtained are satisfactory. If the results are considered unsatisfactory, the system may be re-tested. The following section describes the GUI's developed based on the flowchart models of Figure 3-7.

3.4 GUI's Developed:

3.4.1 Presentation and Explanation of GUI'S:

The following screen shots are intended to familiarise the reader with the two GUI's developed. A brief explanation of the necessary inputs to the GUI will be provided along with an explanation of the outputs generated by the testing procedure.

The first GUI to be discussed is the PRBS based evaluation method. This GUI can be divided into three main sections. The first section involves the selection of the necessary PRBS parameters. Along the left hand side of Figure 3-8 the user can enter in PRBS parameters such as sequence length, pulse amplitude and pulse period. There is also an option to specify the sampling period of the testing scheme. In order to avoid aliasing issues, this sampling period should always be less than the PRBS pulse period. The PRBS sequence length is set by choice of appropriate feedback taps; a list of example taps is displayed on the GUI and a more comprehensive list can be found in Appendix C. Next the user must supply the GUI with the current controller configuration. This may be entered in the form of a transfer function, numerator and denominator are entered, or in the form of controller parameters, any combination of K_c, T_i, T_d and 1/N can be entered. For the example illustrated in Figure 3-8, a proportional controller with a gain of 1 has been entered in transfer function form; this can be seen in the centre column of the GUI. If left unspecified, it is assumed that the current controller is a proportional controller with a gain of 1. It is also possible for the user to enter the current set point and any limitations on the output of the ADC or DAC being used in the testing procedure. The final section of the GUI configuration is located along the right hand side of Figure 3-8. Simulation details based on the parameters entered are displayed here, i.e. necessary simulation time. Subsequent to the completion of the system test, a report on whether or not the controlled variable went outside the range of the ADC/DAC during the test, along with system Input/Output plots, and also any error messages generated are displayed. There is also an option to implement the controller outside Matlab, by unselecting the '*Controller implemented in Matlab button*' and to display the Simulink model being used in the testing procedure; this allows the user flexibility to change any aspects of the test they feel may be unsuitable for their current system configuration.

Figure 3-8 - PRBS Evaluation Tool

The layout of the Relay GUI is very similar to that of the PRBS GUI. In a similar fashion, the relay parameters, such as relay output level, hysteresis width and number of oscillations to be taken, are entered in the panel along the left hand side of the GUI, i.e. left hand side of Figure 3-9. The current controller parameters can be entered, in either transfer function or parameter form, in the centre panel and the simulation details appear along the right hand side of the GUI. Input/Output plots are displayed in the centre of the GUI once the system test has completed. Again the option to specify the sampling period, ADC/DAC limits and to display the Simulink model is available.

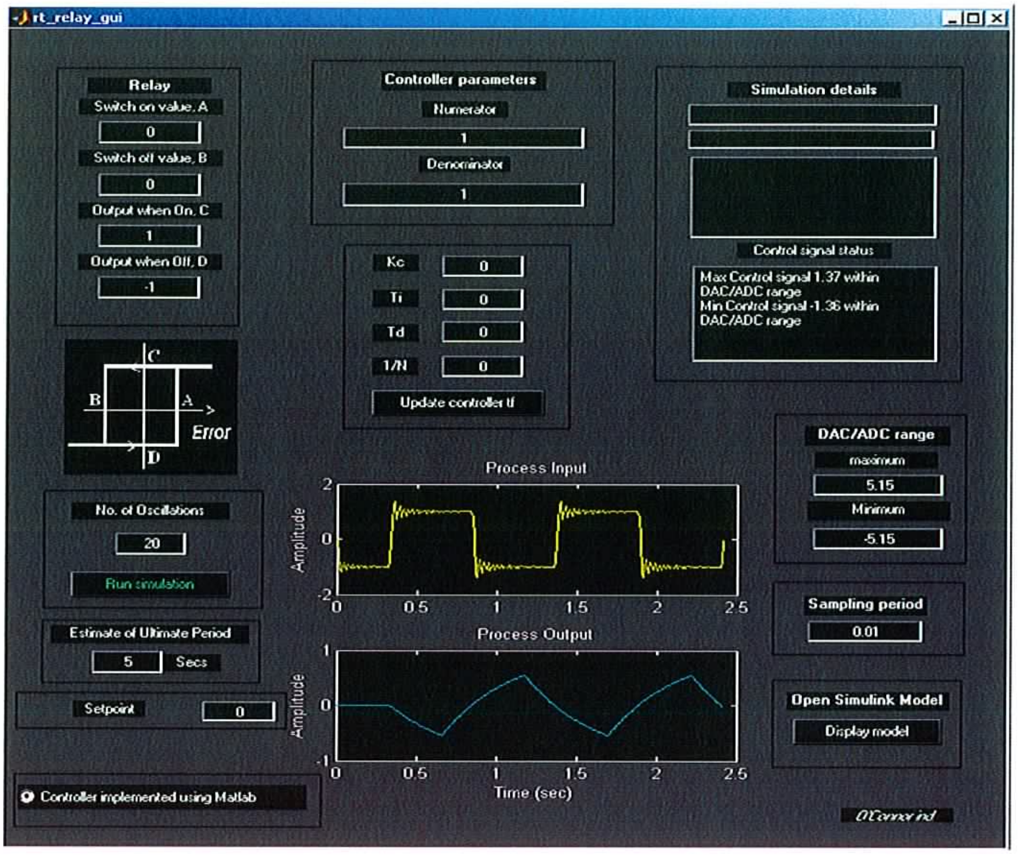

Figure 3-9 - Relay Evaluation tool

In the relay based system testing procedure, it is necessary to perform a short test to determine an accurate estimation of the time needed for one oscillation before the total required relay simulation time may be calculated. The time interval necessary for the short test is based on the estimated ultimate period value entered along the left hand side of Figure 3-9. The short test will take exactly three times the estimated ultimate period value entered. Once this short test has been carried out, a dialog box appears displaying the necessary simulation time. This short test is not a requirement of the PRBS based system-testing procedure.

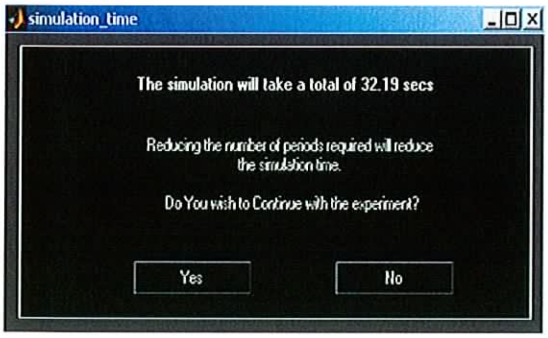

Figure 3-10 - Relay simulation time option box

Figure 3-10 illustrates the option box that appears once the short system test has completed. The necessary simulation time is displayed and the user is given the option to continue with the test or alter the number of oscillations to be taken to adjust the overall simulation time.

Once the testing phase has completed, a results option box with four choices is presented. The user can choose to display the evaluated metrics, display various plots generated during the course of the evaluation process, consult the PI tuning rule database or consult the PID tuning rule database. The following figures illustrate typical results obtained during an evaluation procedure.

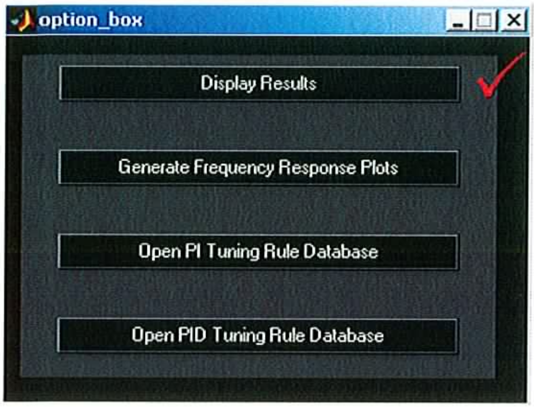

Figure 3-11 - Option Box Choice 1

If the '*Display Results*' push button is pressed a window similar to that of Figure 3-12 will appear. This window contains the estimated model parameters along with the relevant metrics evaluated as a result of the system test. These metrics include rise time, settling time, overshoot, Gm, Pm, L_{cmax} and closed loop system bandwidth.

The frequencies at which the Gm, Pm and closed loop frequency response peak value L_{cmax} are calculated are also displayed, i.e. w_{cg}, w_{cp} and w_r respectively. Details regarding the definitions of these quantities may be found in Chapter 2.

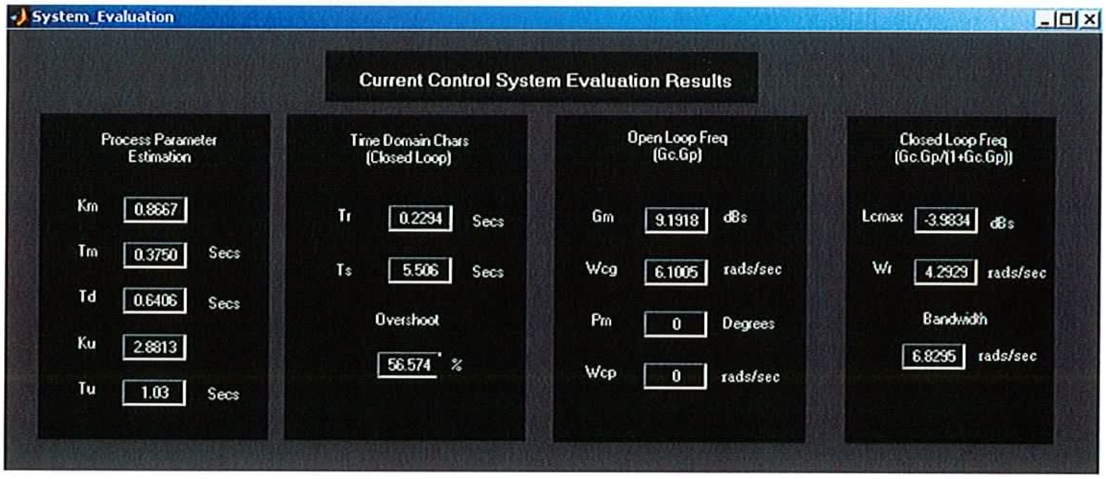

Figure 3-12 - Evaluation Results

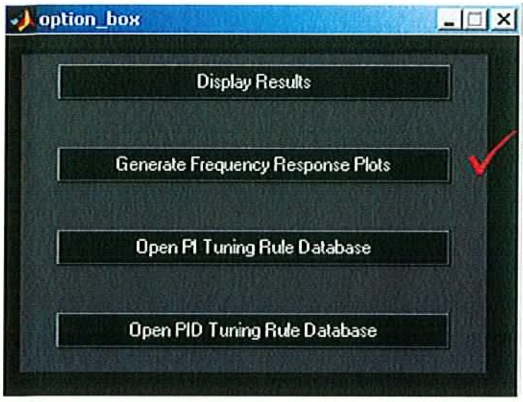

Figure 3-13 - Option Box Choice 2

If the 'Generate frequency response plots' push button is pressed, the plots of Figure 3-14 will be generated. These plots include the open loop frequency response plot, the closed loop frequency response plot, the magnitude of the sensitivity function plot and also a plot of the system input and output obtained during the testing process.

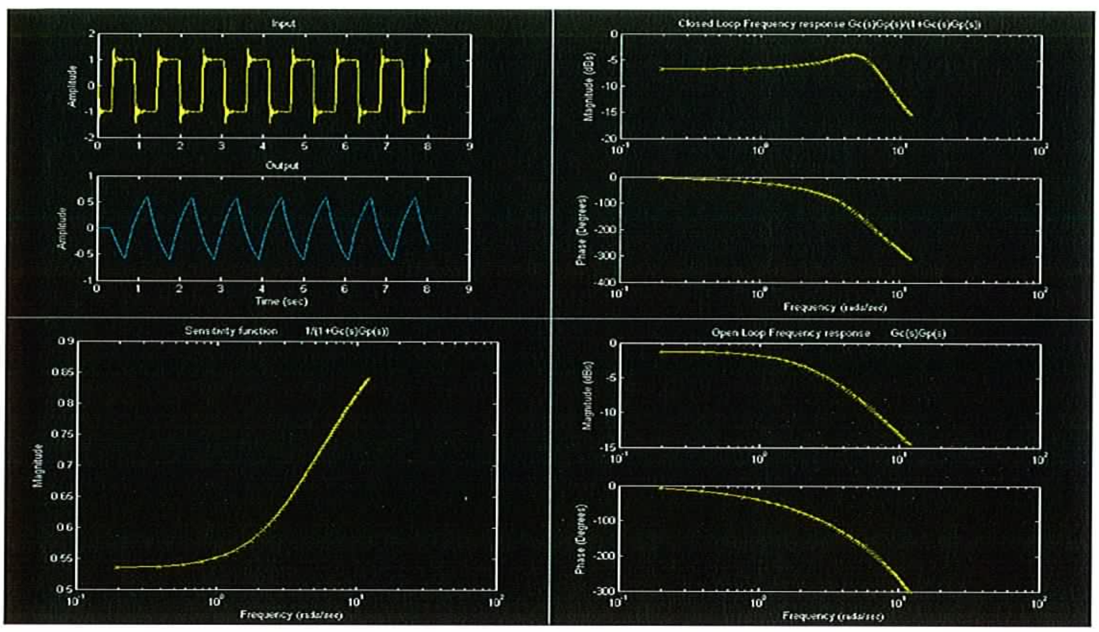

Figure 3-14 - Evaluation Plots

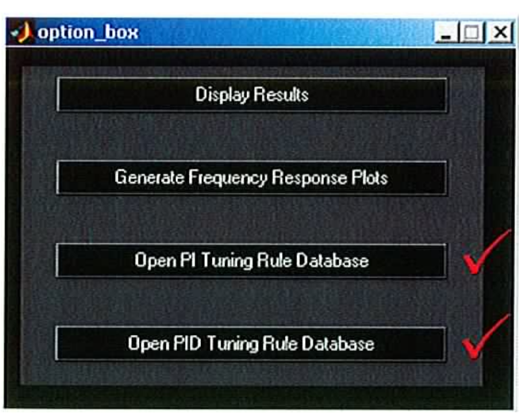

Figure 3-15 - Option Box Choice 3 & 4

Finally, if the PI or PID tuning rule database push buttons are pressed, the displays of Figure 3-16 will be generated. These database displays exhibit the current evaluated metrics for the current control system, a list of alternative tuning rules and their associated metrics based on the current system's Gm and Pm values, along with the estimated model parameters.

The recommended controller structure for the listed alternate tuning rules is also displayed. In addition, there is an option to enter a specific Gm and Pm value and search the database to determine whether there is a tuning rule that will provide the required specifications. The user can specific a tolerance (*'New Tol'*) if no tuning rules are returned on the first scan. When scanning the database, a set of assessment metrics are evaluated using the estimated process model and the controller parameters associated with the suggested alternative tuning rules.

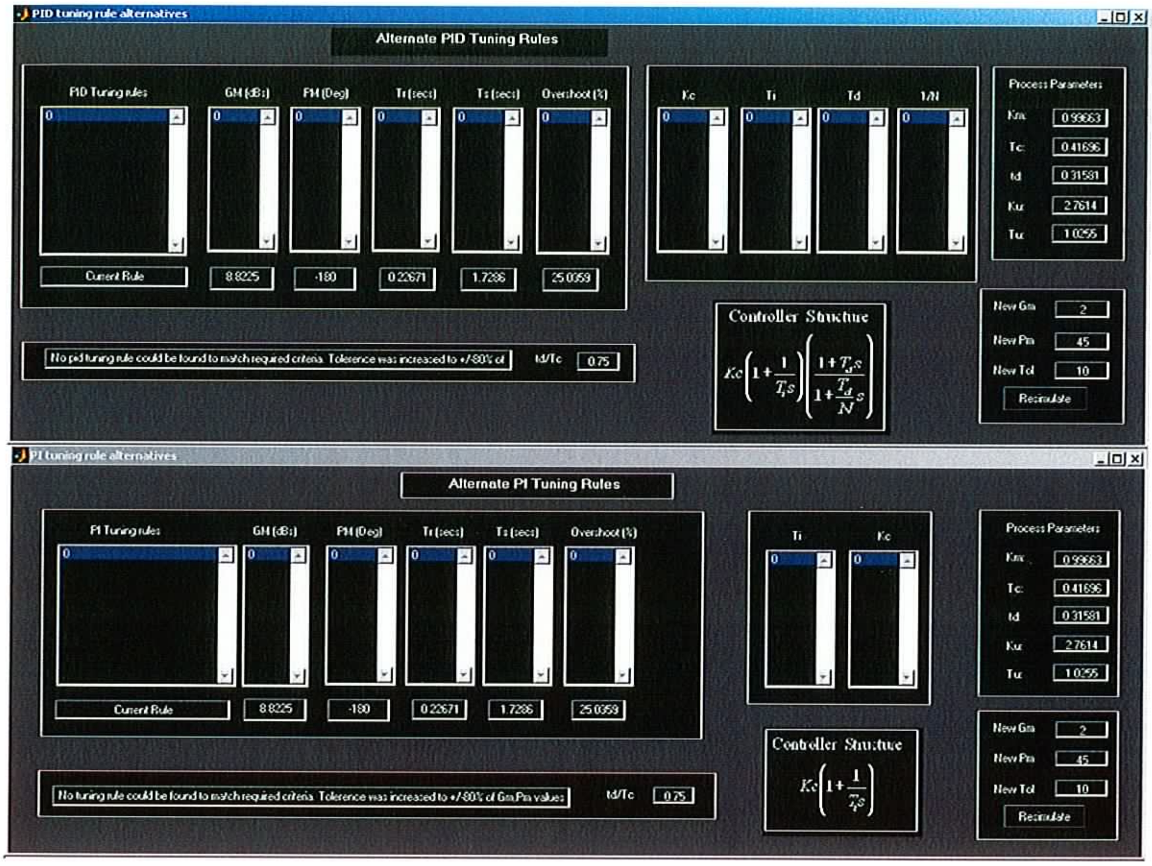

Figure 3-16 - PI/PID Databases

3.5 Summary and Conclusion:

The Chapter presented an assessment and evaluation strategy and discussed the implementation of this strategy in a GUI developed in Matlab. The strategy decided on consisted of two parts, an identification stage and a subsequent evaluation stage. The identification stage involves a system test followed by a signal processing stage to determine the system's frequency response, which could then be used to estimate a process model. The gradient approach was identified as being an appropriate means of process modelling. The evaluation stage entails processing the frequency response information to determine performance and robustness metrics such as gain and phase margins and the closed loop frequency response peak value. Using the estimated process model it was possible to evaluate time domain metrics such as rise time, settling time and overshoot.

A number of identification techniques were reviewed and two particular methods, namely the PRBS approach and relay based approach were identified as being particularly useful. The steps involved in the development of a PI(D) tuning rule database was also discussed. This database was intended to provide a means of benchmarking the current system's performance with a number of possible alternatives. Finally, screen shots describing the operational capabilities of the developed GUI were presented and explained.

Subsequent to the development of the evaluation tool the next step involves the investigation into the limitations and capabilities of each of the GUI's. Both techniques require user inputs before the system-testing phase can begin. For the case of the PRBS based technique, the user must specify a pulse amplitude, a sequence length and also a pulse period. The relay-based approach requires the user to specify the relay output, hysteresis width and number of oscillations to be taken. Chapter 4 will concentrate on developing guidelines for the choice of each of these parameters along with an assessment of the accuracy of each technique.

4 Investigation of Identification Abilities:

4.1 Introduction:

As mentioned in the conclusions of the previous Chapter, both the PRBS and Relay based identification techniques require user inputs before the system-testing phase can begin. As illustrated in Figure 4-1, the PRBS based technique requires the user to specify three parameters, namely the PRBS pulse amplitude $\pm V$, the PRBS pulse period Δt and the sequence length L. The relay-based approach requires four parameters to be specified by the user; the relay output level $\pm\mu$, an estimate of the ultimate period P_u of the system under test, the hysteresis width $\pm\varepsilon$ and M, the number of output oscillations to be taken. This Chapter concentrates on the development of a guideline for the choice of each of these technique parameters along with an investigation into the limitations and capabilities of each of the identification and evaluation methods discussed in Chapter 2.

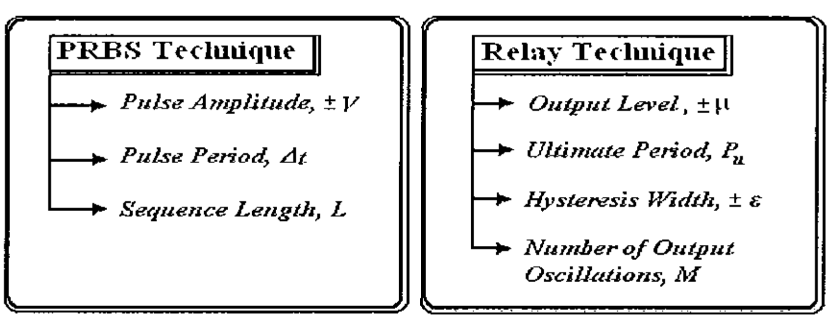

Figure 4-1 - Parameters needed for PRBS and Relay based assessment methods

Section 4.2 contains a synopsis of the results of a number of simulations carried out on various process models using both the PRBS and relay based evaluation methods. For each of these simulations, the parameters of each of the techniques were systematically varied in an attempt to identify an appropriate means of selecting the associated technique parameters. Section 0 contains an investigation into the accuracy of each of the evaluation techniques in the presence of noise. Finally, Section 4.4 contains the results of a number of simulations carried out with the objective of validating the overall evaluation strategy developed in Chapter 2.

4.2 Identifying Optimal Technique Parameters:

In order to develop a guideline for selecting appropriate PRBS and relay technique parameters, a number of simulations were carried out on a variety of process models of various orders. Table 4-1 and Table 4-2 contain a list of the various processes tested.

The first set of simulations were carried out on the first order lag plus delay (FOLPD) models of Table 4-1. For the PRBS and the relay based evaluation procedures, the FOLPD models were tested in the closed loop configurations of Figure 4-2 and Figure 4-3. From Figure 4-2 and Figure 4-3, it can be seen that a proportional controller with a gain of 1 was implemented in this testing phase. This was done in order to identify the most robust means of selecting the required technique parameters. Testing the process models under this type of control resulted in poor closed loop performances, therefore allowing an investigation into the accuracy of both identification techniques in testing under performing closed loop systems. From Table 4-1, it can be seen that for each of the FOLPD models tested, the time constant of the model was kept constant while the time delay was varied. The complete set of results is given in Appendices D and E; a synopsis of these results is presented in sections 4.2.1.2 and 4.2.1.3.

Table 4-1 - Various first order models with varying time delays

Model No.	FOLPD model	Model No.	FOLPD model
1	$\frac{2}{s+1}e^{-0.1s}$	6	$\frac{2}{s+1}e^{-0.6s}$
2	$\frac{2}{s+1}e^{-0.2s}$	7	$\frac{2}{s+1}e^{-0.7s}$
3	$\frac{2}{s+1}e^{-0.3s}$	8	$\frac{2}{s+1}e^{-0.8s}$
4	$\frac{2}{s+1}e^{-0.4s}$	9	$\frac{2}{s+1}e^{-0.9s}$
5	$\frac{2}{s+1}e^{-0.5s}$	10	$\frac{2}{s+1}e^{-1s}$

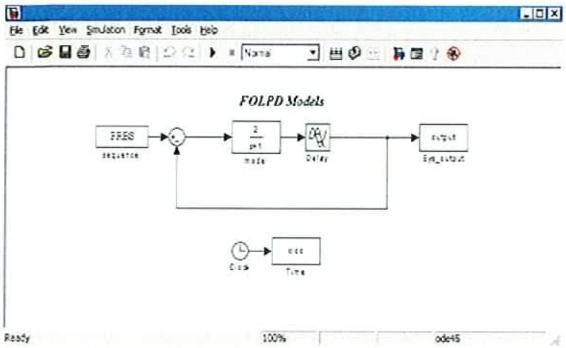

Figure 4-2 - Simulink test set-up for PRBS based testing of FOLPD models

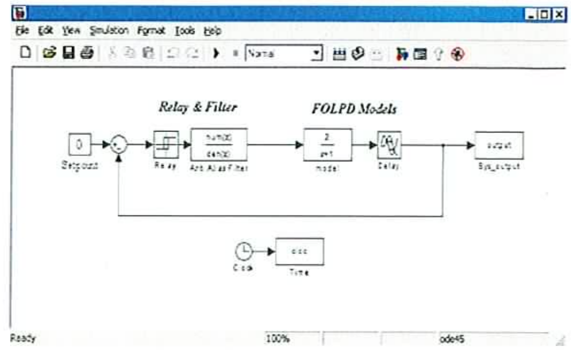

Figure 4-3 - Simulink test set-up for Relay based testing of FOLPD models

The second set of simulations involved the testing of a number of typical process models, Table 4-2, employed by Wang *et al.* in the validation of a variety of relay based approaches to system identification [82]. These typical process models were also tested in a closed loop configuration, illustrated in Figure 4-4 and Figure 4-5. The complete set of results for these simulations can be found in Appendices D and E and a summary of these results is presented in sections 4.2.2.1 and 4.2.2.2 of this Chapter.

Table 4-2 - Typical processes & Generic controllers

Model No.	Typical Process Models	Controllers
1	$\frac{1}{(2s+1)^2}e^{-2s}$	$1.5972\left(1+\frac{1}{9.618s}\right)$
2	$\frac{1}{(2s+1)^5}e^{-2s}$	$1.182\left(1+\frac{1}{22.21s}\right)$
3	$\frac{1}{(s+1)(s^2+s+1)}e^{-0.5s}$	$1.29\left(1+\frac{1}{6.36s}\right)$
4	$\frac{1}{(s+1)(5s+1)^2}e^{-2.5s}$	$2.22\left(1+\frac{1}{19.56s}\right)$
5	$\frac{1}{(s+1)^8}$	$1.11\left(1+\frac{1}{15.17s}\right)$
6	$\frac{1}{(5s+1)}e^{-5s}$	$1.33\left(1+\frac{1}{15.49s}\right)$

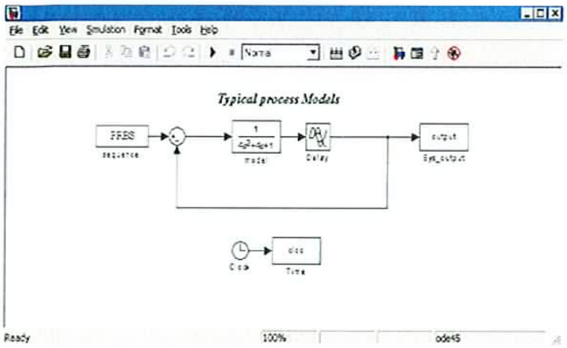

Figure 4-4 - Simulink test set-up for Typical process models

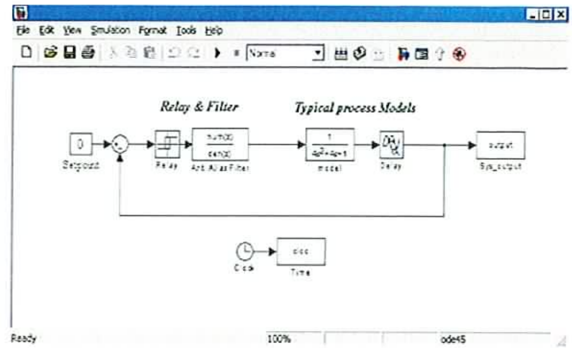

Figure 4-5 - Simulink test set-up for Relay based testing of Typical process models

Finally, a third set of simulations were carried out on the typical process models of Table 4-2, and a selection of generic PI controllers. The controllers were designed, based on the characteristics of the typical process models, using the tuning rule of Edgar *et al.* [87]. The typical process models and their associated controllers were then tested in the closed loop configuration of Figure 4-6 and Figure 4-7.

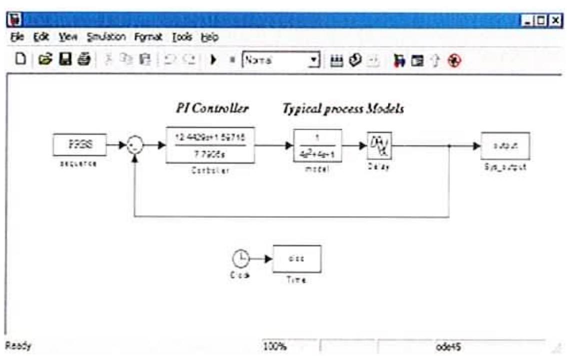

Figure 4-6 - Simulink test set-up for Typical process models and PI controllers

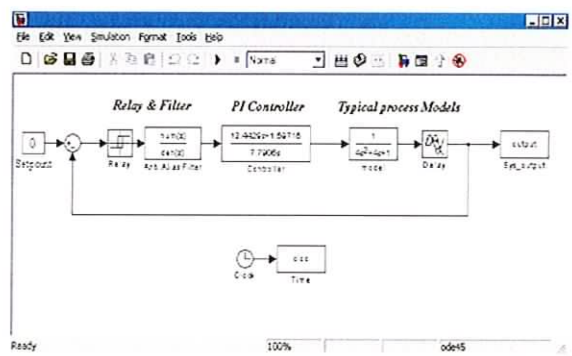

Figure 4-7 - Simulink test set-up for Relay based testing of Typical process models and PI controllers

For each set of simulations carried out, the technique parameters associated with both the PRBS and relay evaluation methods were systematically changed in an attempt to identify an optimal means of selection of the required parameters for the two methods (see Figure 4-1).

4.2.1 Accuracy in Identifying FOLPD Model Characteristics:

The following section contains a synopsis of the results obtained in the testing of the ten FOLPD models of Table 4-1. For each of the models tested, important open loop frequency domain characteristics, such as gain margin and phase margin, and important closed loop frequency domain characteristics, such as closed loop frequency response peak value and closed loop bandwidth, were evaluated, using the assessment techniques, and compared to the actual characteristic values in order to determine the percentage error, and thus the accuracy, of the each of the evaluation methods. A detailed explanation of the testing procedures, along with the full set of results is presented in Appendix D, section D.1 and Appendix E, section E.1.

4.2.1.1 Introduction to PRBS Testing Methods:

For the PRBS evaluation method results displayed in Appendix D, five different PRBS pulse period selection methods were implemented and contrasted, in an attempt to identify the most effective pulse period selection method. The PRBS pulse selection methods and their associated formulae are displayed in Table 4-3. For each of the tests carried out a PRBS pulse amplitude of $\pm$ 1 volt was used. An investigation into the effects of varying the pulse amplitude is discussed in section 4.3 of this Chapter.

From Table 4-3, it can be seen that the selection methods, taken collectively, require a number of different system characteristics in order to calculate the optimal PRBS pulse period. Method 1 was developed by Hang and Sin and requires knowledge of the ultimate period T_u of the system under test, in order to calculate the optimal PRBS pulse period Δt. Method 2, developed by Guinee, requires a priori knowledge of the minimum time constant T_{min} of the system. Method 3 and 4 require information regarding the bandwidth f_{BW} of the system under test in order to calculate Δt. Method 5, developed by Landau, requires information regarding the rise time of the system under test, along with a specification of the PRBS sequence length to be used in the testing procedure, in order to determine the PRBS pulse period. For this particular selection method (selection method 5), N refers to the number of shift registers needed to generate the PRBS sequence of length L (i.e. sequence length $L = 2^N-1$); refer to Appendix A, section A.2 for more information on generating PRBS sequences.

For each of the simulations carried out, the FOLPD models were tested in a closed loop configuration; therefore, the PRBS pulse period values calculated were based on the closed loop system characteristics (i.e. closed loop bandwidth, closed loop rise time and closed loop minimum time constant were used in the Δt formulae of Table 4-3).

Table 4-3 - PRBS pulse period selection methods

Method No.	Authors	Formula	Characteristics needed
1	Hang C.C. & Sin K.K. [88]	$8f_u < f_{PRBS} < 12f_u \Rightarrow \Delta t \approx \frac{10}{T_u}$	Ultimate period (Secs), T_u
2	Guinee R.A. [89]	$\Delta t = \frac{T_{cmin}}{4}$	Minimum time constant (Secs), T_{cmin}
3	Marzzocca C. & Corsi F. [90]	$f_{PRBS} = 5 \times f_{BW} \Rightarrow \Delta t = \frac{1}{5 \times f_{BW}}$	System bandwidth (Hz), f_{BW}
4	Yaacob S.& Mohamed F.A. [91]	$f_{PRBS} = 3 \times f_{BW} \Rightarrow \Delta t = \frac{1}{3 \times f_{BW}}$	System bandwidth (Hz), f_{BW}
5	Landau I.D. [92]	$N \times \Delta t \geq T_r \Rightarrow \Delta t \approx \frac{T_r}{N}$	System rise time (Secs), T_r

From the PRBS pulse period Δt formulae presented in Table 4-3, a number of observations regarding the properties of each of the selection methods can be made. Considering the formula presented for selection method 1, it can be seen that the authors are attempting to concentrate the frequency content of the PRBS test signal to the range over which the ultimate frequency of the system under test occurs. The selection method developed by Guinee, method 2, specifies the pulse period as a fraction of the minimum time constant of the system under test. The objective in specifying the pulse period in this manner is to make the pulse period slow enough to capture the dynamics of the system, but fast enough to keep the overall total test time low. Guinee has specified that the optimal value for the pulse period is 25% of the minimum time constant of the system. In a similar manner to method 1, method 3 and 4, developed by Marzzocca & Corsi and Yaacob & Mohamed respectively, attempt to control the frequency content of the PRBS test signal. In this case, the authors are attempting to ensure that the -3dB point of the power spectrum of the PRBS test signal is outside the bandwidth of the system being tested. By controlling the power content of the PRBS test signal, it is possible to ensure that the system under test is sufficiently excited at all frequencies of interest. This issue is discussed in detail in Appendix A, section A.3.2. Finally, selection method 5 uses the rise time of the system under test to determine the optimal PRBS pulse period. In a similar manner to selection method 2, this method attempts to ensure the dynamics of the system are sufficiently excited while the overall test time is kept as short as possible. Method 5 makes use of the run length property of PRBS sequences, discussed in Appendix A, section A.3.1, to ensure that the PRBS pulse period, and thus overall test time, is kept as short as possible.

For each of the PRBS pulse selection methods of Table 4-3, the PRBS sequence length was varied and the models of Table 4-1 and Table 4-2 were tested in closed loop. Also, according to [74], at least two periods of the PRBS sequence must be applied to the system under test and the data associated with the initial period should be ignored, refer to Appendix D, section D.5. Therefore, for the results obtained in the following tests, two periods of the PRBS sequence were applied to the system under test and the data associated with the initial period was ignored. The results of these simulations are summarised in sections 4.2.1.2 and 4.2.2.1 of this Chapter.

4.2.1.2 PRBS Results for FOLPD Models:

This section contains a summary of the results of Appendix D, section D.1. The information presented in Table 4-4 provides a general overview of the results obtained. For each of the PRBS pulse period selection methods discussed in section 4.2.1.1, the PRBS sequence length was varied and the estimations of the frequency domain characteristics generated were compared to the actual characteristic values in order to determine the percentage error for each of the selection methods. Also the efficiency of each selection method was calculated and the necessary simulation time for each of the methods was recorded. An efficiency metric was calculated for each of the selection methods, by multiplying the necessary simulation times associated with each method, by the overall percentage error values obtained using said method. Therefore, methods that provided low overall percentage error values for short simulation times would generate a small efficiency metric value, and methods with long simulation times and large percentage error values would generate a large efficiency metric value. It was desirable to generate an efficiency number as small as possible. A detailed discussion of how the results were obtained can be found in Appendix D. The Key for Table 4-4, providing the definition for the terms *Very Good*, *Good* etc., is displayed in Table 4-5.

For the results of Table 4-4, the *% Error values* displayed are based on the averages of the *Overall abs % Error* values displayed in Tables D-2 to D-5 of Appendix D. The *Simulation Times* entries are based on the *Sim time(secs)* entries of the aforementioned tables and finally, the *Average Efficiency* entries are based on the averages of the *% Error second* results of the same tables.

Table 4-4 - General summary of results of Appendix D section D.1

FOLPD Models					
Sequence Length	Method No.	% Error Values	Average Efficiency	Simulation Times	Comment on any unusual results
63 bits	5	Very Poor	Very Poor	Very Good	Lowest frequency component insufficient. Poor closed loop frequency resolution
	4	Very Good*	Very Good	Very Poor	-
	3	Good	poor	Poor	-
	2	Good	Mixed	Good	-
	1	Mixed	Good	Mixed	Lowest frequency component insufficient
127 bits	5	Very Poor	Very Poor	Very Good	Poor open loop frequency resolution. Poor closed loop frequency resolution
	4	Very Good*	Very Good	Very Poor	-
	3	Very Good	Mixed	Poor	-
	2	Very Good	Good	Good	-
	1	Good	poor	Mixed	-
511 bits	5	Very Poor	Very poor	Very Good	Poor open loop frequency resolution. Poor closed loop frequency resolution
	4	Very Good*	Poor	Very Poor	-
	3	Very Good	Good	Poor	-
	2	Very Good	Mixed	Good	-
	1	Very Good	Very Good	Mixed	-
1023 bits	5	Poor	Very Poor	Very Good	-
	4	Very Good*	Poor	Very Poor	-
	3	Very Good	Good	Poor	-
	2	Very Good	Mixed	Good	-
	1	Very Good	Very Good	Mixed	-

Table 4-5 - Key for Table 4-4, Table 4-9 and Table 4-10

	% Error Values	Simulation Times	Average Efficiency
Very Good	average % error<10%	Shortest SimulationTimes	lowest
Good	average % error<20%	2nd Shortest	2nd lowest
Mixed	average % error<50%	Mid Range	Mid Range
Poor	average % error<100%	2nd Longest	2nd Highest
Very Poor	average % error>100%	Longest Simulation Times	Highest

* indicates best performer

From Table 4-4, it can be seen that as the PRBS sequence length was varied from 63 bits long to 1023 bits long, selection method 4 consistently performed well with respect to providing an average percentage error value below 10%. As the sequence length increased, selection methods 2 and 3 also improved, moving from *Good* performers to *Very Good* performers. For each of the sequence lengths tested, method 5 consistently performed poorly.

The trade off in obtaining low percentage error values is an increase in the necessary simulation times. Considering the results obtained for selection method 4, it can be seen from Table 4-4 that even though this selection method consistently provided the lowest percentage error values, it also required the longest simulation times. Method 5 on the other hand consistently required the shortest simulation times but provided the worst results.

Examining the efficiency results of Table 4-4, it can be seen that for small sequence lengths, selection method 4 proved to be the most efficient of all selection methods tested. As the sequence length increased, selection method 1 became the most efficient method.

The column of Table 4-4 entitled *Comment on any unusual results*, highlights selection methods that generated abnormally poor results when compared to the other selection methods tested. On further investigation, the root cause of these poor results was determined to be one of three possibilities. Errors in gain margin and phase margin estimation were attributed to either poor frequency resolution in the generated open loop frequency response plots or an inability to identify the 0dB crossing point, as a result of no low frequency content in the PRBS test signal at the frequency where this crossing occurred. Errors associated with the estimations of the closed loop frequency response characteristics resulted from poor frequency resolution around the frequencies where the characteristics were evaluated. It can be seen from Table 4-4 that selection methods 1 and 5 both experienced these frequency resolution and frequency content problems. A detailed explanation of these problems is provided in Appendix D, section D.1.1.

To summarise, based on the results obtained from the FOLPD model tests, it can be seen that the PRBS pulse selection methods that attempt to control the power content of the PRBS test signal to within the bandwidth of the system under test, selection methods 3 and 4, achieve the most accurate results but require the longest simulation times. The other selection methods tested require, in some cases, significantly less testing time but do not achieve the same degree of accuracy in identifying the open and closed loop frequency domain characteristics as methods 3 and 4. It is also worth noting that in order to avoid the frequency resolution and frequency content problems discussed, methods 1 and 5 require the PRBS sequence length to be as long as possible. Unfortunately, increasing the sequence length increases the necessary testing time, thus eliminating the advantage of short simulation times associated with these methods. Overall, PRBS pulse selection method 4 proved to be the most consistently accurate of all the selection methods tested.

Table 4-6 contains the results obtained in the testing of the FOLPD models using PRBS pulse selection method 4 and a sequence length of 63 bits. From this table, it can be seen that the *Overall % Error* results, which is the sum of the *Open Loop % Error* results and the *Closed Loop % Error* results, is relatively small, below 10% in most cases, and, on average, the simulation times are relatively short. The results for the other selection methods can be found in Appendix D, section D.1.

Table 4-6 - Selection method 4 results for FOLPD models, refer to Table D-2 Appendix D

Sequence Length	Method No.	Model No.	Open Loop % Error	Closed Loop % Error	Overall % Error	Simulation Times (secs)	Simulation Times (hr,mins,secs)
63 bits	4	1	2.33	0.88	3.21	68.04	00:01:08
		2	1.36	3.90	5.26	46.62	00:00:46
		3	1.09	6.08	7.18	42.84	00:00:42
		4	1.10	9.23	10.33	46.62	00:00:46
		5	1.28	5.26	6.54	50.40	00:00:50
		6	1.50	5.56	7.05	55.44	00:00:55
		7	1.74	5.46	7.20	60.48	00:01:00
		8	2.02	4.82	6.84	65.52	00:01:05
		9	2.01	4.63	6.64	70.56	00:01:10
		10	6.31	1.46	7.77	75.60	00:01:15

4.2.1.3 Relay Results for FOLPD Models:

This section contains a summary of the relay results of Appendix E, section E.1. As discussed in section 4.1, the relay based evaluation method requires the specification of four technique parameters, namely the relay output level, an estimate of the ultimate period of the system under test, the hysteresis width and the number of output oscillations to be taken. Both the relay output level and the hysteresis width are set based on the noise levels inherent in the system under test. For each of the tests carried out a relay output level of ± 1 volt was used and no hysteresis was incorporated. The issue of noise, and methods of dealing with it, will be discussed in section 0. The estimate of the ultimate period does not have to be too accurate as a short system test is carried out in order to verify the estimate provided before the actual system test begins; this issue is discussed in Chapter 3, section 3.4.1.

Therefore, as shown in the results of Appendix E, section E.1, the effects of varying the number of output oscillation taken, M, was investigated. The number of oscillations taken was varied from 20 to 250 oscillations and the effects on the estimations of the open and closed loop frequency domain characteristics were monitored.

Table 4-7 - Summary of relay results for FOLPD models tested

Model No.	td/Tc ratio	No. of Oscillations Needed Open Loop Error <10% (approx)	No. of Oscillations Needed Closed Loop Error <10% (approx)
1	0.1	98	148
2	0.2	50	101
3	0.3	50	99
4	0.4	99	250
5	0.5	100	151
6	0.6	100	100
7	0.7	150	100
8	0.8	150	100
9	0.9	200	100
10	1	250	100

Table 4-7 contains a general overview of the results of Appendix E, section E.1. The first point that can be made regarding these results is that for models 1 to 5, models with a time delay to time constant ration (t_d/T_c *ratio*) of 0.5 or below, the number of oscillations necessary to obtain an open loop characteristics error (labeled *% Error Gm & Pm* in Table E-1, Appendix E) below 10% is less than the number of oscillations necessary to achieve the same percentage error for the closed loop characteristics (labeled *% Error Lcmax & band* in Table E-1, Appendix E). However, once the t_d/T_c *ratio* goes above 0.5, the situation is reversed, i.e. more oscillations are needed to achieve an approximate error of 10% for the open loop characteristics then for the same level of error for the closed loop characteristics.

The second point that should be made is that traditional relay based identification techniques require significantly less output oscillations, thus significantly less test time and less disturbance to the plant operation, in order to evaluate system characteristics [82]. However, these traditional testing methods tend to focus on identifying one characteristic per test, for example the gain margin of the system. Therefore, multiple tests are necessary to evaluate multiple system characteristics. This testing protocol under discussion identifies a multitude of characteristics in one test; the trade-off is increased testing times and thus increased disturbance to the plants normal operation.

From the results obtained, it was determined that in order to obtain an overall percentage error value (i.e. the sum of the absolute values of the open and closed loop characteristics percentage errors) less than 10%, for FOLPD models with a t_d/T_c *ratio* between 0.1 and 1, it was necessary to obtain at least 250 output oscillations. This requires a significant amount of testing time. A detailed explanation of the full set of relay results may be found in Appendix E, section E.1.

Table 4-8 contains a summary of the results of Table E-1 of Appendix E. Table 4-8 displays the number of oscillations necessary to obtain an overall percentage error below 10%. The results highlighted in yellow illustrate results where an overall percentage error of below 10% was not achieved. In order to reduce this overall percentage error value further, more than 250 output oscillations would need to be taken. The results highlighted in blue indicate the causes of the large overall percentage error values.

Table 4-8 - Relay results for FOLPD models, refer to Table E-1 Appendix E

Model No.	Open loop % error	Closed Loop % error	Overall % error	No. of Output Oscillations Taken	Simulation Times (secs)	Simulation Times (hrs,mins,secs)
1	3.66	9.84	13.50	246.00	162.49	00:02:42
2	2.77	6.27	9.05	151.00	181.49	00:03:01
3	2.23	2.31	4.54	149.00	250.49	00:04:10
4	1.31	13.45	14.76	249.00	529.99	00:08:49
5	2.49	4.36	6.85	201.00	517.99	00:08:37
6	3.46	3.36	6.82	201.00	601.99	00:10:01
7	4.88	3.20	8.08	201.00	679.99	00:11:19
8	5.06	2.56	7.62	249.00	942.49	00:15:42
9	7.59	2.78	10.37	251.00	1045.00	00:17:25
10	12.74	3.42	16.16	250.00	1135.00	00:18:55

Comparing both the overall percentage error results and the simulation times necessary for the relay based approach, Table 4-8, to those of the PRBS based approach, Table 4-6, it can be seen that the PRBS approach is a far superior approach to system evaluation on both counts.

4.2.2 Accuracy in Identifying Typical Process Model Characteristics:

This section contains two sets of results obtained in the testing of the six typical process models of Table 4-2. For the first set of results, each of the typical process models was tested in the closed loop configurations of Figure 4-4 and Figure 4-5. As discussed in section 4.2.1, important open loop and closed loop frequency domain characteristics were evaluated and compared to the actual characteristic values in order to determine the percentage error, and thus the accuracy, of both the PRBS and relay based evaluation methods. A thorough description of the testing procedures, along with the full set of results is presented in Appendix D, section D.2 and Appendix E, section E.2.

The second set of results were obtained in the testing of the six typical process models of Table 4-2 and their associated PI controllers in the closed loop configuration illustrated in Figure 4-6 and Figure 4-7. Again, important open and closed loop frequency domain characteristics were evaluated and compared to their actual values in order to generate percentage error results. A full explanation of the test procedures, along with the complete set of results is presented in Appendix D, section D.3 and Appendix E, section E.3.

4.2.2.1 PRBS Results for Typical Processes:

Table 4-9 presents an overview of the results from the testing of the typical process models of Table 4-2 in closed loop. As in section 4.2.1.2, each of the PRBS pulse period selection methods of Table 4-3 have been graded based on their accuracy in estimating the frequency domain characteristics, their efficiency, and their associated required simulation times.

Table 4-9 - General summary of results of Appendix D section D.2

Typical Process Models					
Sequence Length	**Method No.**	**% Error Values**	**Average Efficiency**	**Simulation Times**	**Comment on any unusual results**
63 bits	5	Very Poor	Very Poor	Good	Poor open loop frequency resolution, Poor closed loop frequency resolution
	4	Very Good*	Good	Very Poor	-
	3	Very Good	Very Good	Poor	-
	2	Very Poor	Poor	Very Good	Poor closed loop frequency resolution
	1	Mixed	Mixed	Mixed	-
127 bits	5	Poor	Very Poor	Good	Poor closed loop frequency resolution
	4	Very Good*	Very Good	Very Poor	-
	3	Very Good	Good	Poor	-
	2	Poor	Poor	Very Good	Poor closed loop frequency resolution
	1	Very Good	Mixed	Mixed	-
511 bits	5	Mixed	Very Poor	Good	-
	4	Very Good	Poor	Very Poor	-
	3	Very Good*	Mixed	Poor	-
	2	Very Good	Good	Very Good	-
	1	Very Good	Very Good	Mixed	-
					* Indicated best performer

From the results presented in Table 4-9, it can be seen that PRBS pulse period selection method 4 consistently performed well with respect to minimizing the percentage error between the estimated and actual frequency domain characteristics. Methods 3 also performed well, and actually generated moderately smaller percentage error results then method 4 when the sequence length was increased to 511 bits. However, the trade off for the accurate results generated by these selection methods is the long simulation times necessary to generate these results. As illustrated, methods 3 and 4 consistently required the longest simulation times. For the shorter length sequences, 63 bits and 127 bits, both selection methods 3 and 4 were the most efficient of all the selection methods tested. However, as the sequence length was increased the efficiency of these methods decreased. Methods 1, 2 and 5 performed relatively poorly for shorter sequence lengths but improved as the sequence lengths were increased. As can be seen, the methods that performed the best required the longest simulation times and vice versa. The results obtained in the testing of the typical process models in closed loop are consistent with those generated in the testing of the FOLPD models, as discussed in section 4.2.1.2.

A number of limitations of particular selection methods are also highlighted in Table 4-9. It can be seen that selection methods 2 and 5 both had issues with respect to poor frequency resolution around the frequencies of interest in both the open and closed loop frequency response results. This issue has been discussed in detail in Appendices D and E and also in section 4.2.1.2 of this Chapter.

In summary, the PRBS pulse period selection based on manipulating the frequency power content of the PRBS test signal to a particular frequency band of interest, i.e. methods 3 and 4, performed best overall, returning the most accurate results and proving most efficient for short sequence lengths. However, these methods required significantly longer testing times than the other selection methods tested.

The results of Table 4-10 present a summary of the results obtained in the testing of the typical process models and their associated PI controllers, in closed loop, displayed in Table 4-2.

Table 4-10 - General summary of results of Appendix D section D.3

Typical Process Models & Associated controllers					
Sequence Length	**Method No.**	**% Error Values**	**Average Efficiency**	**Simulation Times**	**Comment on any unusual results**
63 bits	5	Poor	Very Poor	Good	Poor closed loop frequency resolution
	4	Very Good*	Very Good	Very Poor	-
	3	Very Good	Good	Poor	-
	2	Very Poor	Poor	Very Good	Lowest frequency component insufficient, Poor closed loop frequency resolution
	1	Very Good	Mixed	Mixed	-
127 bits	5	Mixed	Very Poor	Good	Poor closed loop frequency resolution
	4	Very Good*	Very Good	Very Poor	-
	3	Very Good	Good	Poor	-
	2	Mixed	Poor	Very Good	Poor closed loop frequency resolution
	1	Very Good	Mixed	Mixed	-
511 bits	5	Very Good	Very Poor	Good	-
	4	Very Good	Poor	Very Poor	-
	3	Very Good*	Mixed	Poor	-
	2	Very Good	Very Good	Very Good	-
	1	Very Good	Good	Mixed	-
1023 bits	5	Very Good	Poor	Good	-
	4	Very Good	Very Poor	Very Poor	-
	3	Very Good*	Mixed	Poor	-
	2	Very Good	Very Good	Very Good	-
	1	Very Good	Good	Mixed	-
* Indicated best performer					

From Table 4-10, it can be seen that selection methods 3 and 4 provided the most accurate results with respect to reducing the percentage error values calculated. Method 4 performed best for sequence lengths below 511 bits, while method 3 performed better for longer sequence lengths. Both of these methods were efficient for shorter sequence lengths but became less efficient as the sequence lengths were increased. Methods 2 and 5 performed relatively poorly for short sequence lengths, due mainly to issues with poor frequency content of the testing signal and poor open and closed loop frequency response resolution. This issue has been discussed in detail in Appendix D. As the sequence length increased above 127 bits, methods 2 and 5 perform better. These results are consistent with the results obtained in the testing of the FOLPD models and the typical process models on their own. It should be noted that, for this particular set of results, selection method 1 consistently performed well with respect to minimising the percentage error value. This was not the case in the previous tests carried out on the FOLPD models and typical process models on their own.

From Table 4-10, it can be seen that PRBS pulse period selection method 4 appears to be the most attractive of all the selection methods tested. Method 4 provides the best results using the shorter PRBS sequence lengths. This is an attractive feature as short sequence length mean shorter overall simulation times.

Table 4-11 illustrates the results obtained in the testing of the six typical process models in closed loop, using PRBS pulse period selection method 4, and a PRBS sequence length of 63 bits.

Table 4-11 - Selection method 4 results for typical process models, refer to Table D-7 Appendix D

Sequence Length	Method No.	Model No.	Open Loop % Error	Closed Loop % Error	Overall % Error	Simulation Times (secs)	Simulation Times (hr,mins,secs)
63 bits	4	1	0.12	0.58	0.70	325.08	00:05:25
		2	0.15	10.92	11.08	676.62	00:11:16
		3	0.35	4.34	4.69	214.20	00:03:34
		4	0.08	3.44	3.52	812.70	00:13:32
		5	0.08	9.22	9.30	456.12	00:07:36
		6	0.03	6.85	6.88	442.26	00:07:22

Comparing Table 4-11 to Table 4-6, it can be seen that a much longer test time is necessary in the testing of the typical process models than that needed in the testing of the FOLPD models. The main reason for this difference in necessary simulation times is that the typical process models are of much higher orders than the FOLPD models and their dynamics are slower (i.e. longer time constants and longer time delays); this leads to the longer test times. Again, it can be seen that selection method 4 is very accurate in estimating the required characteristics, particularly in estimating the open loop frequency domain characteristics i.e. gain margin and phase margin values. Unfortunately, the relatively long test times needed may make the PRBS testing method less attractive in an industrial environment due to a reluctance on the behalf of many industrial control engineers to permit prolonged disturbances to normal plant operation. The fact that the amplitude of the PRBS testing signal can be kept small, due to the excellent noise rejection capabilities of the PRBS evaluation technique, may help overcome this apprehension. This issue will be discussed further in section 0.

Table 4-12 contains the results obtained in the testing of the typical process models and their associated PI controllers, in closed loop, using selection method 4 and a PRBS sequence length of 63 bits.

Table 4-12 - Selection method 4 results for typical process & Controllers, refer to Table D-11 Appendix D

Typical processes & PI Controllers

Sequence Length	Method No.	Model No.	Open Loop % Error	Closed Loop % Error	Overall % Error	Simulation Times (secs)	Simulation Times (hr,min,secs)
63 bits	4	1	0.42	1.50	1.91	337.68	00:05:37
		2	0.30	1.42	1.71	791.28	00:13:11
		3	0.98	0.57	1.55	234.36	00:03:54
		4	0.25	0.62	0.87	694.26	00:11:34
		5	0.36	2.02	2.38	541.80	00:09:01
		6	0.27	0.51	0.78	525.42	00:08:45

Comparing Table 4-12 and Table 4-11, it can be seen that when the PI controllers are introduced into the closed loop, the necessary simulation times does not increase significantly but there is a noticeable improvement in the overall percentage error results (with one exception).

4.2.2.2 Relay Results for Typical Processes:

Table 4-13 provides a summary of the results of the relay tests on the typical process models in closed loop. The full set of results are given in Appendix E, section E.2. As was the case for Table 4-8, Table 4-13 illustrates the number of oscillations necessary to obtain an overall percentage error value below 10%, where possible. The results highlighted in yellow indicate instances where an overall percentage error of below 10% was not achieved. In order to reduce this overall percentage error value further, more than 250 output oscillations would need to be taken. The blue entries illustrate the causes of the high overall percentage error values. As can be seen, it is large errors in the closed loop frequency responses that lead to the large overall percentage error values. Increasing the number of oscillation taken would reduce the closed loop percentage error further.

Table 4-13 - Relay results for typical process models, refer to Table E-3 Appendix E

Typical process models

Model No.	Open loop % error	Closed Loop % error	Overall % error	No. of Output Oscillations Taken	Simulation Times (secs)	Simulation Times (hr,mins,secs)
1	1.28%	8.64%	9.92%	200	2,878	00:47:58
2	1.79%	15.55%	17.34%	250	8,295	02:18:15
3	1.60%	34.46%	36.05%	251	2,410	00:40:10
4	1.46%	5.76%	7.23%	150	4,429	01:13:49
5	1.91%	10.04%	11.95%	251	5,677	01:34:37
6	1.26%	14.17%	15.43%	250	5,600	01:33:20

Comparing the necessary simulation times for the relay based evaluation procedure to those of the PRBS based approach, refer to Table 4-11, it can be seen that the PRBS based approach requires significantly less testing time than that of the relay approach. The large simulation times necessary for the relay evaluation technique make this approach to system evaluation untenable in an industrial environment.

Table 4-14 contains a summary of the results generated in the relay testing of the typical process models and their associated PI controllers in closed loop.

Table 4-14 - Relay results for typical process models & Controllers, refer to Table E-4 Appendix E

Model No.	Open loop % error	Closed Loop % error	Overall % error	No. of Output Oscillations Taken	Simulation Times (secs)	Simulation Times (hr,mins,secs)
1	4.94%	3.05%	8.00%	250	3,987	01:06:27
2	5.27%	3.81%	9.08%	250	8,990	02:29:50
3	5.59%	3.50%	9.09%	251	2,590	00:43:10
4	6.56%	2.81%	9.37%	250	8,312	02:18:32
5	4.98%	3.99%	8.97%	251	6,127	01:42:07
6	5.23%	4.40%	9.63%	200	4,946	01:22:26

Comparing the results of Table 4-14 to those of Table 4-13, it can be seen that after the introduction of the PI controller into the closed loop it was possible to achieve an overall percentage error of below 10% in all cases. In some cases, model 4 in particular, this required a significant increase in the necessary simulation times. However, as already discussed, with respect to the results for the typical process models in closed loop on their own, the simulation times necessary for the relay based approach make it an inappropriate evaluation procedure for testing in an industrial setting.

4.3 Accuracy in The Presence of Noise:

Based on the results obtained in section 4.2, a number of simulations were carried out using the PRBS and relay evaluation techniques in an attempt to assess the abilities of each of the techniques to deal with varying levels of noise. The typical process models and their associated PI controllers (see Table 4-2) were tested in the closed loop configuration of Figure 4-8 and Figure 4-9. For the PRBS testing procedure, PRBS pulse selection method 4 (refer to Table 4-3) was used and the effects of varying the PRBS sequence length was investigated (see Appendix D, section D.4 for the full set of results obtained for this testing procedure). For the relay based evaluation procedure, 250 output oscillations were recorded and processed in order to generate the results, detailed in full in Appendix E section E.4. To investigate the ability of each technique to deal with noise, a band limited white noise source was introduced into the test circuit, as shown in Figure 4-8 and Figure 4-9, and the noise power was varied from 10% to 50% of the output signal power. For each of the tests carried out, the amplitude of the PRBS test signal injected into the closed loop systems was limited to ± 1 volt. Similarly, the output amplitude of the relay was limited to ± 1 volt. The key objective in this test phase was to identify the minimum PRBS pulse amplitude and relay output level necessary to acquire meaningful results.

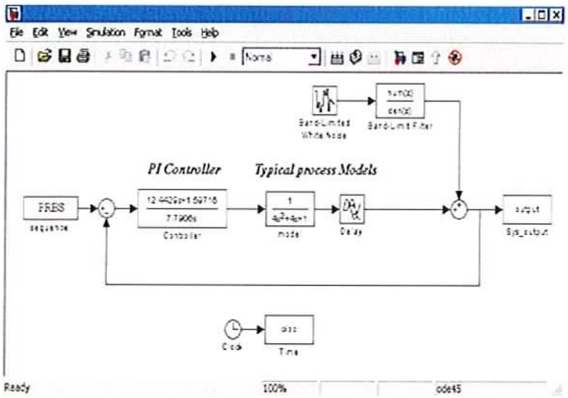

Figure 4-8 - Simulink test set-up for Typical process models and PI controllers in the presence of noise

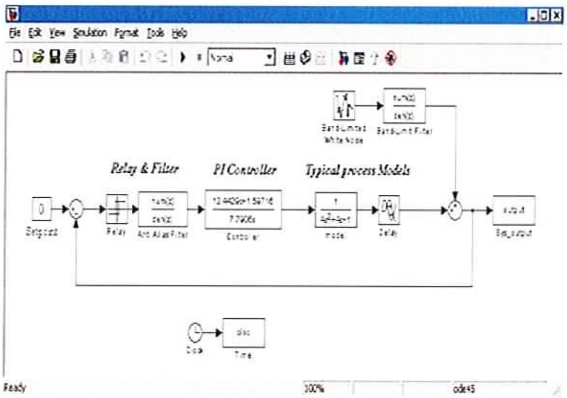

Figure 4-9 - Simulink Relay based test set-up for Typical process models and PI controllers in the presence of noise

The following sections will present a review of the results obtained for both the PRBS and relay based evaluation techniques in the presence of noise.

4.3.1 PRBS Results for Typical Processes & Associated Controllers:

Figure 4-10 to Figure 4-12 illustrate the results obtained using the PRBS evaluation technique for noise levels varying from 10% to 50% of the closed loop output signal power. The figures illustrate the overall absolute percentage error obtained in the estimation of the open and closed loop frequency domain characteristics, versus the noise power levels. A detailed explanation of these figures is given in Appendix D, section D.4.

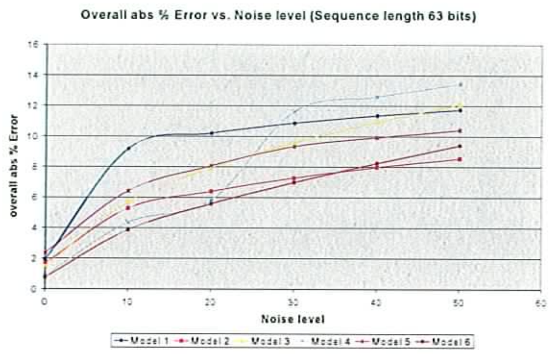

Figure 4-10 - Percentage error for varying noise levels, sequence length of 63 bits

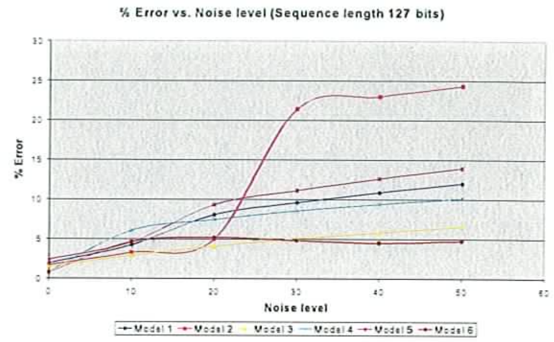

Figure 4-11 - Percentage error for varying noise levels, sequence length of 127 bits

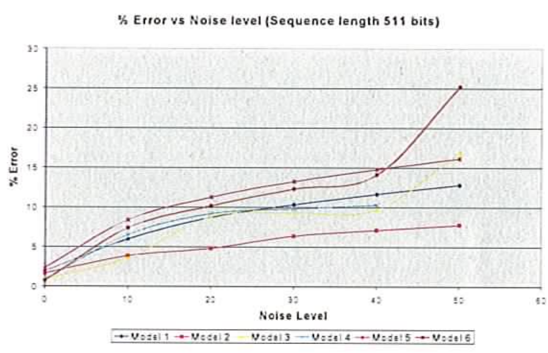

Figure 4-12 - Percentage error for varying noise levels, sequence length of 511 bits

Comparing Figure 4-10 to Figure 4-12, it can be seen that as the PRBS sequence length is increased from 63 bits to 511 bits, the overall percentage error values also increase. This may be due to the fact that increasing the PRBS sequence length, increases the total simulation test time, therefore, providing more opportunity for the low frequency noise to effect the system output and thus the results.

Using the results displayed in Figure 4-10, it can be seen that using a sequence length of 63 bits and PRBS pulse period selection method 4, it is possible to obtained open and closed loop frequency domain characteristic results with an overall percentage error below 15%. This result is an indication of the extremely effective noise rejection capabilities of the PRBS based approach to system evaluation. It is therefore possible to inject a low amplitude PRBS test signal into a controlled plant and obtain accurate estimations of the plants characteristics, with minimal disturbance to the normal operation of the closed loop system.

Table 4-15 illustrates the results obtained using the PRBS evaluation technique for a 50% noise level. It can be seen that even at this extreme noise level the overall percentage error values are still below 15%. The full set of results for the noise tests can be found in Appendix D, Table D-15.

Table 4-15 - PRBS results for 50% noise level, refer to Appendix D Table D-15

Typical processes & PI Controllers (50% Noise)

Sequence Length	Method No.	Model No.	Open Loop % Error	Closed Loop % Error	Overall % Error	Simulation Times (secs)	Simulation Times (hr,mins,secs	Noise Level (%)
63 bits	4	1	3.79	7.96	11.74	337.68	00:05:37	50%
		2	7.29	1.26	8.55	791.28	00:13:11	
		3	6.58	5.52	12.09	234.36	00:03:54	
		4	7.81	5.63	13.44	694.26	00:11:34	
		5	2.47	7.94	10.41	541.80	00:09:01	
		6	6.52	2.88	9.40	525.42	00:08:45	

4.3.2 Relay Results for Typical Processes & Associated Controllers:

Figure 4-13 and Figure 4-14 illustrate the results obtained using the relay evaluation technique for noise levels varying from 10% to 50% of the closed loop output signal power. For the results of Figure 4-13, no hysteresis was employed in the relay and for the results of Figure 4-14, hysteresis of twice the noise band was employed. The full set of results are given in Appendix E, section E.4.

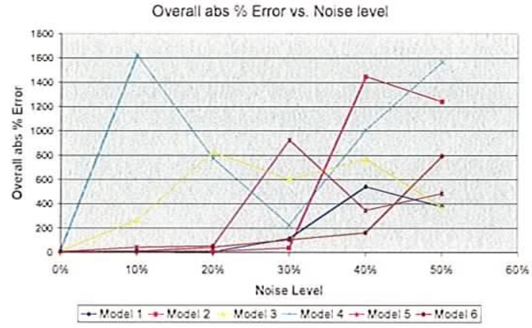

Figure 4-13 - Percentage error for varying noise levels with no hysteresis employed

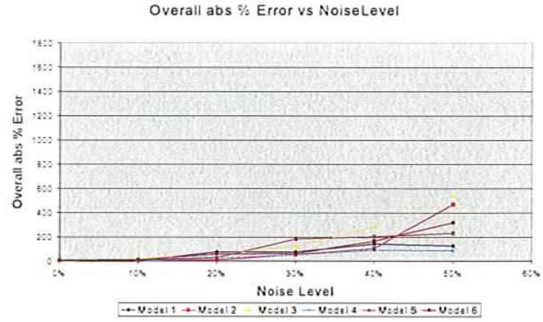

Figure 4-14 - Percentage error for varying noise levels with hysteresis of twice the noise band employed

Comparing Figure 4-13 and Figure 4-14, it can be seen that employing a hysteresis of twice the noise band drastically reduces the effect the noise has on the percentage error values obtained. Table 4-16 contains the results obtained from the relay tests, using hysteresis of twice the noise band, for a noise power level of 50%. Comparing the results of Table 4-16 with those obtained for the PRBS evaluation technique, Table 4-15, it can be seen that even after the application of the hysteresis in the relay, the overall percentage error values obtained for the relay approach are still unacceptably large when compared to those obtained using the PRBS evaluation technique. It can also be seen that employing hysteresis has the effect of increasing the necessary simulation times. Comparing the required relay simulation times to those necessary for the PRBS approach, it can be seen that the relay approach requires an unacceptable testing period; the accuracy of the final results is also much poorer for the relay approach.

Table 4-16 - Relay results for 50% noise level, refer to Appendix E Table E-6

Typical processes & PI Controllers (50% Noise)

Model No.	Open loop % error	Closed Loop % error	Overall % error	No. of Output Oscillations Taken	Simulation Times (secs)	Simulation Times (hr,mins,secs	Noise Level (%)
1	72.41	55.38	127.79	250.00	10782.00	02:59:42	
2	198.36	272.44	470.80	250.00	32157.00	08:55:57	
3	121.46	411.71	533.17	250.00	8442.50	02:20:42	50%
4	51.45	35.73	87.17	250.00	19715.00	05:28:35	
5	26.31	206.79	233.10	250.00	23462.00	06:31:02	
6	64.36	255.21	319.57	250.00	24522.00	06:48:42	

4.4 Validating the Evaluation Strategy:

Once a method of calculating the optimal technique parameters for both the PRBS and relay based evaluation techniques had been identified, the next step was to carry out a number of simulations with the objective of validating the overall assessment strategy developed in Chapter 3. Based on the results of section 4.2, the optimal technique parameters for both the PRBS and relay based assessment techniques were employed in the testing of typical process models 2 and 4, and their associated PI controllers; refer to Table 4-2. Models 2 and 4 were chosen based on their ultimate gain characteristics. Excluding model 5 of Table 4-2 as it does not contain a time delay, model 2 has the smallest ultimate gain specification while model 4 has the largest ultimate gain specification. This was considered justification for choosing these two models in this testing phase. The objective of this testing phase was to incorporate all the stages of the evaluation strategy, i.e. system test, characterise system, generate process model, determine metrics and benchmark generated metrics using the PI(D) database, in order to determine whether or not improved performance could be achieved with the application of the strategy developed.

The evaluation procedure was as follows: typical process models 2 and 4, and their associated PI controllers, were tested using both the PRBS and relay based evaluation procedures, the frequency domain metrics were calculated and a FOLPD model of the process was determined using the frequency response information generated in the testing procedure. The PI(D) databases were then consulted to determine if there were any rules in the databases that would provide similar gain and phase values to the current control system. Tuning rules were then extracted from the database based on their ability to achieve three objectives, namely reduce the closed loop rise time below its current value, reduce the closed loop settling time below its current value and finally, reduce the percentage overshoot below its current value. In order to generate a manageable and representative set of results three tuning rules for each of these objectives were recorded. These recorded tuning rules were then applied to typical process models 2 and 4 in order to determine whether the rules generated by the database actually achieved their stated objectives of minimising these time domain characteristics. If the tuning rules suggested by the PI(D) databases resulted in improved time domain performance where specified, then the evaluation strategy would be validated. The following sections contain a summary of the results contained in Appendix F.

4.4.1 PRBS Based Approach:

Based on the results of section 4.2, PRBS pulse period selection method 4 was used in the testing of typical process models 2 and 4, and their associated PI controllers, in closed loop. To keep the simulation times short, a PRBS sequence length of 63 bits was employed. The full set of results for these tests can be found in Appendix F, section F.1.

From the results of Appendix F, section F.1, it can be seen that when the PI or PID database suggested a tuning rule that would achieve the specified objective, be it minimising rise time, minimising settling time or minimising overshoot, the specified objective was achieved upon application of the suggested rule. Table 4-17 contains a review of some of the results obtained from the testing of typical process model 2 and its associated controller. As can be seen, only results pertaining to the objectives of minimising closed loop settling time and closed loop percentage overshoot are displayed. This is because the database did not contain any tuning rule that would reduce the closed loop rise time below the current value.

Table 4-17 - Strategy validation using PRBS evaluation technique, typical process model 2 PI controller suggestion results

Objective: Minimise Settling time

Tuning rules	Settling Times (secs)
Current Edgar et al. pg52	84.5
Rovira et al. pg31	49.6
Schneider pg34	61.4
Schneider pg34	55.9

Objective: Minimise Overshoot

Tuning rules	% Overshoot
Current Edgar et al. pg52	18
Schneider pg34	0
St Clair pg21	0
Bi et al pg37	0

From Table 4-17, it can be seen that a significant reduction in either the settling time or the percentage overshoot was obtained when the suggested tuning rules were implemented. The current system characteristics are highlighted in yellow in Table 4-17 and the achieved values are listed below the current values. A detailed explanation of the testing protocol, and its associated problems, and the full set of results can be found in Appendix F, section F.1.

4.4.2 Relay Based Approach:

Following from the results of section 4.2, 250 output oscillations were taken and processed in order to obtain the results of the relay based testing of the typical process models 2 and 4, and their associated PI controllers, in closed loop. The full set of results for these tests can be found in Appendix F, section F.2.

From the results of Appendix F, section F.2, it can be seen that when the tuning rules suggested by the PI(D) database were applied to the typical process models (typical process models 2 and 4), the specified objectives were achieved in all but one case. The single inaccurate result was obtained due to a process-model mismatch problem. This model mismatch issue was discussed in connection with both the PRBS and relay based evaluation measures in Appendix F.

Table 4-18 contains an example set of results obtained in the testing of typical process model 2 using the relay-based approach. In a similar manner to the result obtained in the PRBS based testing scheme, significant reductions in either the closed loop settling time or the percentage overshoot were obtained upon application of the tuning rules suggested by the database. Again, no tuning rules could be found in the database that could reduce the closed loop rise time below its current value for this particular example.

Table 4-18 - Strategy validation using relay evaluation technique, typical process model 2 PI controller suggestion results

Objective: Minimise Settling time

Tuning rules	Settling Times (secs)
Current: Edgar et al. pg52	84.5
Rovira et al. pg31	50
Schneider pg34	55.4
Gorecki et al. pg38	49

Objective: Minimise Overshoot

Tuning rules	% Overshoot
Current: Edgar et al. pg52	18
St Clair pg21	0
Schneider pg 34	0
Cluett & Wang pg41	0

4.5 Summary and Conclusion:

Subsequent to the development of the evaluation strategy in Chapter 3, it was recognised that it was necessary to develop a guideline for selecting the technique parameters associated with the PRBS and relay based evaluation protocols. The techniques parameters associated with the two identification schemes were identified in the introduction of this Chapter and a review of the results of an investigation into the optimal methods of selecting these parameters was presented in section 4.2. For the case of the PRBS evaluation technique, from the assessment of five different pulse period selection methods, see Table 4-3, it was determined that a PRBS pulse period selection method developed by Yaacob and Mohamed was the most robust and returned the most accurate simulation results for the systems investigated. For the case of the relay based identification technique, it was determined that it was necessary to obtain at least 250 output oscillations in order to obtain accurate open and closed loop frequency response estimations. Although this number varied depending on the dynamics of the system under test, it was decided that in the majority of cases taking 250 oscillations returned reasonably accurate results. From section 4.2, it was also determined that, with respect to carrying out plant tests in an industrial environment, both the PRBS and relay based testing techniques had serious limitations due to the necessary testing times required. The relay based approach, in particular, was identified as being totally unfeasible due to extremely long test times and, in some cases, only moderately accurate results. The PRBS based technique, on the other hand, required significantly less testing time than that of the relay based approach; however, the test times involved, with respect to the typical process models tested in particular, were still unacceptably long.

In section 4.3, an investigation into the ability of both the PRBS and relay based techniques to deal with varying noise levels was performed. From the results of these simulations, it was determined that the relay based approach struggled to deal with high noise levels, even with the employment of hysteresis. The PRBS based approach, on the other hand, was particularly robust in the presence of high noise levels. From the results of the simulations carried out it was determined that the power of the PRBS test signal should be at least twice the noise power inherent in the system under test. This would mean an apparent doubling of the effects of system noise during the PRBS testing period. This result could be used to justify the long simulation times necessary, from an industrial plant testing stand point, as the disturbance to the plants normal operation could be considered minimal if this low amplitude PRBS test signal was used in the evaluation procedure.

Finally, section 4.4 presented a review of the results obtained from a number of simulations carried out with the objective of validating the overall evaluation strategy developed in Chapter 3. The GUI's developed in Chapter 3 were applied to two of the typical process models, namely models 2 and 4, and their associated PI controllers in closed loop, and the results generated by the PI(D) database were recorded and evaluated. The results generated by the PI(D) database, which are suggested alternative tuning rules, were evaluated based on their ability to achieve certain specified objectives, i.e. minimise closed loop rise time, closed loop settling time and closed loop percentage overshoot. From the results reviewed in this section, it was determined that through the application of the evaluation strategy developed in Chapter 3, it was possible to achieve improved closed loop performance.

Following the completion of the simulation work and analysis of the generated data, a number of issues relating to the work carried out should be highlighted. Firstly, it should be noted that the dynamics of many of the systems tested through the course of the work could be considered similar and not broadly representative of a wide range of industrial processes, which might, for example, have time-constants of the order of minutes or hours, or robotics applications where time constants are in the order of milli-seconds. Prior to conducting the simulations, it was considered reasonable to select a set of test models that would not require an unreasonable amount of simulation time. This meant choosing models with relatively short time constants and time delays thus ruling out models with time constants in the order of hours. It should also be noted that, with respect to the PRBS testing, during the testing phases the sequences length used were limited to the range of 63 bits up to 1023 bits. Again, prior to the commencement of the model testing this was considered a reasonable range. In some instances it may have been beneficial to use a wider range of sequences lengths; however in the majority of cases the range used was adequate. Also, the argument may be made that the closed loop systems tested through the course of the work were unrealistically poor performers and thus may not have provided a fair basis for comparison of the techniques investigated. This is illustrated best by considering the results shown in Table 4-11. From this set of results it can be seen that in identifying the closed-loop frequency response directly, from which the open-loop frequency response is obtained (indirectly), better estimates of the open-loop characteristics are generated than the closed-loop characteristics calculated. One would expect the opposite result. This inaccurate result is due mainly to the shape of the frequency response plots associated with the models tested resulting from the extremely poor performing closed loop. However, as mentioned in section 4.2, one of the key objectives of the testing carried out, in particular with respect to the PRBS based approach, was to identify the most robust method for specifying the technique parameters associated with each of the techniques investigated. Therefore, while the closed loop systems tested can be considered extremely poor performing loops, the fact that it some methods were able to more accurately identify the characteristics of these loops than others verifies the validity of using these closed loop system in this testing phase. Thus while there were limitations with the simulation work carried out, the results obtained can certainly be considered useful in attaining the specified objectives of each of the testing phases.

Therefore, the overall conclusion of this Chapter is that the evaluation strategy developed through the course of the research is valid.

5 Conclusion:

5.1 Conclusions of Research:

This thesis presents, in detail, the development of a software tool to automate the evaluation of the performance and robustness of a closed loop control system.

Resulting from a comprehensive review of a variety of performance and robustness measures and an investigation into currently available assessment software packages, it was decided to group a number of evaluation metrics into five assessment categories namely, time domain, frequency domain, MVC benchmarking, statistical analysis and problem specific assessment measures.

An assessment strategy was then developed incorporating some of these evaluation metrics. The strategy consisted of two main stages, namely an identification stage followed by a subsequent evaluation step. The identification stage involved the application of a test signal and the subsequent manipulation of the system's response in an effort to characterise its dynamic behaviour and develop a process FOLPD model. Subsequent to a review of a variety of identification techniques, two particular methods, namely the PRBS and relay-based approaches, were identified as being suitable for incorporation into the identification stage of the assessment strategy developed. The subsequent evaluation stage involved the processing of the test data and the application of the assessment metrics in order to characterise the system's performance. It was decided to direct the main focus of the evaluation stage on time domain and frequency domain assessment measures, as these quantities are the most widely accepted and easily understood of all the metrics investigated. In addition to the assessment metrics investigated, a PI(D) tuning rule database was developed. This database was developed with the intention of providing a means of benchmarking the current system's performance with a number of possible alternatives. A simple graphical user interface (GUI) was developed in order to facilitate a user-friendly approach to the implementation of the evaluation strategy. The GUI's developed may be found in the accompanying CD, located at the back of this thesis.

It was identified that both the PRBS and relay based identification techniques required user inputs before the system-testing phase could begin. For the case of the PRBS based technique, the user was required to specify a pulse amplitude, a sequence length and also a pulse period. The relay-based approach required the specification of the relay output level, hysteresis width, number of output oscillations to be taken, as well as an estimation of the ultimate period of the system to be tested.

For the case of the PRBS identification technique, based on the results of a comparative study of five different pulse period selection methods, it was determined that a selection method developed by Yaacob and Mohamed [91] was the most robust and returned the most accurate simulation results for the models tested. For the relay based identification technique, it was determined that it was necessary to obtain at least 250 output oscillations in order to obtain accurate estimation of open and closed loop frequency response metrics, although this number varied depending on the dynamics of the system under test. It was also determined that, with respect to carrying out plant tests in an industrial environment, both the PRBS and relay based testing techniques had serious limitations due to the necessary testing times required. The relay based approach, in particular, was identified as being totally unfeasible, and while the PRBS based technique required significantly less testing time than the relay based approach, the test times involved were still unacceptably long for application in an industrial environment. For example, when the necessary test times, in the simulations carried out, are compared to the dynamics of the systems tested, it can be seen that the ratio of time constant to necessary test time is very high. Therefore, if the time constants of the systems tested were in the region of hours as opposed to seconds (as may be the case in many industrial applications) the necessary system test times would be expected to increase accordingly resulting in very long test times and thus lengthy disturbances to normal plant operation.

As a result of an investigation into the ability of both the PRBS and relay based techniques to deal with varying noise levels, it was determined that the relay based approach struggled to deal with high noise levels, even after the employment of hysteresis. In contrast, the PRBS based approach was identified as being particular robust in the presence of high noise levels. From an industrial plant testing perspective, the PRBS's accuracy in the presence of noise could be used to justify the long simulation times necessary, as the disturbance to the plants normal operation could be kept to a minimum if low amplitude PRBS pulses were employed.

As already mentioned, the PRBS based approach to system identification outperformed the relay based approach in both accuracy and necessary test time. Based on a comprehensive analysis of the results obtained during the testing of a variety of process models, it was possible to produce a guideline for determining the technique parameters necessary to develop an effective and robust PRBS test signal. It was determined that the most robust PRBS test signal was produced when the PRBS pulse period Δt was calculated based on the closed loop bandwidth of the system under test using the formula developed by Yaacob and Mohamed [91], i.e. $f_{PRBS} = 3f_{BW} \Rightarrow \Delta t = 1/3f_{BW}$. While it was not possible to specify an upper limit on the necessary amplitude of the PRBS test signal V, as this limit is entirely system dependant (i.e. how large a disturbance is allowable, how much can the system be disturbed before it is driven into a region of non-linearity…etc.) it was possible to determine a lower limit on this amplitude value. It was determined that the power of the PRBS test signal should be at least twice the noise power inherent in the system under test in order to avoid excessive distortion to the frequency response plots obtained. With respect to the necessary PRBS sequence length L, based on the results of the simulations carried out, it was determined that, when using the formula developed by Yaacob and Mohamed, a sequence length of at least 63 bits provided accurate results in the majority of cases. It is fair to say that it would be advantageous to carry out more research in order to develop a more comprehensive means of determining appropriate PRBS sequence lengths based on the dynamics of the system under test. This issue is discussed later on in this section, in relation to the limitations of the research conducted. It was also determined that at least two length (i.e. one repetition) of the PRBS test signal should be applied to the system under test and the data associated with the first sequence should be ignored. Test results showed that a more accurate estimate of the frequency response of the system was obtained when the data associated with the initial test sequence was ignored and only the data generated by the second sequence (i.e. the repetition) was processed.

Based on an analysis of the effectiveness of the overall assessment and evaluation strategy developed, it was determined that through the application of the evaluation strategy, it was possible to achieve improved closed loop performance in simulation.

Therefore, based on the work carried out through the course of the research, it can be seen that the strategy and software tool developed can be successfully applied to a variety of closed loop systems in order to assess their current level of performance and, when possible, suggest improvements. While the tools developed have their limitations, from an industrial testing perspective, the PRBS evaluation GUI in particular, is still a far more attractive evaluation technique when compared to traditional frequency response identification techniques, such as sine wave testing.

As already discussed in the conclusions of Chapter 4, a number of limitations of the research conducted were identified subsequent to the testing phase of the project. Firstly, the argument could be made that the dynamics of the models used in the testing phase were very similar and therefore the results obtained do not adequately reflect a wide enough range of typical processes that would be encountered in the majority of industrial environments. A comparison of a much wider range of typical process models with a significantly more diverse range of dynamics (i.e. time constants ranging from milli-seconds to hours) would have provided a much richer and meaningful set of results. Therefore, it would be appropriate to direct any future testing of this evaluation strategy at comparing process models with significantly diverse dynamics in order to more thoroughly evaluate the limitations of the assessment scheme developed. Secondly, it should be noted that in the majority of the tests carried out, the closed loop system was an extremely poor performing loop. In the case of the PRBS based technique, it may be advantageous the re-evaluate the selection methods investigated using closed loop systems whose characteristics were not so extreme so as to determine whether or not the overall results vary significantly. It would also be beneficial to carry out a more thorough investigation into determining an appropriate means of selecting the PRBS sequence length. For the simulations carried out through the course of this research, the PRBS sequence lengths used were limited to a range between 63 and 1023 bits. A more thorough investigation would involve linking the choice of PRBS sequence length to the dynamics of the system under test. It may also be interesting to compare the tool developed through the course of this research against any of the industrial equivalents mentioned in the literature review detailed in Chapter 2. Unfortunately due to licensing issues it was not possible to carry out this comparison during the course of this project.

5.2 Future Work:

One of the main limitations of the research conducted during the course of this thesis is the lack of real-time testing of the GUI's developed and thus the overall evaluation strategy. Therefore, the first stage of any future work should be to carry out real-time testing of the GUI's developed, to account for all of the limitations discussed in the last paragraph of the previous section. While the simulation work carried out provided useful information regarding a number of aspects associated with the implementation of the techniques developed, it is only through real-time testing that all the issues and limitations associated with the evaluation tool will become apparent.

From the results discussed in Chapter 4 of this thesis, it was determined that the PRBS based system evaluation protocol proved particularly accurate when the correct pulse period selection method was employed. However, even though the necessary testing times associated with this technique were quite small (when compared to traditional frequency domain identification methods, such as sine wave testing, and even the relay based approach evaluated in Chapter 3), it was established that the required test times were unacceptably long for application of this technique in an industrial environment. As discussed in Chapter 3, the most successful PRBS pulse period selection technique involved focusing the power content of the PRBS test signal to within the bandwidth of the system under test. This focusing of power content resulted in relatively large PRBS pulse periods when compared to the alternative PRBS pulse period selection methods investigated.

In [93] a comparison of a number of perturbation signals, for linear system identification in the frequency domain, is discussed. In this paper, two classes of perturbation signals are considered, namely computer-optimised test signals, and pseudo random signals such as the PRBS signal. Computer-optimised test signals are designed to match specified power spectra as closely as possible, while pseudo random signals, on the other hand, have a fixed power spectra. The computer optimised test signals considered include multisine sum of harmonic (SOH) signals, discrete interval binary (DIB) signals and multilevel multiharmonic (MLMH) signals. The pseudo random signals investigated include pseudo random binary (PRB) signals and pseudo random multilevel (PRML) sequences. Figure 5-1 to Figure 5-3 illustrate typical time domain signals and their associated power spectra for different length computer optimised test signals, while Figure 5-4 and Figure 5-5 illustrate typical time domain signals and their associated power spectra for different length pseudo random signals.

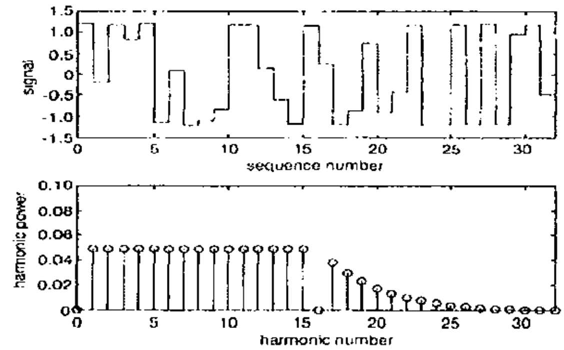

Figure 5-1 - SOH signal with period L = 32 bits, [93]

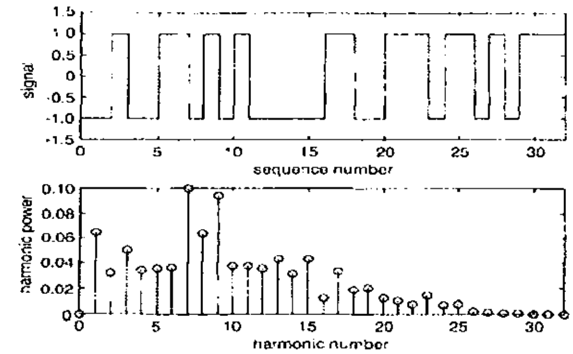

Figure 5-2 - DIB signal with period L= 32 bits, [93]

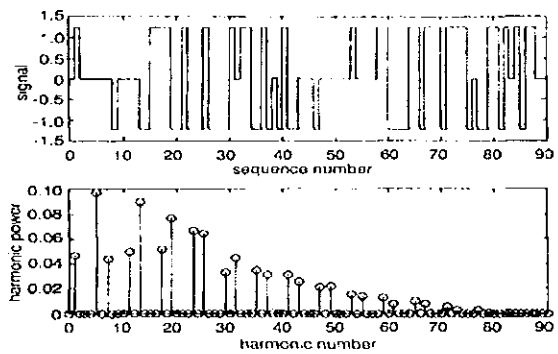

Figure 5-3 - MLMH signal with period L= 90 bits, [93]

From Figure 5-1 to Figure 5-3, it can be seen that while the computer optimised time domain test signals illustrated look similar to those of the PRBS test signal discussed in Chapters 2 and 3, their associated power spectra differ significantly.

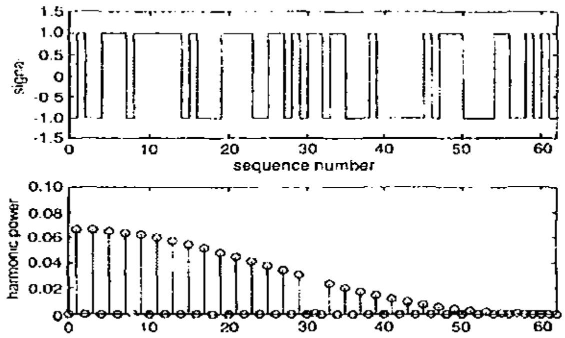

Figure 5-4 - PRB signal with period L= 62 bits, [93]

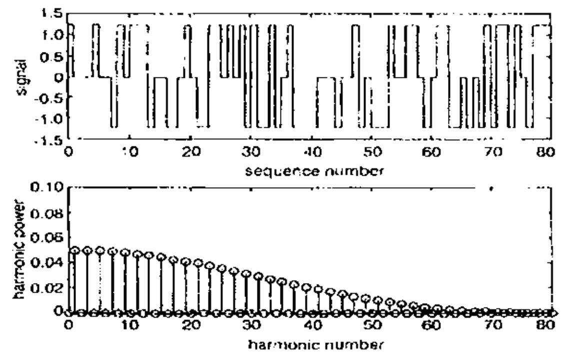

Figure 5-5 - PRML signal with period L= 80 bits, [93]

The PRBS test signal incorporated in the developed evaluation strategy, had a power spectrum similar to that of Figure 5-4. Figure 5-5 illustrates the time domain signal, and associated power spectrum, of a slightly modified version of the PRBS test signal.

Because the power spectra of these test signals differs significantly from the PRBS test signal, it may be possible to focus more of the test signal power to within the bandwidth of the system under test while keeping the pulse period of the time domain signal small. Therefore, it may be advantageous to focus future work on the investigation into the effects of replacing the PRBS test signal incorporated in Chapters 2 and 3, with computer optimised test signals or modified pseudo random test signals in order to determine whether it is possible to obtain the same degree of accuracy with smaller pulse periods, and thus shorter overall testing times.

As mentioned in Chapter 3, in an attempt to keep the research manageable, only a brief review of possible modelling techniques was carried out. It was necessary to develop a FOLPD model of the process under investigation, in order to implement the tuning rules of the PI(D) database developed. However, when evaluating time domain characteristics, such as the rise time and settling time, a more accurate result would be obtained if a higher order model of the process were developed. As discussed in Appendix F, the process-model mismatch, resulting from the modelling of a high order process with an FOLPD model, led to erroneous results being generated by the PI(D) database. Therefore, an investigation into the viability of incorporating a higher order modelling technique into the evaluation strategy would be beneficial.

Finally, the strategy developed through the course of the research involved the application of a test signal and subsequent offline processing of the data generated. It may be beneficial to investigate the possibility of developing an on-line evaluation protocol. If it were possible to process the data effectively, in an ongoing manner, while the system testing was happening, it may be possible to reduce the necessary testing times even further. For example, if it happened that all necessary frequency domain characteristics had been identified after the test was only mid way through, it would be possible to end the test at that stage, thus reducing the total disturbance applied to the system under test.

A: Identification Techniques: PRBS and Relay Based Approaches:

A.1 Introduction To PRBS System Testing:

An introduction to pseudo random binary signals (PRBS) and a comparison to a number of other identification techniques is provided in section 3.3.1.1 of Chapter 3. The following Appendix is intended to provide the reader with a general overview of a literature review style investigation carried out into Pseudorandom binary sequence based system testing.

A.2 Generating the Pseudorandom Binary Sequence:

Pseudorandom Binary Sequences are most commonly generated using feedback shift register circuits [74]. Certain outputs of the shift register are modulo-2 added and fed back into the register. Modulo-2 addition can be defined as follows, $1 \oplus 1 = 0$; $0 \oplus 0 = 0$; $0 \oplus 1 = 1$; $1 \oplus 0 = 1$. Only certain outputs, or taps, can generate what is known as a maximal length sequence [77]. An N-stage shift register can generate a maximal length sequence of $L = (2^N - 1)$ bits. Generator polynomials can be developed and used to describe the characteristics of any particular maximal length sequence. For example, the polynomial $1+x^1+x^4$ describes a sequence of length 2^4-1, generated by modulo-2 adding the outputs from stages 1 and 4 of the shift register circuit, and feeding the result back to the input of first stage (see Figure A-1).

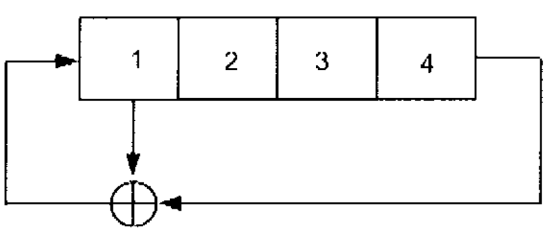

Figure A-1 - Generation of a PRBS signal using 4 shift registers (N=4) [77].

The procedure for PRBS generation can be summarised as follows [77]:

Step 1 Initialise the value of each register at either 1 or 0. Different initialisations lead to different binary sequences, i.e. different sequence orders of 1's and 0's, but autocorrelation functions (see section 3.3.1.1 of Chapter 3 for an explanation of the autocorrelation function) are the same regardless of the initialisation. (At least one of the registers must have an initial value of 1 to avoid the circuit becoming locked in the all zero state).

Step 2 Perform the binary summation between the registers corresponding to the tap associated with the particular sequence that is being generated (i.e. for the example cited the outputs from register 1 and 4 would be modulo-2 added).

Step 3 Shift all the register values to the right by one.

Step 4 Enter the summation result from Step 2 into the first register.

Step 5 Repeat Steps 2 – 4 until 2^N-1 data points are obtained. The data history of any register can be used as the basic sequence corresponding to one period.

A.3 Theory and Properties of PRBS's:

The following sections contains details relating to several unique properties associated with PRBS sequences.

A.3.1 Run Length:

A succession of consecutive l's or 0's within a sequence is called a 'run' and the number of 1's and 0's is the run length. A sequence of length 2^N- 1 contains one run of N '1's, and one run of N-1 '0's (the run of 1's and run of 0's are not necessarily consecutive to each other) [77]. Also, the numbers of 1's and 0's contained within any sequence, differ by only one. For example a sequence of length 31 (= 2^5 - 1) contains sixteen 1's, fifteen 0's, one run of 5 successive 1's and one run of 4 successive 0's.

A.3.2 Spectra of a PRBS Sequence:

The (two-sided) power spectral density of a non-periodic signal is the Fourier transform of its autocorrelation function [75]. For a periodic signal, this Fourier transform is a line spectrum, with values defined only at (cyclic) frequencies f = k/T Hz, where k is an integer and T is the period of the signal. For the case of a PRBS signal, T = LΔt, where L is the sequence length and Δt is the pulse period. Figure A-2 shows the envelope of the normalized power spectral density (PSD) with respect to frequency for a PRBS signal of length 31 bits.

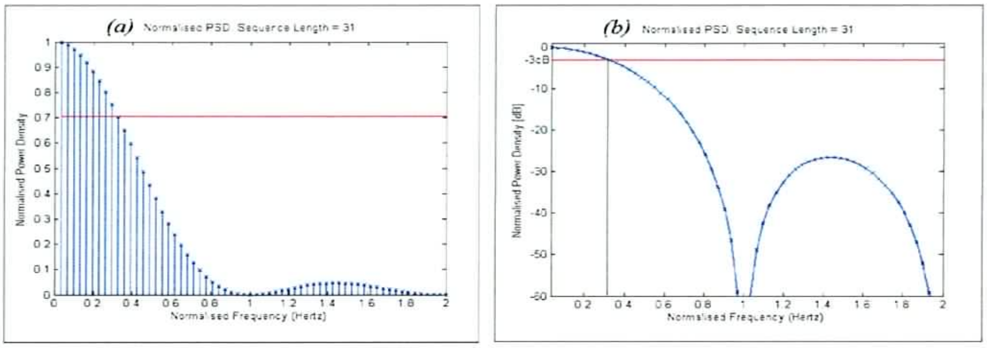

Figure A-2 - (a) Normalized PDS of a PRBS signal, (b) Normalized PDS of a PRBS signal in dBs.

If the power spectrum is measured on a spectrum analyser, values are given for positive frequencies only, and may be calculated using the following formulae [75]:

$$S_{xx}(0) = \frac{V^2}{L^2},$$
$$S_{xx}\left(f = \frac{k}{L\Delta t}\right) = 2V^2 . \frac{L+1}{L^2} . \frac{\sin^2(k\pi / L)}{(k\pi / L)^2}, \quad \text{k=1, 2, 3, ...} \tag{A.3.1}$$

where S_{xx} is the power spectrum of the PRBS signal. From Figure A-2, it can be seen that the power spectrum of a PRBS sequence has a sinc squared envelope and exhibits an upper -3 dB cutoff frequency at one-third of the clock frequency f_c (i.e. for this example $f_c = 1/\Delta t = 1$ Hz) [94]. From 0 to f_c, the spectrums frequency content has a spacing of $1/L\Delta t$ Hz. Thus, the frequency spacing may be reduced by choosing a longer sequence length.

A.4 Choosing PRBS Parameters:

Accurate evaluation requires that the PRBS probing signal excite the process sufficiently near the gain crossover frequency [88]. In the case of linear systems, this effectively means that the signal must span the bandwidth of the system being identified. In other words, the PRBS amplitude V, interval Δt, and length L have to be selected carefully. Presently, there is a surplus of both confusing, and in some cases contradictory, information regarding the methodology for choosing PRBS parameters. The following section attempts to consolidate the various ideas in a concise manner.

A.4.1 Choosing Pulse Amplitude

According to [88], in order not to cause undue disturbance to normal on-line plant operation, which could result in non-linear behaviour, the perturbation of the process must to be limited, for example, to 5% of full span. For open loop evaluation, the static gain K_m of the process has to be known in order to set the perturbation amplitude appropriately, if trial and error is to be avoided.

For closed-loop evaluation, the PRBS amplitude directly controls the magnitude of the output perturbations. However, in some cases (i.e. cases where the initial PID parameters resulted in an underdamped closed-loop system) the value of the amplitude used may have to be half of the allowable magnitude of perturbations, to give tolerance for excessive overshoots. Although it is possible to choose the magnitude of the PRBS to be very low it should chosen with the key aim of achieving output variations larger than the residual noise of the system.

A.4.2 Choosing Pulse Period & Sequence Length

According to [88], the choice of the PRBS bit interval Δt depends on the dynamics of the process. Good accuracy requires that the ultimate frequency $1/T_u$ lies between 1/12 (8.33%) and 1/8 (12.5%) of the PRBS bandwidth $1/\Delta t$. The upper bound on Δt avoids errors associated with the roll-off in PRBS bandwidth. On the other hand, the lower bound ensures that the PRBS provides sufficient excitation around T_u. The PRBS length L determines the frequency resolution of the test signal, for example a sequence length of 63 bits gives a frequency resolution of 1.6% ((1/63)*100) within the PRBS bandwidth. If Δt is chosen such that it lies within the recommended range, the error in T_u is less than 10%.

As discussed in [89], the PRBS period T_{PRBS} (= $L\Delta t$) must be greater than the plant settling time Ts, which is generally taken as the dead time plus 4 to 5 times the longest time constant T_{cmax} of the system. Generally the value of T_{PRBS} should be chosen in the region 1.25 Ts to 1.5 Ts. The bit interval Δt should be less than 25% of the smallest plant time constant T_{cmin}.

In [90], it is purported that in order to test a circuit with cut-off frequency or bandwidth of f_b, f_{PRBS} (= $1/\Delta t$) should be chosen conveniently greater than f_b. It is stated that f_{PRBS} should be equal to at least $5f_b$. For instance, assuming f_b = 10Hz, then $\Delta t \leq 0.02$s.

In [91], it is explained that the choice of PRBS frequency depends on the highest frequency to be identified, and should be set at three times this maximum frequency. This is mainly due to the fact that pseudo-random sequences have a spectral density that is constant up to approximately $0.3f_{PRBS}$. In order to correctly identify the steady state gain of the plant, the duration of at least one of the pulses (e.g. the maximum duration pulse or run length of consecutive 1's, see A.3.1) must be greater than the rise time t_r of the plant [92]. For the case where the sampling period is equal to the pulse period, the maximum duration pulse of a particular sequence is NT_{samp}, where N is the number of shift registers needed to generate the sequence and T_{samp} is the sampling period, therefore the following condition results:

$$N \times T_{samp} > T_r \qquad \text{(A.4.1)}$$

Using (A.4.1), one may determine N and therefore the length of the sequence (i.e. $L = 2^N-1$).

In a large number of practical situations, a submultiple of the sampling frequency is chosen as the clock frequency for the PRBS [92]. This enables an increase in the maximum duration pulse time without having to increase the sequence length. For example if:

$$f_{PRBS} = \frac{f_{samp}}{p}, \quad (\text{i.e. } \Delta t = p \times T_{samp}) \qquad p=1, 2, 3, \ldots \qquad \text{(A.4.2)}$$

then (A.4.1) becomes:

$$p \times N \times T_{samp} > t_r \qquad \text{(A.4.3)}$$

Note that dividing the clock frequency of the PRBS (i.e. reducing f_{prbs}) will reduce the frequency range corresponding to a constant spectral density (see A.3.2). In general, this will not affect the quality of identification, because in many cases when this solution is considered, the plant to be identified is of a low pass nature. However it is recommended to choose $p \leq 4$.

A.5 Introduction To Relay Based System Testing:

Conventionally, in order to identify two or more points on the process frequency response using relay based testing methods, additional linear components (or varying hysteresis width) had to be introduced and additional relay tests needed to be performed

[82]. These methods tended to be time consuming and also the resulting estimation was still approximate in nature. Subsequent modifications to the relay based evaluation techniques have tended to focus primarily on identifying multiple points on the process frequency response from a single relay test [82].

The following sections provide a brief history of the various relay-based techniques that have evolved in recent times in order to review the various advantages and disadvantages associated with each technique. This section will rely heavily on information provided by [82].

A.6 Relay Basics:

A standard 'ideal' relay has only one parameter to resolve; the relay output amplitude μ. A large value for μ is desirable in order to improve the accuracy in the system identification process. However, large signal disturbances tend to make the controlled variable deviate further from its set point or may even force the system into a region of non-linear operation. Therefore, the choice of μ can be considered a trade-off between identification precision and system constraints, and it also depends on the level of measurement noise on the controlled variable. According to [82], the relay output level should be adjusted such that the oscillation amplitude of the process is about three times as large as its noise band. If this adjustment is not possible, μ may be set to 3 – 10 % of the maximum range of the manipulated variable.

Before a relay test is performed, the noise band in the process should be measured. If necessary, the hysteresis width, ε, should be adjusted to avoid spurious switching in the relay output. Typically ε is chosen to be twice the noise band of the system so that reliable stationary oscillations can be produced.

Figure A-3 provides a graphical illustration of the concepts discussed in this section. Consider the case where the initial relay output is at μ_-. As the error (relay input) becomes more positive (moves along the x-axis of Figure A-3 to the right) the relay output will stay at μ_- until a threshold value ε_+ is reached. Once this value is reached the relay output will switch from μ_- to μ_+. After a short period of time the error signal should start to decrease and move along the x-axis of Figure A-3 to the left. The relay output will stay at μ_+ until the threshold ε_- is reached, only then will its output change from μ_+ to μ_-.

A.7 Describing Function:

Consider the case where an ideal relay (no hysteresis i.e. ε_- and ε_+ of Figure A-3 are set to zero) is used and the closed loop system is set up as illustrated in Figure A-4.

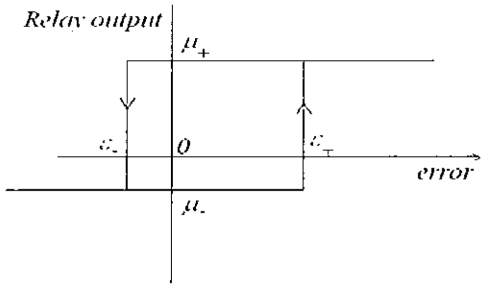

Figure A-3 - General relay function

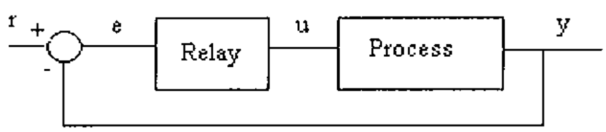

Figure A-4 - Relay feedback system (RFS)

When the controlled variable, y, lags behind the manipulated variable, u, by $-\pi$ radians, the closed loop system output will oscillate with a period P_u [95]. Figure A-5b illustrates the typical manipulated variable/output signals that one would expect to find for a system set up as illustrated in Figure A-4. For a relay of magnitude μ inserted in the feedback loop (r set equal to zero), initially, the manipulated variable u will increase to a value of μ. As the controlled variable y starts to increase (after a time delay D), the relay will switch to $-\mu$ and as a result u will now equal $-\mu$. This process of switching will continue resulting in a limit cycle of period P_u, which is approximately equal to the ultimate period (see Figure A-5).

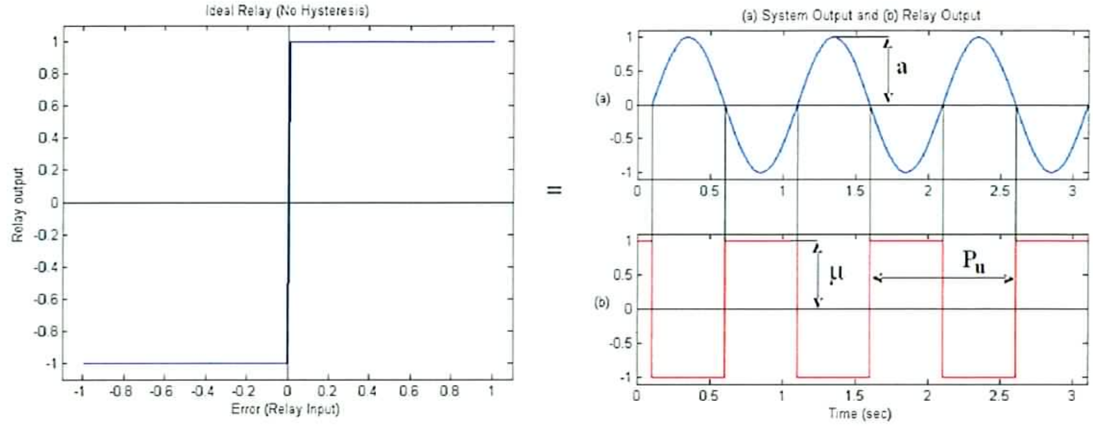

Figure A-5 – Ideal Relay function (Left) and typical system response plots (Right).

Therefore the approximate ultimate frequency of the process can be determined from:

$$\omega_u = \frac{2\pi}{P_u} \tag{A.7.1}$$

Using the Fourier series expansion, the amplitude of the controlled variable a can be considered to be the result of the primary harmonic of the relay output. An approximation of the ultimate gain of the process can be determined from:

$$K_u = \frac{4\mu}{\pi a} \tag{A.7.2}$$

where μ is the height of the relay output (see Figure A-3) and a is the amplitude of oscillation. The above analysis gives an estimation of the process frequency response at one frequency point only. In order to obtain frequency response information at frequencies other than the ultimate frequency the relay test must be repeated and the hysteresis level of the relay must be adjusted. Varying the hysteresis level has the effect of moving the point of intersection of the Nyquist curve of the process with the negative inverse describing function of the relay. Figure A-6 illustrates this concept graphically.

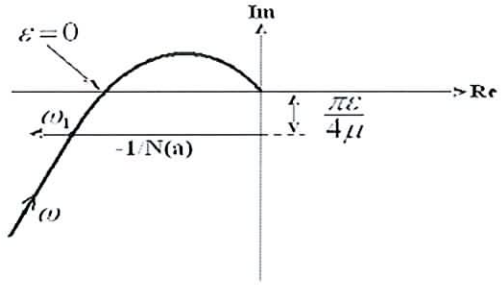

Figure A-6 – Intersection of Process Nyquist Curve and Negative inverse describing function of Hysteretic relay

Figure A-7 illustrates a typical closed loop system output when a relay with hysteresis is used in the circuit described by Figure A-4. It should be noted that the period of oscillation of the relay output is no longer an approximation for the ultimate period of the process. This new period is associated with a new frequency labelled ω_1 in Figure A-6.

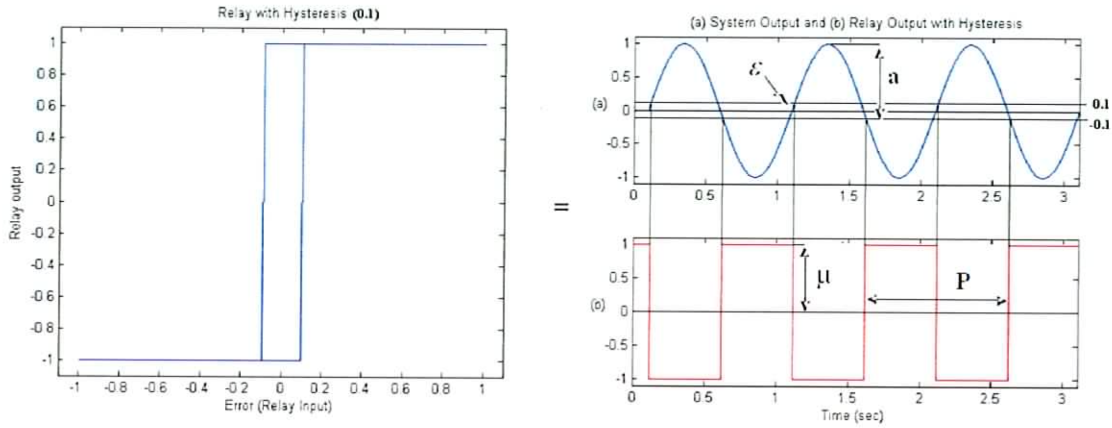

Figure A-7 – Relay with Hysteresis function (Left) and typical system response plots (Right).

Performing multiple relay tests is not only time consuming but it may mean upsetting normal plant operation for an unacceptable period of time. Also, the describing function analysis ignores harmonics beyond the fundamental component of the relay output. As a result only approximate values of ω_u and K_u are determined.

A.8 Parasitic Relay:

As discussed in the previous section A.7, an ideal relay test, can only excite the process at an approximation of its ultimate frequency ω_u, as well as at odd integer multiples of this frequency (i.e. $3\omega_u$, $5\omega_u$...) due mainly to the square wave nature of the relay output. However, due to the low pass nature of most practical processes, the signal to noise ratios at these harmonics are too low to enable meaningful estimation of the process frequency response at these points. Effectively, the only significant information obtained from this test is the processes approximate ultimate point information [82]. The frequency response information between zero and ω_u is very important for an understanding of the process dynamics. In order to obtain more information about this region, a modification to the relay test is required. This modification includes the addition of a 'parasitic' relay. Figure A-8 below illustrates the proposed arrangement for this modified test.

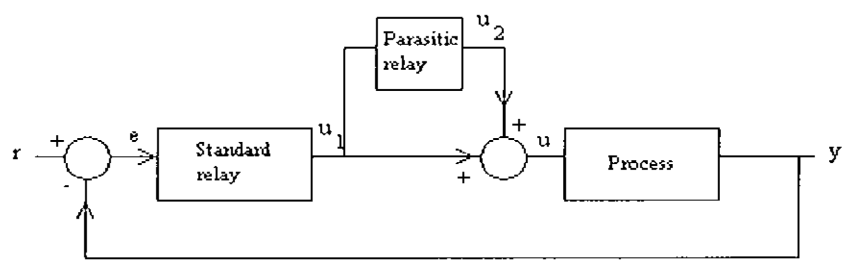

Figure A-8 - Modified relay feedback system

The block diagram of Figure A-8 works as follows. The standard relay operates as usual with its output amplitude being +/- μ_1 (i.e. +/- depending on the relay input). This relay tends to excite the process mainly at frequency ω_u (i.e. an approximation of the processes ultimate frequency). In order to provide the additional excitation, at frequencies other than ω_u, a parasitic relay with output amplitude $\alpha\mu_1$ set to oscillate at twice the period of u_1 is introduced and superimposed on the output of the standard relay. The parasitic relay may be realised using the following identity:

$$\begin{cases} u_2(0) = \alpha\mu_1; & \\ u_2(k) = -\alpha\mu_1.\text{sign}(u_2(k-1)), & \text{if } u_1(k-1) > 0 \text{ and } u_1(k) < 0; \\ u_2(k) = u_2(k-1), & \text{otherwise} \end{cases} \quad (A.8.1)$$

the following table may be used to help explain equation (A.8.1).

Table A-1 -Standard relay and corresponding parasitic relay outputs

k	0	1	2	3	4	5
$u_1(k)$	$1\mu_1$	$-1\ \mu_1$	$1\ \mu_1$	$-1\ \mu_1$	$1\ \mu_1$	$-1\ \mu_1$
$u_2(k)$	$\alpha\ \mu_1$	$-\ \alpha\mu_1$	$-\ \alpha\mu_1$	$\alpha\mu_1$	$\alpha\mu_1$	$-\ \alpha\mu_1$

Figure A-9 illustrates the typical manipulated variable, ($u_1 + u_2$), that can be obtained using this modified relay set-up. It should be noted that the maximum input to the process will be $\mu_1 + \alpha\ \mu_1$. This maximum input level should be carefully selected in order to avoid an unnecessary or unacceptable level of disturbance to normal plant operation during the course of the test.

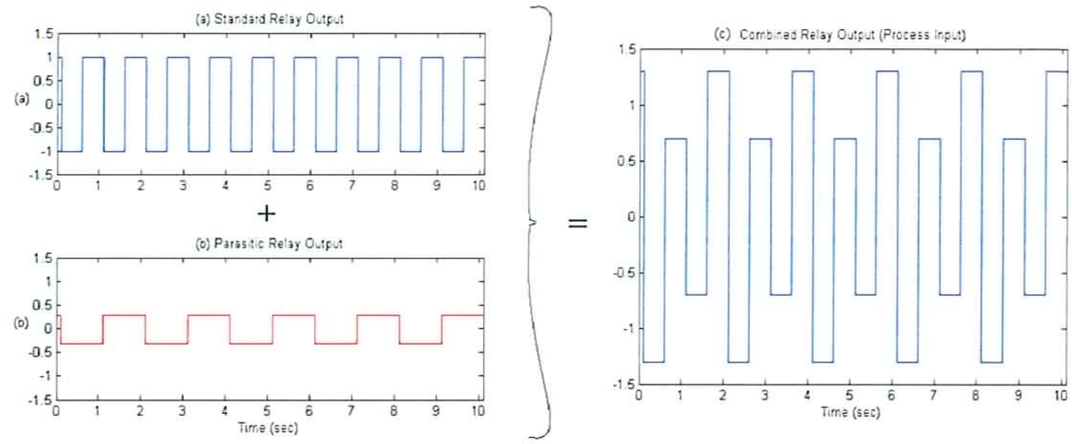

Figure A-9 – (a) standard relay output u_1 in Figure A-8, (b) Parasitic relay output u_2 in Figure A-8 and (c) Combined relay outputs corresponding to the manipulated variable u illustrated in Figure A-8.

The constant α should be large enough to sufficiently stimulate the process; however, it should also be small enough so that the parasitic relay does not change the period of oscillation generated by the standard relay by too much. Using this set-up, the process is stimulated by two different excitations whose frequencies are ω_u (due to the standard relay) and $\omega_u/2$ (due to the parasitic relay). It is also possible to use more than one parasitic relay in a relay test and find more points on the frequency response in one relay test. If y_S and u_s are one period of the controlled variable and manipulated variable respectively, then for a linear process the frequency response may be found using the following equation [82]:

$$G(j\omega_i) = \frac{FFT(y_s)}{FFT(u_s)} \qquad i = 1, 2, 3, ..., \qquad \text{(A.8.2)}$$

where,

$$\omega_i = \frac{(2i-1)2\pi}{2^l T_u} \qquad l=0, 1, 2, ... \qquad \text{(A.8.3)}$$

Here ω_i are the basic and odd harmonic frequencies in u_s and y_S. Since this method uses spectrum analysis instead of the describing function (section A.7), it will lead to accurate process frequency response estimation.

A.9 Cascade Relay:

Using the parasitic set-up of section A.8, it should be noted that the amplitude of the parasitic relay could not be selected freely [82]. As already stated, the amplitude should be large enough to adequately excite the process, but small enough to prevent it from interfering with the period of oscillation caused by the standard relay. Because it is recommended to keep this α value small, the resultant estimation at $0.5\omega_u$ tends to be sensitive to measurement noise. An alternative to the parasitic relay test is the cascade relay feedback test. This test consists of a master relay in the outer loop and a slave relay in the inner loop, as shown in Figure A-10.

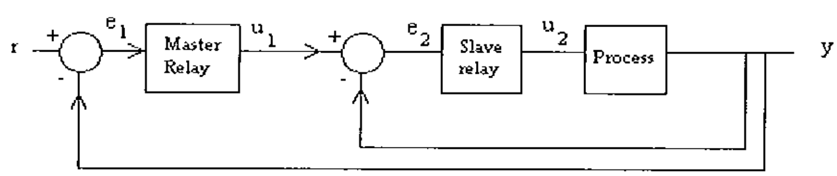

Figure A-10 - Cascade relay feedback system

The slave relay is a standard relay with output amplitude of $+/-d_2$. As discussed earlier this type of relay configuration, in a non-cascaded implementation, will excite the process at the frequency ω_u. In order to provide further excitations to the process at other frequencies, a master relay is introduced into the outer loop. This relay has an output amplitude of d_1 and bias of μ_1. As in the case of the parasitic relay, this master relay will operate at a frequency of $0.5\omega_u$. The master relay can be realised by the following identity:

$$u_1(k) = \begin{cases} -u_1(k\text{-}1) + 2\mu_1, & \text{if } e_1(k\text{-}1) < 0 \text{ and } e_1(k) > 0; \\ u_1(k\text{-}1) - 2d_1, & \text{if } e_1(k\text{-}1) > 0 \text{ and } e_1(k) < 0; \\ u_1(k\text{-}1), & \text{Otherwise.} \end{cases} \qquad (A.9.1)$$

Table A-2 may help explain equation (A.9.1)

Table A-2 – Master relay input e_1 and output u_1, assume initial error is -1 and initial master relay output is $(-2d_1+\mu_1)$ for k=0

k	0	1	2	3	4
e_1	-1	1	-1	1	-1
u_1	$-2d_1 + \mu_1$	$2d_1 + \mu_1$	μ_1	μ_1	$-2d_1 + \mu_1$

An example of typical relay outputs for the cascade relay feedback circuit can be seen in Figure A-11.

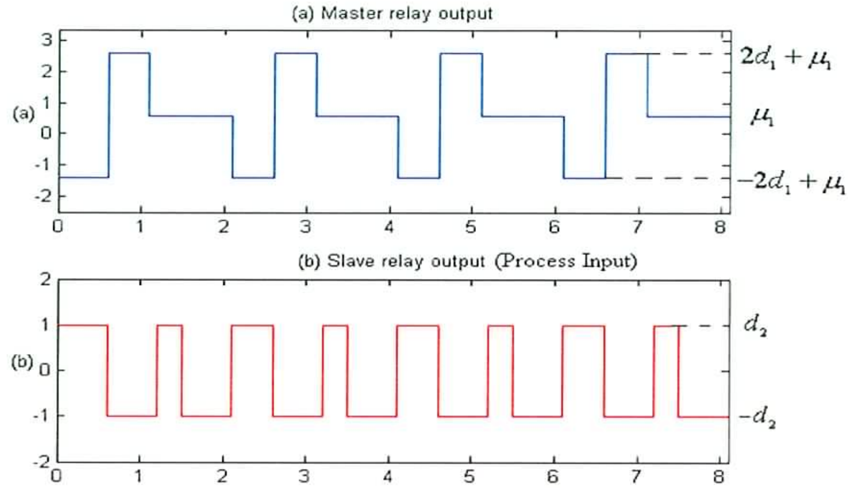

Figure A-11 - Cascade relay feedback system relay output signals

Using this set-up, the process is stimulated by two different excitations whose frequencies are ω_u and $\omega_u/2$. The bias μ_1 is introduced to reduce possibly unnecessary switching due to noise and disturbances (i.e. load noise disturbances will not cause any relay switching unless its amplitude is larger than μ_1). As in the case of the parasitic relay set-up, this method may be extended to obtain more points on the frequency response by using more than one cascade outer loop. Equations (A.8.2) and (A.8.3) may be used to obtain the frequency response from the manipulated variable u_2 and output y in the same manner as the method described in section A.8.

A.10 Decomposition Method:

This method of frequency response estimation involves the decomposition of the manipulated variable and output of a standard relay feedback system (see Figure A-4) into two parts, namely their transient and steady state parts. Figure A-12 gives an example of a typical decomposition of a standard relay feedback system's output. A similar decomposition procedure is carried out on the manipulated variable (i.e. the relay output).

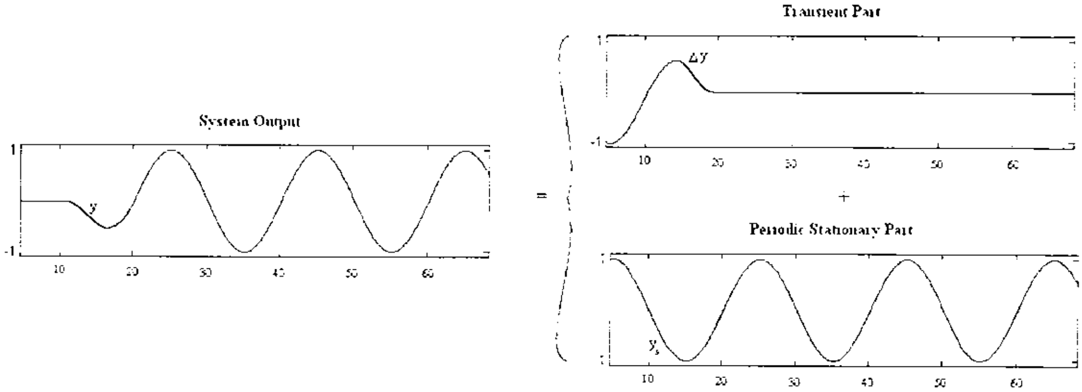

Figure A-12 - Signal Decomposition, transient and steady state

According to [82], applying the FFT algorithm directly to the manipulated variable and output in order to determine the process frequency response yields inaccurate results. In order to verify this point, the author decomposes a typical input/output signal set into its transient and stationary parts. The point is then made that because the transient part of the signals is non-periodic its frequency spectrum is a continuum. This point may be highlighted by considering a non-periodic signal to be a periodic signal with infinite period. As the period of a signal increases, the fundamental frequency decreases and harmonically related components become closer in frequency. Thus, as the period becomes infinite, the frequency components form a continuum [79]. The stationary part of the decomposed signals is, however, periodic in nature. As a result, its Fourier transform is a line spectrum, with values only at (cyclic) frequencies f = k/T Hz, where k is an integer and T is the period of the signal. This concept is illustrated in Figure A-13. As a result since these two spectra have different meanings, summing the two of them together does not have any practical sense and will not produce the correct process frequency response.

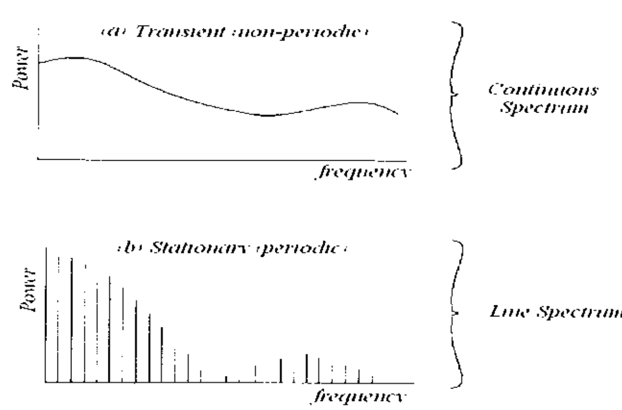

Figure A-13 – (a) Continuous spectra of non-periodic signal, (B) Line spectra of Periodic signal

The following formula is proposed by [82] as a means of avoiding this frequency spectra mismatch. Once the output and input signals have been decomposed into their transient parts, Δy and Δu, and their stationary parts, u_s and y_s, respectively, the following formula can then be applied to these resulting signals in order to determine the process frequency response:

$$G(j\omega) = \frac{\Delta Y(j\omega) + \dfrac{1}{1-e^{-j\omega T_u}} \displaystyle\int_0^{T_u} y_s(t) e^{-j\omega t} dt}{\Delta U(j\omega) + \dfrac{1}{1-e^{-j\omega T_u}} \displaystyle\int_0^{T_u} u_s(t) e^{-j\omega t} dt}, \qquad (A.10.1)$$

Equation (A.10.1) may be interpreted in the following manner. $\Delta Y(j\omega)$ and $\Delta U(j\omega)$ are simply the Fourier transform of the output and input transients Δy and Δu respectively and T_u is the ultimate period of the system. The second term in the above equation involves determining the Laplace transform of the stationary signals and setting $s = j\omega$. This formula allows us to determine the Fourier transform of the periodic signal at frequencies other than the singularity frequencies (the singularity frequencies are those corresponding to k/T Hz as defined earlier). For more information on the theory behind this technique see [82].

A.11 Comparison of Relay Based Evaluation Methods:

The authors in [82] provide a detailed comparison of each of the relay based evaluation methods discussed throughout this appendix with respect to accuracy in determining ultimate point information. A number of different model structures are considered, from first order lag plus delay models (FOLPD) through to more complicated models such as multi-lag high order models with non-minimum phase plus dead time. Also, each technique is considered in the presence of varying levels of noise. The results of these experiments may be summarised as follows; in a comparison between the describing function, parasitic relay and the cascade relay approach, the cascade relay based scheme proved to be the most accurate evaluation technique, especially in the presence of high levels of noise, with the parasitic based approach coming in a close second. A similar comparison was made between the decomposition method and the weighting method (see section 3.3.1.2 of Chapter 3 for details regarding the weighting method). Again using a set of different model structures and varying noise levels, it was determined that the weighting method yielded slightly better estimation results in comparison to the decomposition method.

Therefore, to summarise, while the describing function, parasitic relay and cascade relay approach were relatively uncomplicated in their arrangement and proved to be reasonably accurate in their estimations of ultimate point information, they yielded little or no information regarding a processes frequency response at frequencies other than the ultimate frequency or fractions of this frequency. To overcome this shortfall, two additional techniques were presented, namely the decomposition method and the weighting method. While both these techniques provided accurate ultimate point information, they had the added functionality of also supplying process response information at a variety of additional frequencies. As already stated, when comparing these techniques the weighting method was shown to provide slightly more accurate results.

B: Process Trainer Testing:

The following Appendix details the results obtained in a number of preliminary system identification tests carried out at the beginning of the research project. This Appendix is tangential to the information contained in the main body of the thesis.

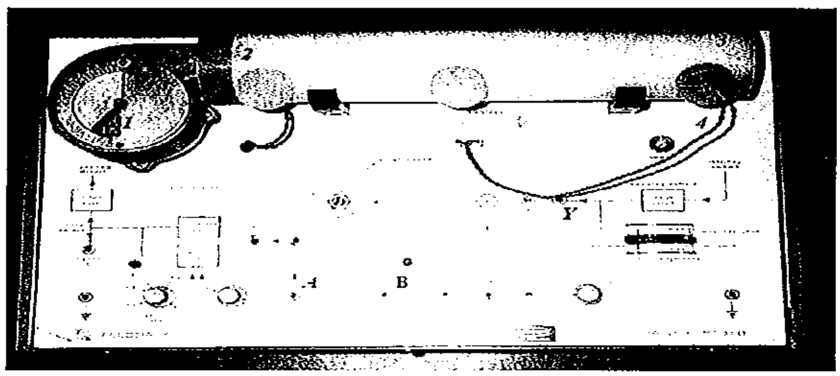

Figure B-1 - Picture of the process Trainer PT326 Serial No. 326/74/5

B.1 Model Estimation: FOLPD Model

The step test on the process trainer was carried out under the following *standing conditions*:

- Throttle control is set to 4.
- Proportional Band (PB) is bypassed
- External control (A & B) is connected and continuous control is on (see Figure B-1 for location of points A & B).
- The thermistor (heat sensing element) is placed at its furthest point from the heating grid (point 3 on Figure B-1).
- The Bridge is then balanced so the set value and measured value both read 30°C

The object was to test the process trainer over the operating range of 30°C to 40°C. The following plots and results were obtained:

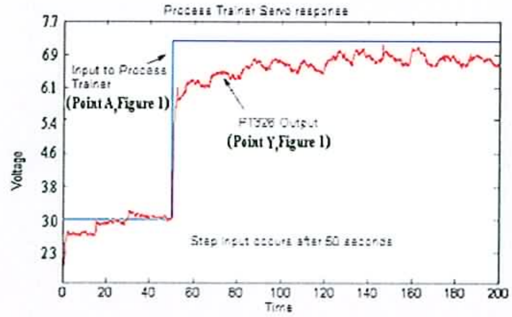

Figure B-2 - Step Test results for the Process Trainer taken over 200 seconds.

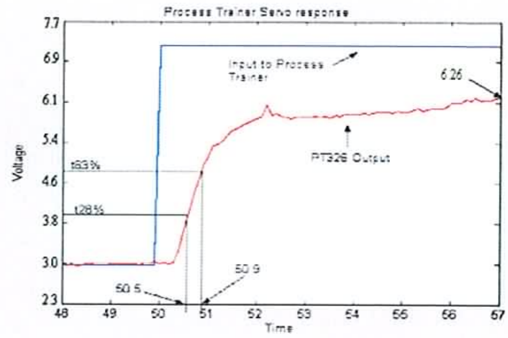

Figure B-3 - Step tests results for the first seven seconds after the step occurred.

The step input to the system occurred after 50 seconds, had an offset of 3.07 V corresponding to the voltage required to bring the system to a temperature of 30°C, and a final value of 7.26 V corresponding to the voltage required to bring the system output to a temperature of 40°C. Therefore the system was tested over the operating range 30 – 40°C.

Using the data in Figure B-3, it is possible to use a number of graphical approaches to modelling the open loop system. It was decided that the two-point method was appropriate as it is a simple and effective means of determining an accurate FOLPD process model. The following formulae were used [96]:

$t_{28\%} = t_d + T_c/3$...eqn (1) $\qquad$ $t_{63\%} = t_d + T_c$...eqn (2)

where $t_{28\%}$ is the time taken to reach 28% of the final change in the process output value, $t_{63\%}$ is the time taken to reach 63% of the final change in the process output value, t_d is the model time delay and T_c is the time constant of the model. From Figure B-3, $t_{28\%}$ is determined by calculating the time at which the system output has a value that is 28% of 3.19 plus the initial value of 3.07V (i.e. 0.28*(6.26V-3.07V) + 3.07V). The value for $t_{63\%}$ is determined in a similar manner.

From Figure B-3 above, $t_{28\%}$ was calculated to be 50.5 – 50 = 0.5 seconds. Also, $t_{63\%}$ was determined to be 50.9 – 50 = 0.9 seconds. Using eqn (1) and (2) given above, t_d was determined to be 0.3 seconds and T_c was calculated to be 0.6 seconds. The value for K_m is determined from the formula:

$$K_m = \frac{Final - initial\ value\ of\ the\ Output}{Final - initial\ value\ of\ the\ Input} = \frac{0.405 - 0.199}{0.47 - 0.199} = 0.76$$

The FOLPD model takes the following form:

$$FOLPD = \frac{K_m e^{-s\tau_m}}{1+sT_m} = \frac{0.76e^{-0.3s}}{1+0.6s} \quad ...(A)$$

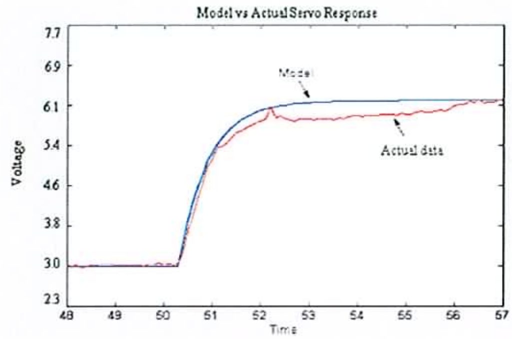

Figure B-4 - Model response versus the actual system response over the time interval that the model was designed for (i.e. first 7 seconds after the step input).

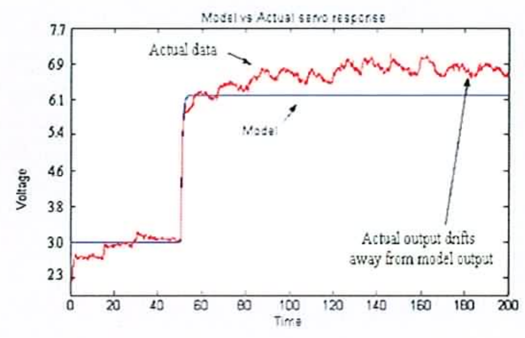

Figure B-5 - model response versus the actual response over 200 seconds.

From Figure B-4, it may be seen that over the interval for which the model was developed, i.e. the first 7 seconds after the step input occurs, the model quite accurately represents the actual response of the system. However, over a longer period of time the actual system output is seen to drift away from the models steady state value. This would seem to suggest that there is an extra element in the process trainer that the model does not take into account. However, as a basic representation of the process, the FOLPD model obtained will be sufficient for obtaining PI controller parameters in the next section.

B.2 Controller Design: PI Controller

Using the FOLPD model a PI controller was designed in order to give the compensated system a Phase Margin (Pm) of 45° and a Gain Margin (Gm) of 2. The PI controller has the following form:

$$PI\ Controller = k_c\left(1+\frac{1}{T_i}\right) = \frac{k_c}{T_i}\frac{(1+sT_i)}{s}$$

In order to obtain a compensated system Pm of 45° and Gm of 2, the controller gain k_c was determined using the following formula [87]:

$$k_c = \frac{aT_m}{k_m \tau_m} = \frac{0.785(T_m)}{k_m \tau_m} = \frac{0.785(0.6)}{0.76(0.3)} = 2.066$$

The time constant T_i of the controller was set equal to the time constant of the model T_m. Therefore, the transfer function for the PI controller was calculated to be:

$$G_c(s) = 2.066\left(1 + \frac{1}{0.6s}\right) = \frac{3.44 + 2.066s}{s}$$

The Simulink parameters were calculated as follows:

$$P = k_c, \qquad I = \frac{k_c}{T_i} = \frac{2.066}{0.6} = 3.44$$

Using the system model $G_m(s)$ and the PI controller $G_c(s)$, designed, it was possible, using Matlab, to calculate the Bode plot of the compensated system. Using the Bode plot, the Pm and Gm of the compensated system may be determined and compared with the design specification, i.e. Gm of 2 and Pm of 45°.

The following m-file was written in order to develop the Bode plot of the compensated system:

```
% PI controller
        num=[2.066 3.44];
        den=[1 0];
        PI=tf(num,den);

% Model Transfer Function
        num1=[0.76];
        den1=[0.6 1];
        proc=tf(num1,den1,'td',0.3);

% Open Loop Transfer Function
        ol=PI*proc;

% Compensated System Bode Plot
        margin(ol);
        bode(ol,'k-')

% Uncompensated System Bode Plot
        margin(proc);
        bode(proc,'k+')
```

M-file 1 – m-file to determine Bode plot of compensated and uncompensated system.

Using M-file 1 the following results were obtained:

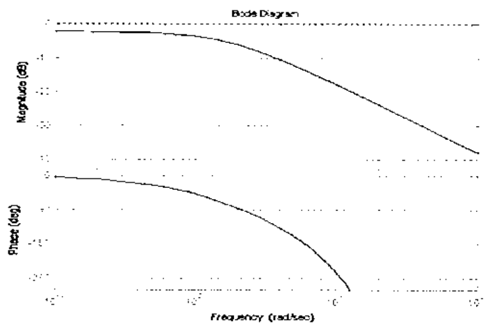

Figure B-6 – Bode plot of the uncompensated system ($G_m(s)$).

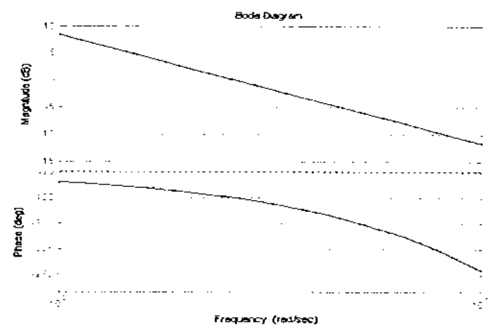

Figure B-7– Bode plot of the compensated system ($G_c(s)G_m(s)$).

Using the margin function in Matlab, the Gm and Pm of the uncompensated system was calculated to be:

Gain Margin = 14 dBs at ω = 6.12 rad/sec, Phase Margin = Inf.

From the Bode plot, Figure B-6, it may be seen that the magnitude of the system never crosses the 0dBs line. This would explain the infinite Phase Margin obtained.

Again, using the margin function in Matlab, the Gm and Pm of the compensated systems was calculated to be:

Gain Margin = 1.9969 at ω = 5.2252 rad/sec,

Phase Margin = 44.9505 at ω = 2.6162 rad/sec

A comparison of the uncompensated system versus the compensated system can be seen in Figure B-8.

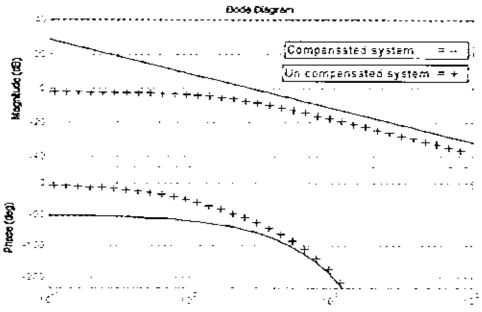

Figure B-8 – Comparison of compensated versus uncompensated system Bode plots

The effects of introducing the PI controller on the magnitude and phase of the open loop system can be seen in Figure B-8.

B.3 Effects of Increasing Pm and Gm

In order to examine the effects of increasing both Pm and Gm, three PI controllers were designed based on the model of the Process Trainer PT326:

$$G_m(s) = \frac{k_m e^{-s\tau}}{1+sT_m} = \frac{0.76e^{-0.3s}}{1+0.6s}$$

The k_c and T_i parameters for each of the PI controllers designed, were determined using the formulae in Table B-1.

Table B-1 - Determining Controller parameters

Gm	Pm	K_c	T_i
1.5	30°	$1.047(T_m)/k_m\tau_m$	T_m
2.0	45°	$0.785(T_m)/k_m\tau_m$	T_m
3.0	60°	$0.524(T_m)/k_m\tau_m$	T_m

Using Table B-1, the following transfer functions were determined for each of the PI controllers:

Table B-2 - PI Controller Transfer Functions for different Pm and Gm

Gm	Pm	PI Controller Transfer Function
1.5	30°	$G_c(s) = k_c\left(1+\frac{1}{sT_i}\right) = 4.59\left(1+\frac{1}{0.6s}\right) = \frac{4.59+2.76s}{s}$
2.0	45°	$G_c(s) = k_c\left(1+\frac{1}{sT_i}\right) = 2.066\left(1+\frac{1}{0.6s}\right) = \frac{3.44+2.066s}{s}$
3.0	60°	$G_c(s) = k_c\left(1+\frac{1}{sT_i}\right) = 1.38\left(1+\frac{1}{0.6s}\right) = \frac{2.3+1.38s}{s}$

Using Simulink and Matlab the performance of the compensated systems, for each of these three PI controllers, was examined in both the time and frequency domain.

B.3.1 Simulation Results:

Using an m-file similar to M-file 1, the following Bode plots were obtained:

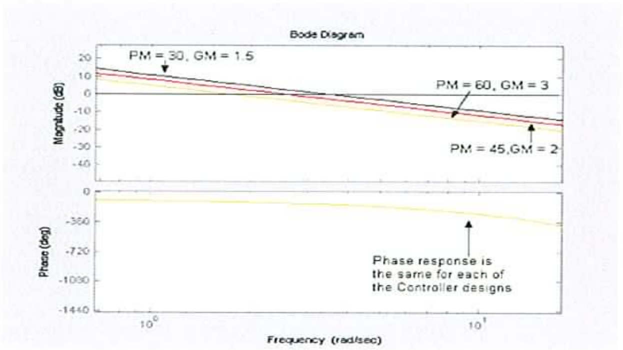

Figure B-9 - Bode plot comparison for each of the controller designs.

The following circuit was set up in Simulink in order to evaluate the closed loop time domain performances for each of the different controller designs:

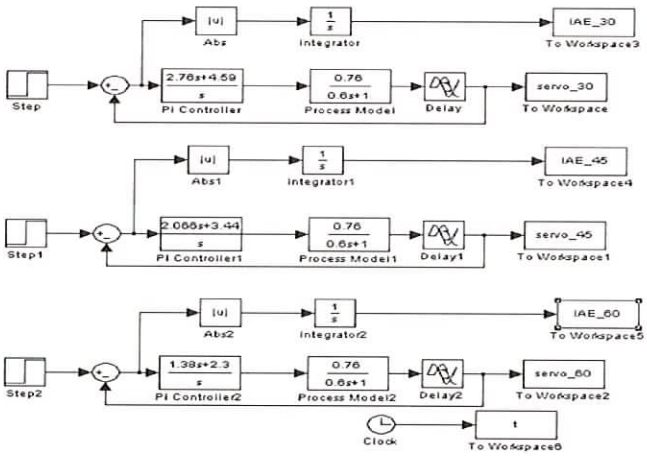

Figure B-10 - Circuitry used to evaluate servo responses of compensated systems.

The following servo response plot was obtained using the Simulink models of Figure B-10:

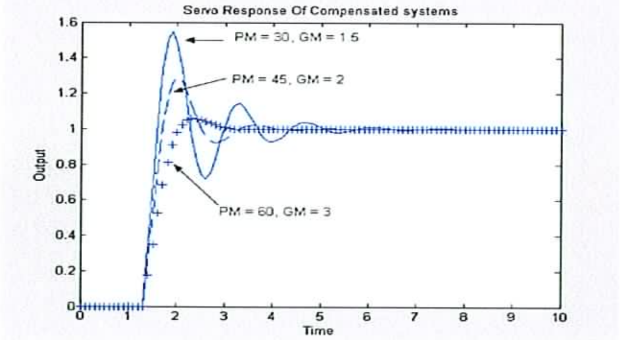

Figure B-11 - Servo responses of compensated systems

Table B-3 - IAE, ISE, ITSE and ITAE values for the compensated systems servo response.

Gm	Pm	IAE	ISE	ITSE	ITAE
1.5	30^o	0.9283	0.5314	0.7683	1.826
2.0	45^o	0.6845	0.4610	0.5932	1.071
3.0	60^o	0.6427	0.4963	0.6312	0.9018

Using a Simulink model similar to that shown in Figure B-10, the following regulatory responses were obtained (note: for the regulatory performance analysis the disturbance was applied between the controller and the process model):

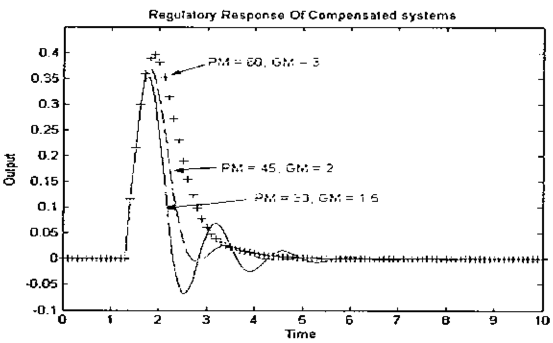

Figure B-12 - Regulatory responses of compensated systems.

Table B-4 - IAE, ISE, ITSE and ITAE values for the compensated system's regulatory response.

Gm	Pm	IAE	ISE	ITSE	ITAE
1.5	30^o	0.2916	0.0604	0.1111	0.65
2.0	45^o	0.2925	0.0754	0.1393	0.5824
3.0	60^o	0.4348	0.1180	0.2357	0.94

B.4 Frequency Response Calculation: Open Loop

After obtaining a first order lag plus delay (FOLPD) model of the Process trainer PT 326 (section B.1) and designing a PI controller to achieve a specific Gain margin (Gm) and Phase margin (Pm) (section B.2), it was decided to carry out a frequency response experiment to evaluate the performance of the control system. Using the controller designed to give a compensated system Pm of 45^o and a Gm of 2, a series of sine waves were applied to the open loop system in order to determine the system's open loop frequency response. The PI controller was implemented in Matlab and the system was set up as shown in Figure B-13.

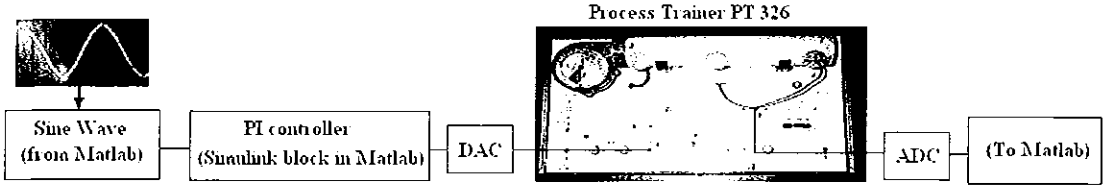

Figure B-13 - Process trainer frequency response testing circuit

The output and input to the process trainer were connected to the AD 512 acquisition card and were stored in the computer so any necessary calculations could be made at a later time.

The procedure for the frequency response test was as follows:

1. The *standing conditions* as discussed previously were set up, with the exception that the bridge was balanced at 34°C as opposed to 30°C. This was necessary because sine wave inputs were used to obtain the frequency response of the system. These inputs cause the system to oscillate around the 'operating point'. If the 'operating point' were set to 30°C, on the minimum peak (or trough) of the input sine wave, the system would be operating outside the conditions for which the model, and subsequently the controller, were designed (see sections B.1 and B.2), i.e. the system would be operating in a region below 30°C. Therefore it was necessary to increase this 'operating point' to 34°C to avoid this happening.

2. The next step was to set up the PI controller and necessary software in Simulink so that the output from the controller could be sent to the process trainer and, similarly, the output from the trainer could be sent to the Matlab workspace to be evaluated.

3. Once steps 1 and 2 had been carried out, the frequency response of the open loop system could be assessed. The frequencies of the sine waves applied to the open loop system ranged from 0.4 rad/sec to 5.5 rad/sec. This frequency range was chosen after evaluating the Matlab simulation Bode plots of the controller design stage (section B.2); see Figure B-7. The main objective in obtaining the open loop frequency response of the control system was to calculate the Pm and Gm of the compensated system and compare them to the design specifications.

Therefore, by inspection of Figure B-7 it may be deduced that the information we are concerned with lies within this frequency range.

4. A series of input sine waves with frequencies ranging from 0.4 – 5.5 rad/sec were then applied to the input of the PI controller and the output of the process trainer was recorded. By calculating the ratio of the FFT of the system input to the FFT of the controlled variable (Process trainer output) for each of the applied sine waves it was possible to develop a Bode plot of the open loop system. Altogether, over the frequency range mentioned, 25 inputs were applied to the system. An m-file was written in order to obtain the Bode plot from this data.

5. Due to the presence of the integrator associated with the PI controller applied to the process trainer some signal processing of the controlled variable was necessary before the system Bode plot could be obtained. It may be seen in Figure B-14 that the process trainer output has a tendency to drift away from its mean value; this is due to the integrating element within the PI controller applied to the process trainer. Therefore the signal processing stage involved de-trending this output signal of the drift term, ensuring the signal has a zero-mean value; see Figure B-14 and Table B-5. The following figures should help to clarify the stages involved in obtaining the Bode plot.

The following results were obtained after processing the data acquired subsequent to the application of a sine wave of frequency 5.5 rad/sec:

- Sine wave input amplitude 0.25 V.
- Sine wave frequency of 5.5 rad/sec.
- Test duration was 200 sec.

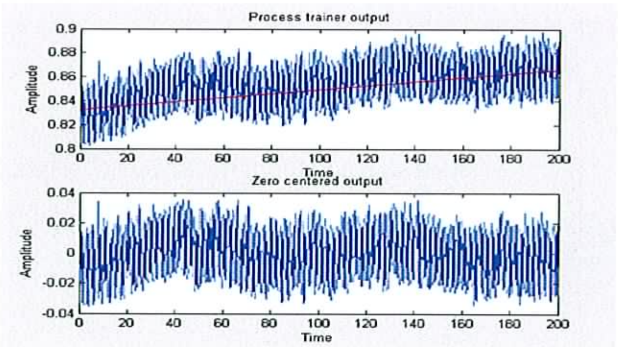

Figure B-14 - process trainer output with non-linearity and adjusted output

Table B-5 - Mean value of trainer output before and after adjustment

Process Trainer Output	Before Adjustment	After adjustment
Mean	0.8544 Volts	7.3072e-017 Volts

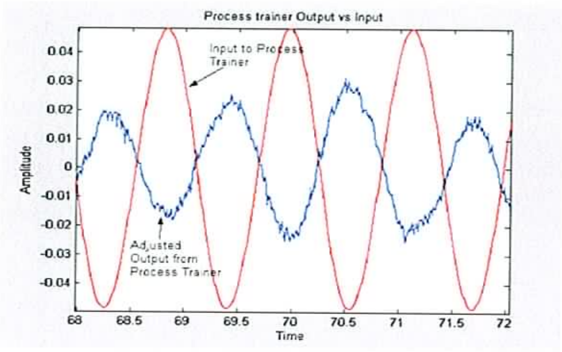

Figure B-15 - Comparison of input vs. output of open loop system.

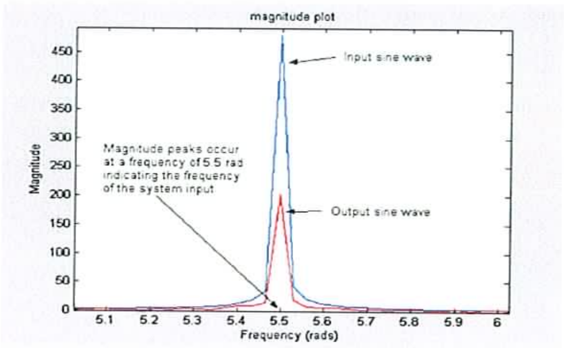

Figure B-16 - Comparison of input signal magnitude and output signal magnitude

Table B-6 - ratio value of output versus input magnitude for an input frequency of 5.5 rad/sec

Input Frequency	Magnitude ratio (Output : Input magnitude)
5.5 rad/sec	-7.4889 dBs

For each of the input sine waves of different frequencies the magnitude ratio is determined through the use of the FFT. Figure B-15 and Figure B-16 show the time domain process trainer input and output signals and their corresponding magnitude plots obtained using the FFT. Table B-6 illustrates the magnitude response results obtained using the information illustrated in Figure B-16. Each magnitude ratio is then plotted against its corresponding input frequency to obtain the magnitude response of the system, see Figure B-17. In all, 25 sine waves were applied to the system with a test time of 200 sec per sine wave, this meant a total experimental test time of 1hr 23 mins.

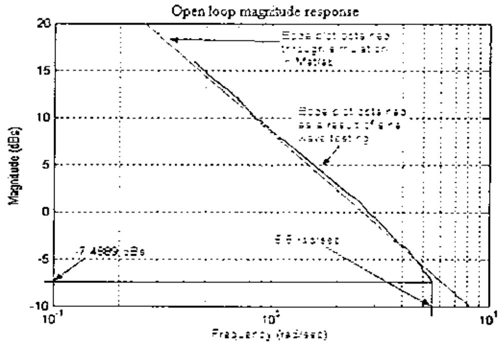

Figure B-17 - Comparison of magnitude response obtained through simulation and magnitude response obtained through sine wave testing

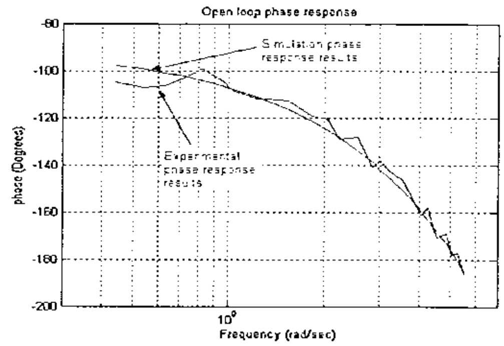

Figure B-18 - Comparison of phase response obtained through simulation and phase response obtained through sine wave testing

Using Figure B-17 and Figure B-18, it is possible to obtain the actual Pm and Gm of the compensated system and compare these values to the design specifications of the system. Figure B-19 illustrates the process involved in obtaining both Pm and Gm.

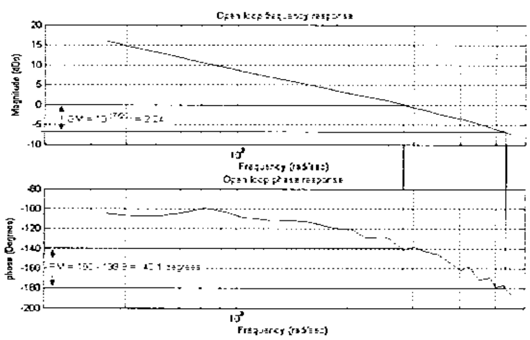

Figure B-19 - Bode plot obtained as a result of sine wave testing

From Figure B-19, it may be seen that the actual Pm of the experimental data was calculated to be 40.1° and the actual Gm was calculated to be 2.24. These calculations do not exactly meet the design specification of a Pm equal to 45° and a Gm of 2, but they are quite close. Discrepancies between the desired Pm and Gm and the actual values may be due to disturbances to the process trainer during the sine wave testing or they may in fact be due to non-linearities associated with the process trainer that were not taken into account during the FOLPD modelling and subsequent controller design stage (see secion B.1 and section B.2).

The m-file used to obtain the Bode plot of Figure B-19 is given below:

```
T=0.01;                    % Sample time
sim_time=200;
N=sim_time*(1/T);
```

```
t=0:T:200;
[b,c]=size(t);                    % b is set to the last sample number i.e. N
l=1:1:c;
OU=25;

% Adjusting the output voltage to centre it on zero volts

output1=outputs*2;                % Doubles the output to make up for the divider circuit
[n,a]=size(output1);
for k=1:1:25
   y1(k)=mean(output1(1:(20/T),k));       % Sets y1 equal to the average output value
                                             obtained for the first 10 seconds
   y2(k)=mean(output1((n-20/T):n,k));     % Sets y2 equal to the average output for
                                             the last 10 seconds of the experiment
end

x1=t(1);                                  % x1 is set to the initial value of t i.e. 0
x2=t(c);                                  % x2 is set equal to the last sample value of t

for k=1:1:25
   m(k)=(y2(k)-y1(k))/(x2-x1);            % Equation to determine the slope of a line
   y(k,l)=m(k)*t+y1(k);
end

for k=1:1:25
   adjust_out(l,k)=output1(:,k)-y(k,:)';   %Creates a matrix containing the 25 outputs
   ff(k)=mean(adjust_out(:,k));
   adjusted_out(l,k)=adjust_out(l,k)-ff(k);
end

for k=1:1:25
   gg(k)=mean(inputs(:,k));
   adjusted_in(l,k)=inputs(l,k)-gg(k);
end

% Calculation of magnitude response of the system

for k=1:1:25
   fft_in(l,k)=fft(adjusted_in(:,k));
   fft_out(l,k)=fft(adjusted_out(:,k));
   mag_in(l,k)=abs(fft_in(l,k));
   mag_out(l,k)=abs(fft_out(l,k));
   ratio(k)=max(mag_out(1:N/2,k))/max(mag_in(1:N/2,k)); % The ratio of max
                                                          output to max input
                                                          magnitude is
                                                          calculated

   ratio_in_dB(k)=20*log10(ratio(k)) ;           %the ratio is converted to decibels
   [pp(k),kk(k)]=max(mag_out(1:N/2,k));
   radians(k)=(kk(k)/N)*(1/T)*2*pi;
```

```
    phase_in(l,k)=angle(fft_in(l,k));
    phase_out(l,k)=angle(fft_out(l,k));
    phase_difference(k)=phase_in(kk(k),k)-phase_out(kk(k),k);
    phase_difference1(k)=(phase_difference(k)*360);
end

magnitude_ratio=ratio_in_dB(OU);
input_frequency=radians(OU);
freq_range=0:1/N:0.5;
rad=freq_range*(1/T)*2*pi;
[a,b]=size(rad);


% Calculation of Phase response of system

T=0.01;
N=200*(1/T);

for k=1:1:25;
   t=1:1:N+1;
   ff(k,t)=fft(inputs(:,k)');
   mag(k,t)=abs(ff(k,t));
   [a1(k),b1(k)]=max(mag(k,1:(N)/2));
   w(k)=(b1(k)/(N))*(1/T)*2*pi;
   period(k)=(2*pi)/w(k);
end


for k=1:1:25;
   t2=1:1:2*(N+1)-1;
   v(:,k)=xcorr(outputs(:,k),inputs(:,k));
   [d1,d2]=size(v);
   [a2(k),b2(k)]=max(v(:,k));
   sample(k)=(N+1)-(b2(k));
   phase_diff(k)=((sample(k)/(1/T))/period(k))*360;

    if phase_diff(k)>0
      phase_diff(k)=phase_diff(k)-360;
   end

end

% Plots magnitude and phase response on the same figure

figure
subplot(2,1,1), semilogx(radians,ratio_in_dB)
grid
xlabel('Frequency (rad/sec)')
ylabel('Magnitude (dBs)')
title('Open loop frequency response')
```

```
axis([0.3,6,-10,20])

subplot(2,1,2), semilogx(w,phase_diff)
grid
xlabel('Frequency (rad/sec)')
ylabel('phase (Degrees)')
title('Open loop phase response')
axis([0.3,6,-200,-80])
```

M-file 2– m-file to determine Bode plot of compensated using sine wave test data.

The m-file given above uses the data from the 25 different frequency inputs and outputs from the sine wave testing.

C: PRBS Feedback Taps Configuration:

The following table contains a list of possible feedback tap configurations that can be used to generate maximal length sequences of length 2^n-1 bits:

Table C-1 - List of possible feedback tap configurations for generating maximal length sequences [97]

n	Generator Polynomial	n	Generator Polynomial
2	[2 1 0]	21	[21 19 0]
3	[3 2 0]	22	[22 21 0]
4	[4 3 0]	23	[23 18 0]
5	[5 3 0]	24	[24 23 22 17 0]
6	[6 5 0]	25	[25 22 0]
7	[7 6 0]	26	[26 25 24 20 0]
8	[8 6 5 4 0]	27	[27 26 25 22 0]
9	[9 5 0]	28	[28 25 0]
10	[10 7 0]	29	[29 27 0]
11	[11 9 0]	30	[30 29 28 7 0]
12	[12 11 8 6 0]	31	[31 28 0]
13	[13 12 10 9 0]	32	[32 31 30 10 0]
14	[14 13 8 4 0]	33	[33 20 0]
15	[15 14 0]	34	[34 15 14 1 0]
16	[16 15 13 4 0]	35	[35 2 0]
17	[17 14 0]	36	[36 11 0]
18	[18 11 0]	37	[37 12 10 2 0]
19	[19 18 17 14 0]	38	[38 6 5 1 0]
20	[20 17 0]	39	[39 8 0]
40	[40 5 4 3 0]	47	[47 14 0]
41	[41 3 0]	48	[48 28 27 1 0]
42	[42 23 22 1 0]	49	[49 9 0]
43	[43 6 4 3 0]	50	[50 4 3 2 0]
44	[44 6 5 2 0]	51	[51 6 3 1 0]
45	[45 4 3 1 0]	52	[52 3 0]
46	[46 21 10 1 0]	53	[53 6 2 1 0]

D: PRBS Results:

D.1 FOLPD Models - Closed Loop:

The following results were obtained during the testing of 10 simulated first order lag plus delay (FOLPD) models listed in Table D-1. For these models, the time constant (T_c) and steady state gain (K_m) was kept constant and the time delay (t_d) was varied from 10% to 100% of T_c.

Table D-1 - Various first order models with varying time delays

Model No.	FOLPD model	Model No.	FOLPD model
1	$\frac{2}{s+1}e^{-0.1s}$	6	$\frac{2}{s+1}e^{-0.6s}$
2	$\frac{2}{s+1}e^{-0.2s}$	7	$\frac{2}{s+1}e^{-0.7s}$
3	$\frac{2}{s+1}e^{-0.3s}$	8	$\frac{2}{s+1}e^{-0.8s}$
4	$\frac{2}{s+1}e^{-0.4s}$	9	$\frac{2}{s+1}e^{-0.9s}$
5	$\frac{2}{s+1}e^{-0.5s}$	10	$\frac{2}{s+1}e^{-1s}$

The FOLPD models were tested in a closed loop configuration as illustrated in the Simulink model of Figure D-1.

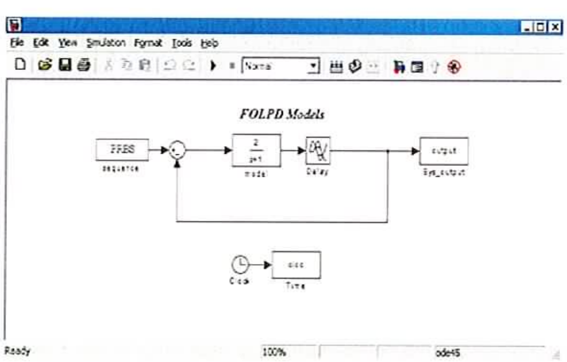

Figure D-1 - Simulink test set-up for FOLPD models

It can be seen in Figure D-1 that the PRBS testing signal was injected into the closed loop system on top of any existing set point. During the simulation the controlled variable and PRBS input were recorded and stored so that they could be processed off-line once the simulation had completed.

The system characteristics evaluated as a result of the simulation were then compared to the closed loop system's actual characteristics and absolute percentage error values were calculated using equation (D.1.1). For each of the simulations detailed in the following Appendix a fixed sampling period of 0.01 seconds was used and the frequency response plots were generated using sampled system input and output response data (refer to Chapter 3 section 3.3.1.1 for a detailed discussion of how frequency response plots were generated).

$$\partial x = abs\left(\frac{x_0 - x}{x}\right) \times 100 \qquad \text{(D.1.1)}$$

where the true value of the characteristic is x and the measured or inferred value is x_0. The characteristics evaluated were gain margin (Gm), phase margin (Pm), closed loop frequency response peak value (L_{cmax}) and closed loop system bandwidth (Bw). A definition and explanation of these terms may be found in Chapter 2, section 2.2.

D.1.1 Sequence Length = 63 bits:

The results in Table D-2 were obtained during the testing of the FOLPD models of Table D-1. For the results illustrated in Table D-2, a PRBS sequence length of 63 bits was used. Five different methods for selecting an appropriate PRBS pulse period were investigated and compared. For more information regarding these methods see Chapter 4. The columns of Table D-2 may be defined as follows:

- *GM est* is the estimate of the gain margin
- *Gm act* is the actual open loop system gain margin
- *PM est* is the estimate of the phase margin
- *Pm act* is the actual open loop system phase margin

- *% Error Gm & Pm* is the sum of the absolute percentage error in the gain margin and phase margin results calculated using equation (D.1.1)

- *Lcmax est* is the estimate of the peak in the closed loop frequency response
- *Lcmax act* is the actual peak value
- *Band est* is the estimate of the closed loop system bandwidth
- *Band act* is the actual closed loop system bandwidth

- *% Error Lcmax & band* is the sum of the absolute percentage error in the Lcmax and bandwidth results calculated using equation (D.1.1)

- *Overall abs % Error* is the sum of *% Error Gm & Pm* and *% Error Lcmax & band*

- t_d/T_c *ratio* is the time delay to time constant ratio of the model being tested

- *period (secs)* is the PRBS pulse period used in each test
- *Sim time (secs)* is the total simulation time necessary to carry out each test

- *% Error second* is the overall percentage error multiplied by the necessary simulation time (*Overall abs % Error* × *Sim time (secs)*)
- *Normalised error sec* is a normalised version of the *% Error second* column (normalised by the largest value in the column *% Error second* for each of the individual methods)

Therefore, each column represents a set of characteristics and each row represents a model tested for each of the five methods. For example, the first 10 rows represent the results obtained in testing models 1-10 using PRBS pulse period selection method 1, the second 10 rows represent the results obtained in testing models 1-10 using PRBS pulse period selection method 2...etc. Unusual results obtained through the course of the testing procedure have been highlighted and will be discussed. Also, a number of plots have been generated in order to assist in the interpretation of the results obtained.

Table D-2 - Results obtained for FOLPD model in closed loop using a sequence length of 63 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	% Error second	Normalised error sec
	18.61	18.25	180.00	110.08	65.50	-4.99	-3.52	4.99	3.90	69.73	135.23	0.10	0.04	5.04	2.79	0.196
Method 1	12.72	12.57	100.78	100.15	1.85	-3.65	-3.52	5.70	5.60	5.42	7.26	0.20	0.07	8.82	0.38	0.027
Hang C.C. and Sin K.K.	9.45	9.38	90.32	90.23	0.84	-2.53	-2.50	5.44	6.14	12.56	13.40	0.30	0.11	13.86	1.38	0.097
	7.25	7.21	80.10	80.31	0.79	-0.18	-0.11	4.99	5.73	80.51	81.30	0.40	0.14	17.64	11.27	0.792
(Tu needed)	5.61	5.59	69.91	70.38	1.07	2.31	2.40	4.69	5.20	13.54	14.61	0.50	0.17	21.42	2.30	0.162
	4.33	4.32	59.84	60.46	1.30	4.55	4.97	4.49	4.73	13.04	14.34	0.60	0.20	25.20	2.61	0.183
	3.29	3.29	49.94	50.54	1.35	7.60	7.66	3.90	4.34	10.90	12.25	0.70	0.23	28.98	2.51	0.176
	2.43	2.43	39.86	40.61	1.96	10.53	10.57	3.84	4.00	4.45	6.42	0.80	0.26	32.76	1.16	0.081
	1.71	1.70	30.53	30.69	1.33	13.91	13.93	3.56	3.72	4.37	5.71	0.90	0.28	35.28	1.22	0.086
	1.11	1.07	17.20	20.76	20.99	17.23	18.17	3.22	3.48	12.74	33.73	1.00	0.31	39.06	3.95	0.277
	18.62	18.25	110.49	110.08	2.42	-3.57	-3.52	3.59	3.90	9.25	11.67	0.10	0.25	31.50	2.31	0.162
Method 2	12.72	12.57	100.31	100.15	1.38	-3.53	-3.52	5.59	5.60	0.58	1.97	0.20	0.25	31.50	0.15	0.010
Gunes R.A.	9.47	9.38	90.15	90.23	1.00	-2.45	-2.50	5.98	6.14	4.13	5.13	0.30	0.25	31.50	1.03	0.073
	7.26	7.21	80.01	80.31	1.05	-0.14	-0.11	5.59	5.73	26.86	27.90	0.40	0.25	31.50	6.71	0.472
(Tmin needed)	5.62	5.59	69.90	70.38	1.21	2.36	2.40	4.79	5.20	9.68	10.89	0.50	0.25	31.50	2.42	0.170
	4.34	4.32	59.84	60.46	1.48	4.95	4.97	4.39	4.73	7.38	8.86	0.60	0.25	31.50	1.85	0.130
	3.30	3.29	49.60	50.54	1.77	7.05	7.66	3.99	4.34	16.10	17.87	0.70	0.25	31.50	4.02	0.283
	2.44	2.43	40.19	40.61	1.44	10.35	10.57	3.59	4.00	12.34	13.78	0.80	0.25	31.50	3.09	0.217
	1.68	1.70	29.68	30.69	4.25	11.15	13.93	3.59	3.72	23.47	27.72	0.90	0.25	31.50	5.87	0.412
	1.17	1.07	17.71	20.76	24.08	17.52	18.17	3.19	3.48	11.88	35.97	1.00	0.25	31.50	2.97	0.209
	18.62	18.25	110.45	110.08	2.39	-3.55	-3.52	3.74	3.90	4.92	7.31	0.10	0.32	40.32	1.58	0.111
Method 3	12.73	12.57	100.30	100.15	1.39	-3.53	-3.52	5.44	5.60	3.25	4.64	0.20	0.22	27.72	0.71	0.050
Marzocca C. and Corsi F.	9.46	9.38	90.20	90.23	0.93	-2.48	-2.50	5.98	6.14	3.14	4.08	0.30	0.20	25.20	0.63	0.044
	7.26	7.21	80.02	80.31	1.07	-0.15	-0.11	5.44	5.73	37.74	38.81	0.40	0.22	27.72	8.30	0.583
(Bw needed)	5.62	5.59	69.89	70.38	1.26	2.27	2.40	4.99	5.20	9.32	10.58	0.50	0.24	30.24	2.24	0.157
	4.34	4.32	59.86	60.46	1.35	4.72	4.97	4.43	4.73	11.36	12.70	0.60	0.27	34.02	3.07	0.215
	3.30	3.29	49.88	50.54	1.73	7.14	7.66	4.13	4.34	11.65	13.38	0.70	0.29	36.54	3.38	0.237
	2.43	2.43	39.77	40.61	2.38	10.13	10.57	3.86	4.00	7.65	10.03	0.80	0.31	39.06	2.37	0.167
	1.69	1.70	29.11	30.69	5.24	12.58	13.93	3.52	3.72	15.06	20.30	0.90	0.34	42.84	5.12	0.360
	1.15	1.07	17.87	20.76	21.12	16.36	18.17	3.32	3.48	14.42	35.54	1.00	0.36	45.36	5.19	0.365
	18.62	18.25	110.39	110.08	2.33	-3.53	-3.52	3.88	3.90	0.88	3.21	0.10	0.54	68.04	0.47	0.033
Method 4	12.73	12.57	100.27	100.15	1.36	-3.53	-3.52	5.39	5.60	3.90	5.26	0.20	0.37	46.62	1.44	0.101
Yaacob S. and	9.47	9.38	90.10	90.23	1.09	-2.46	-2.50	5.87	6.14	6.08	7.18	0.30	0.34	42.84	2.07	0.145
Mohamed F.A.	7.26	7.21	80.00	80.31	1.10	-0.12	-0.11	5.66	5.73	9.23	10.33	0.40	0.37	46.62	3.42	0.240
	5.62	5.59	69.90	70.38	1.28	2.37	2.40	4.99	5.20	5.26	6.54	0.50	0.40	50.40	2.10	0.148
(Bw needed)	4.34	4.32	59.86	60.46	1.50	4.90	4.97	4.53	4.73	5.56	7.05	0.60	0.44	55.44	2.45	0.172
	3.30	3.29	49.86	50.54	1.74	7.57	7.66	4.16	4.34	5.45	7.20	0.70	0.48	60.48	2.62	0.184
	2.44	2.43	39.94	40.61	2.02	10.49	10.57	3.84	4.00	4.82	6.84	0.80	0.52	65.52	2.51	0.176
	1.70	1.70	30.21	30.69	2.01	13.88	13.93	3.56	3.72	4.63	6.64	0.90	0.56	70.56	2.59	0.182
	1.10	1.07	21.43	20.76	6.31	17.98	18.17	3.49	3.48	1.46	7.77	1.00	0.60	75.60	0.87	0.061
	19.19	18.25	111.44	110.08	6.38	-3.67	-3.52	2.17	3.90	48.39	54.77	0.10	0.09	11.59	4.45	0.313
Method 5	12.56	12.57	100.41	100.15	0.32	-3.50	-3.52	3.08	5.60	45.62	45.93	0.20	0.07	8.19	2.97	0.208
Landau I.D.	9.77	9.38	86.88	90.23	7.83	-2.78	-2.50	3.64	6.14	51.75	59.58	0.30	0.06	6.93	2.85	0.200
	7.29	7.21	180.00	80.31	125.25	-0.37	-0.11	3.79	5.73	268.58	393.83	0.40	0.05	6.68	14.23	1.000
(Tr needed)	5.69	5.59	180.00	70.38	157.47	0.95	2.40	3.79	5.20	87.59	245.06	0.50	0.05	6.68	4.64	0.682
	4.43	4.32	53.43	60.46	14.24	0.75	4.97	3.64	4.73	107.94	122.18	0.60	0.06	6.93	5.94	0.872
	2.39	3.29	180.00	50.54	283.36	2.39	7.66	1.85	4.34	126.13	409.49	0.70	0.05	6.80	6.81	1.000
	1.62	2.43	180.00	40.61	376.63	5.11	10.57	1.85	4.00	105.30	481.92	0.80	0.05	6.80	5.69	0.999
	1.27	1.70	180.00	30.69	511.80	7.38	13.93	1.76	3.72	99.81	611.62	0.90	0.06	7.18	5.69	1.000
	1.33	1.07	180.00	20.76	791.45	11.82	18.17	1.79	3.48	83.49	874.94	1.00	0.06	7.06	4.68	0.914

Figure D-2 displays a plot of *Overall abs % error* vs. t_d/T_c *ratio* for each of the PRBS pulse period selection methods. Each of the five selection methods are represented by a different colour on the plot, for example, selection method 5 is represented by a violet line while selection method 1 is represented by a yellow line. From this figure it can be determined that, for each of the FOLPD models tested, selection method 4 returned a consistently low *Overall abs % error* while selection method 5 on the other hand provided consistently poor results. Selection methods 2 and 3 were reasonably accurate with *Overall abs % error* values hovering around the 10% value. Method 1 was reasonably accurate for the FOLPD models with longer time delay values. It should be noted that the y-axis of this plot has a log scale.

Figure D-3 is a plot of *Normalised error sec* vs. t_d/T_c *ratio*. This figure is designed to emphasize the efficiency of each of the selection methods. Methods that are efficient at providing low *Overall abs % error* will have a consistently low *Normalised error sec* value and inefficient methods will generate a large value. Again, it should be noted that the y-axis of this plot has a log scale. From Figure D-3 it can be seen that although selection method 4 required the longest simulation times it was reasonably efficient when compared to the other selection methods. Methods 2 and 3 were relatively similar when it comes to efficiency and method 1 performed well for the models with higher time delay values. Method 5 was the most inefficient for this particular sequence length.

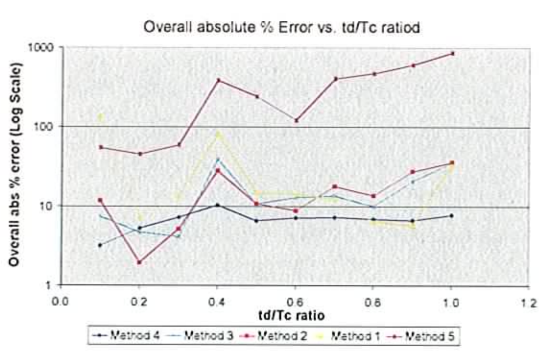

Figure D-2 - Overall percentage error for parameter selection methods

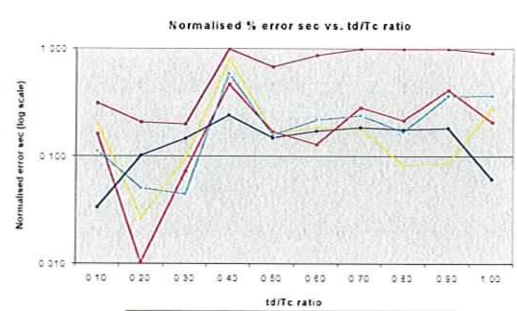

Figure D-3 - Simulation time weighted by (multiplied by) Percentage error for FOLPD models tested, sequence length of 63bits

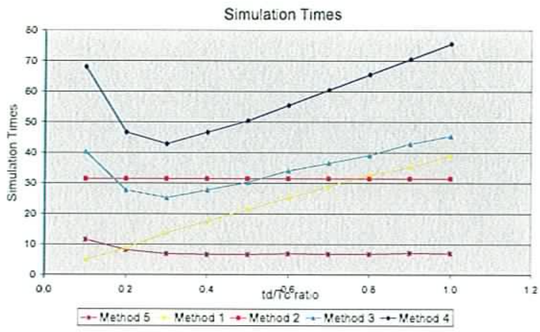

Figure D-4 - Simulation times for parameter selection methods

Figure D-4 is a plot of *Sim time (secs)* vs. t_d/T_c *ratio*. Form this plot it can be seen that the methods that provided the lowest *Overall abs % error* required the longest simulation times.

For example, selection method 4 provided consistently low *Overall abs % error* values; however Figure D-4 shows how this method requires the longest simulation times. Ideally a selection method would be required to provide a low *Overall abs % error* value for a short *Sim time (secs)* value.

From further analysis of the results displayed in Table D-2 there were a number of unusual entries identified. These results were identified as meriting further investigation. These entries have been highlighted in gold, blue and yellow. For the results highlighted in gold, a seemingly uncharacteristically large value for *% Error Gm & Pm* was obtained due to an inaccurate estimation of the system phase margin. For each of these cases the phase margin was inaccurately estimated to be 180 degrees. In investigating the result obtained using selection method 1 for model 1's phase margin, a closer examination of the calculated open loop frequency response (see Figure D-5) showed that for this particular method no 0dB crossing point had been identified.

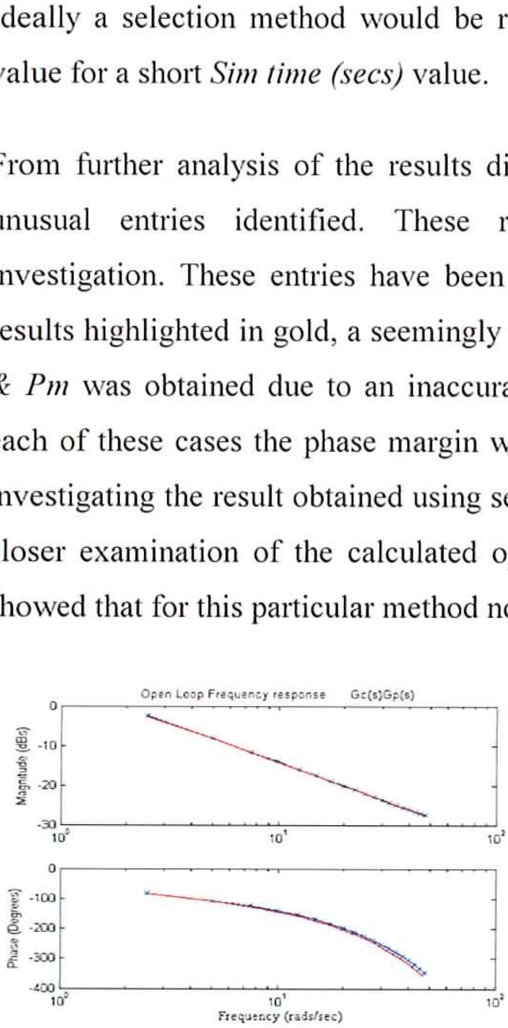

Figure D-5 - Open loop frequency response obtained for FOLPD model 1 using Selection method 1

As can be seen in Figure D-5, the open loop frequency response never crosses the 0db line therefore no estimate of the phase margin could be made. The default response for this situation is to estimate the system's phase margin as 180 degrees.

No crossing point was identified because the lowest frequency component of the PRBS sequence was not small enough to excite the process at the frequency where this 0dB crossing occurs. Lowering the PRBS pulse period or increasing the PRBS sequence length will improve the chances of capturing this 0dB crossing point. However, this will result in an increase of the total experiment time. On further investigation into the unusual results (highlighted in gold) obtained for selection method 5, it was determined that these entries were as a result of a similar error in identifying the 0dB crossing point.

A second set of unusual results were identified and highlighted in yellow in Table D-2. This set of unusual results is associated solely with selection method 5. Very large *% Error Lcmax & band* were recorded as a result of inaccurate estimates of the Lcmax value. On further investigation it was determined that the resolution of the closed loop frequency response plots obtained was not sufficient at the frequencies around the closed loop peak value to determine an accurate estimation of this peak value. Taking model's 5, 6, 7 and 8 and using selection method 5 the following closed loop frequency response plots were obtained.

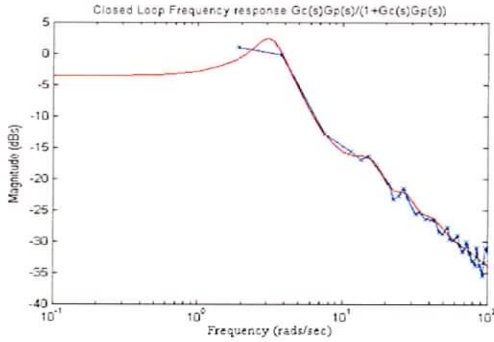

Figure D-6 - Closed loop frequency response plot obtained for model 5 using selection method 5

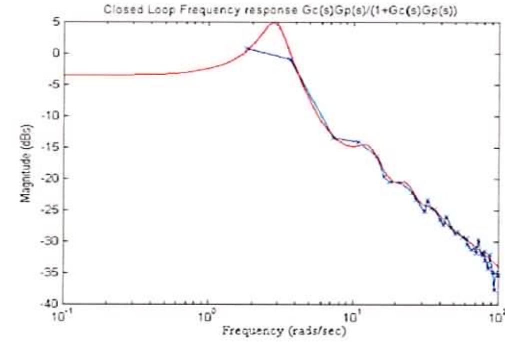

Figure D-7 - Closed loop frequency response plot obtained for model 6 using selection method 5

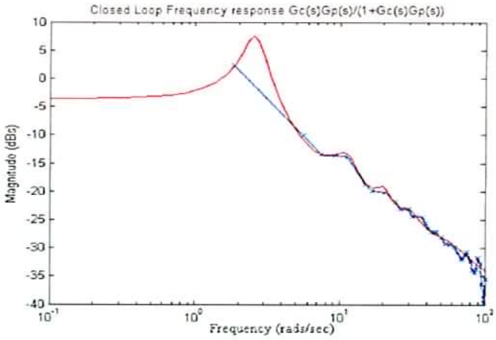

Figure D-8 - Closed loop frequency response plot obtained for model 7 using selection method 5

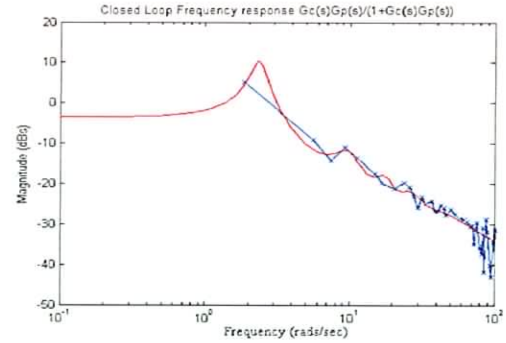

Figure D-9 - Closed loop frequency response plot obtained for model 8 using selection method 5

Figure D-6 to Figure D-9 illustrate the closed loop frequency response results obtained for models 5 through 8 using selection method 5 in blue, compared to the actual results in red. For each of the figures shown, it can be seen that the resolution around the peak is extremely poor. In order to increase the accuracy of the closed loop peak estimation the sequence length would need to be increased thus increasing the simulation time.

A number of entries were highlighted in blue in Table D-2. For this particular set of results, large *% Error Lcmax & band* values determined were not entirely due to inaccuracies using the selection methods. In this case, model 4 has a very small peak in its closed loop frequency response, therefore even slight inaccuracies in its estimation lead to large percentage errors. The narrow, and often negative, closed loop peak values resulted from the fact that the processes simulated were put under proportional control, with a controller gain of 1, which resulted in an overall closed loop gain of below 1. For this particular case, a more suitable measure of accuracy might be the absolute difference as opposed to the percentage error. However, for this table and all subsequent tables, the percentage error will be used as a measure of precision. It should be noted that no scientific formula or specific cut off point was used to identify the table entries highlighted in blue. These highlighted entries are intended to be representative only of the issue arising as a result of using percentage error as a performance defining criteria and thus are entirely opinion oriented.

Finally, it should be noted that for each of the selection methods tested, the *Overall abs % error* is more heavily weighted by the *% Error Lcmax & band* as opposed to the *% Error Gm & Pm*. For each of the selection methods and for each of the models tested the *% Error Gm & Pm* is consistently lower than the *% Error Lcmax & band*. This may be due to the nature of the closed-loop versus open-loop response. The open-loop is relatively well behaved in that it is gradually decaying. In contrast, the closed-loop is rapidly changing, increasing to the peak and then decreasing. Because it is changing so quickly (10dB's over two rad in Figure D-8) it is harder to pick out the peak value.

D.1.2 Sequence Length = 127 bits:

The following results were obtained in a similar manner to those of section D.1.1 using a PRBS sequence length of 127 bits. See section D.1.1 for a detailed explanation of the contents of Table D-3 and Figure D-10 to Figure D-12.

Table D-3 - Results obtained for FOLPD model in closed loop using a sequence length of 127 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	% Error second	Normalised error sec
	18.62	18.25	111.54	110.08	3.35	-3.93	-3.52	3.71	3.90	16.53	19.88	0.10	0.04	10.16	0.66	0.008
Method 1	12.72	12.57	100.54	100.15	1.61	-3.55	-3.52	4.95	5.60	12.54	14.15	0.20	0.07	17.78	0.88	0.011
Hang C.C. and Sin K.K.	9.47	9.38	90.14	90.23	1.04	-2.45	-2.50	5.85	6.14	6.25	7.29	0.30	0.11	27.94	0.69	0.009
	7.26	7.21	79.99	80.31	1.09	-0.14	-0.11	5.65	5.73	30.61	31.70	0.40	0.14	35.56	4.29	0.053
(Tu needed)	5.62	5.59	69.90	70.38	1.25	2.35	2.40	4.95	5.20	7.04	8.29	0.50	0.17	43.18	1.20	0.015
	4.34	4.32	59.86	60.45	1.45	4.90	4.97	4.70	4.73	1.99	3.46	0.60	0.20	50.80	0.40	0.005
	3.30	3.29	49.87	50.54	1.73	7.64	7.66	4.30	4.34	1.14	2.87	0.70	0.23	58.42	0.26	0.003
	2.44	2.43	39.94	40.61	2.03	10.39	10.57	3.81	4.00	6.52	8.55	0.80	0.26	66.04	1.70	0.021
	1.70	1.70	30.03	30.69	2.41	13.75	13.93	3.53	3.72	6.27	8.68	0.90	0.28	71.12	1.76	0.022
	1.07	1.07	20.30	20.76	2.45	15.84	18.17	3.35	3.45	16.50	18.98	1.00	0.31	78.74	5.12	0.064
	18.62	18.25	110.38	110.08	2.32	-3.53	-3.52	3.76	3.90	3.95	6.28	0.10	0.25	63.50	0.99	0.012
Method 2	12.73	12.57	100.24	100.15	1.33	-3.52	-3.52	5.54	5.60	1.17	2.50	0.20	0.25	63.50	0.29	0.004
Guinee R.A.	9.47	9.38	90.10	90.23	1.08	-2.45	-2.50	6.13	6.14	1.62	2.70	0.30	0.25	63.50	0.40	0.005
	7.26	7.21	79.95	80.31	1.13	-0.10	-0.11	5.54	5.73	12.10	13.22	0.40	0.25	63.50	3.02	0.038
(Tmin needed)	5.62	5.59	69.90	70.38	1.28	2.38	2.40	5.15	5.20	1.86	3.13	0.50	0.25	63.50	0.46	0.006
	4.34	4.32	59.85	60.46	1.50	4.97	4.97	4.55	4.73	3.84	5.34	0.60	0.25	63.50	0.96	0.012
	3.30	3.29	49.87	50.54	1.75	7.65	7.66	4.16	4.34	4.36	6.12	0.70	0.25	63.50	1.09	0.014
	2.43	2.43	39.95	40.61	1.96	10.49	10.57	3.96	4.00	1.84	3.79	0.80	0.25	63.50	0.46	0.006
	1.70	1.70	30.13	30.69	2.13	13.84	13.93	3.56	3.72	4.93	7.05	0.90	0.25	63.50	1.23	0.015
	1.09	1.07	20.45	20.76	3.20	18.05	18.17	3.36	3.45	4.00	7.20	1.00	0.25	63.50	1.00	0.012
	18.62	18.25	110.38	110.08	2.32	-3.53	-3.52	3.87	3.90	1.14	3.45	0.10	0.32	81.28	0.36	0.005
Method 3	12.73	12.57	100.25	100.15	1.34	-3.52	-3.52	5.40	5.60	3.76	5.10	0.20	0.22	55.88	0.83	0.010
Marzocca C. and Corsi F.	9.47	9.38	90.10	90.23	1.08	-2.45	-2.50	5.94	6.14	4.93	6.01	0.30	0.20	50.80	0.99	0.012
	7.26	7.21	79.99	80.31	1.13	-0.10	-0.11	5.62	5.73	9.96	11.09	0.40	0.22	55.88	2.19	0.027
(Bw needed)	5.62	5.59	69.90	70.38	1.29	2.41	2.40	5.15	5.20	1.38	2.67	0.50	0.24	60.96	0.33	0.004
	4.34	4.32	59.86	60.45	1.51	4.94	4.97	4.58	4.73	3.66	5.17	0.60	0.27	68.58	0.99	0.012
	3.30	3.29	49.87	50.54	1.77	7.66	7.66	4.27	4.34	1.73	3.50	0.70	0.29	73.66	0.50	0.006
	2.44	2.43	39.95	40.61	1.98	10.37	10.57	3.99	4.00	2.12	4.10	0.80	0.31	78.74	0.66	0.008
	1.70	1.70	30.10	30.69	2.26	13.79	13.93	3.64	3.72	3.19	5.44	0.90	0.34	86.36	1.08	0.013
	1.07	1.07	20.26	20.76	2.63	16.39	18.17	3.44	3.45	11.09	13.72	1.00	0.36	91.44	3.99	0.050
	18.62	18.25	110.37	110.08	2.31	-3.52	-3.52	3.85	3.90	1.45	3.77	0.10	0.54	137.16	0.78	0.010
Method 4	12.73	12.57	100.23	100.15	1.32	-3.52	-3.52	5.45	5.60	2.19	3.51	0.20	0.37	93.98	0.81	0.010
Yaacob S. and	9.47	9.38	90.09	90.23	1.09	-2.45	-2.50	6.11	6.14	2.07	3.16	0.30	0.34	86.36	0.70	0.009
Mohamed F.A.	7.26	7.21	79.98	80.31	1.14	-0.10	-0.11	5.62	5.73	11.50	12.64	0.40	0.37	93.98	4.25	0.053
	5.62	5.59	69.90	70.38	1.30	2.41	2.40	5.19	5.20	0.58	1.89	0.50	0.40	101.60	0.23	0.003
(Bw needed)	4.34	4.32	59.86	60.45	1.52	4.98	4.97	4.72	4.73	0.34	1.86	0.60	0.44	111.76	0.15	0.002
	3.30	3.29	49.88	50.54	1.76	7.65	7.66	4.33	4.34	0.44	2.20	0.70	0.48	121.92	0.21	0.003
	2.44	2.43	39.95	40.61	2.03	10.47	10.57	4.00	4.00	1.06	3.10	0.80	0.52	132.08	0.55	0.007
	1.70	1.70	30.09	30.69	2.32	13.76	13.93	3.71	3.72	1.51	3.83	0.90	0.56	142.24	0.85	0.011
	1.07	1.07	20.30	20.76	2.55	18.02	18.17	3.46	3.48	1.29	3.83	1.00	0.60	152.40	0.77	0.010
	18.69	18.25	100.16	110.08	11.43	-2.39	-3.52	3.14	3.90	51.78	63.22	0.10	0.08	20.07	4.09	0.051
Method 5	12.41	12.57	86.53	100.15	14.91	-1.10	-3.52	4.42	5.60	89.72	104.64	0.20	0.06	14.22	5.02	0.062
Landau I.D.	9.07	9.38	71.17	90.23	24.40	0.12	-2.50	5.28	6.14	118.63	143.03	0.30	0.05	11.94	5.58	0.069
	6.10	7.21	63.84	80.31	35.86	1.76	-0.11	3.23	5.73	1747.85	1783.71	0.40	0.05	11.68	80.40	1.000
(Tr needed)	4.63	5.59	52.80	70.38	42.21	4.17	2.40	3.23	5.20	111.73	153.93	0.50	0.05	11.68	5.14	0.670
	3.35	4.32	40.47	60.45	55.41	6.26	4.97	4.22	4.73	36.69	92.10	0.60	0.05	11.94	1.72	0.225
	1.99	3.29	27.10	50.54	85.69	5.03	7.66	3.17	4.34	61.37	147.06	0.70	0.05	11.94	2.88	0.376
	2.13	2.43	27.09	40.61	45.65	9.59	10.57	3.23	4.00	28.44	74.09	0.80	0.05	11.68	1.31	0.171
	1.29	1.70	36.53	30.69	43.27	-1.67	13.93	3.04	3.72	130.36	173.62	0.90	0.05	12.45	6.39	0.833
	0.17	1.07	1.15	20.76	178.93	1.92	18.17	1.03	3.45	159.76	335.69	1.00	0.05	12.19	7.67	1.000

From Figure D-10, it may be seen that selection method 4 provided the smallest *Overall abs % error* when compared to the other selection methods. Selection methods 2 and 3 provided reasonably accurate results with an average *Overall abs % error* below 10%. Selection method 1 was accurate for models with longer time delays and selection method 5 performed the worst.

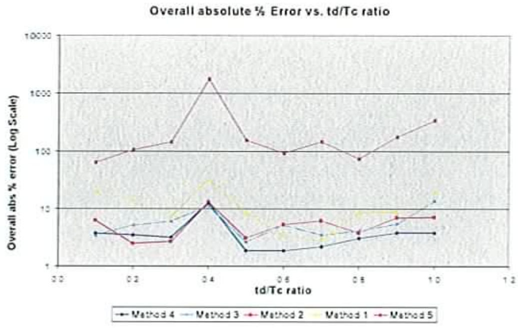

Figure D-10 - Overall percentage error for parameter selection methods

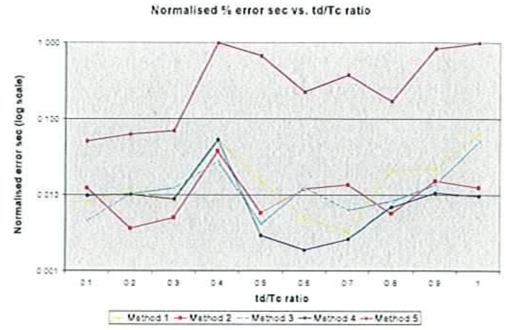

Figure D-11 - Simulation time weighted by (multiplied by) Percentage error for FOLPD models tested, sequence length of 127 bits

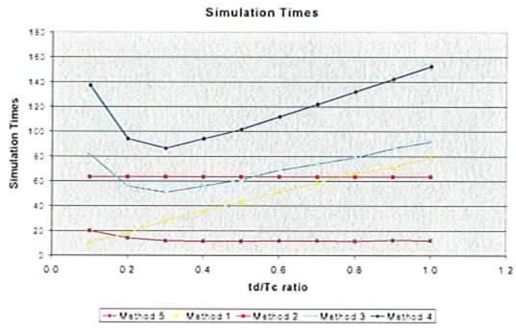

Figure D-12 - simulation times for parameter selection methods

As can be seen in Figure D-12, method 4 required the longest simulation times while method 5 required the shortest. The other methods lie in between these two extremes. It should also be noted that while the simulation times have increased (in comparison to those of Table D-2) the shape of the plots in Figure D-12 are similar to those of Figure D-4.

From Figure D-11, it can be seen that selection method 4 was the most efficient for this particular sequence length. Selection methods 1, 2 and 3 performed reasonably well while selection method 5 was the most inefficient.

As in the previous section, a number of results in Table D-3 were highlighted as meriting further investigation. Two entries associated with selection method 5 (highlighted in gold) provided a relatively large *% Error Gm & Pm* value due to inaccurate estimations of the PM. In contrast to the results of Table D-2, these inaccuracies in the phase margin estimation did not appear to be as a result of an inability to identify the 0dB crossing point as already discussed (see section D.1.1). After further investigation it was determined that these inaccuracies were due to poor frequency resolution around the 0 dB crossing point. This issue is highlighted in Figure D-13 and Figure D-14.

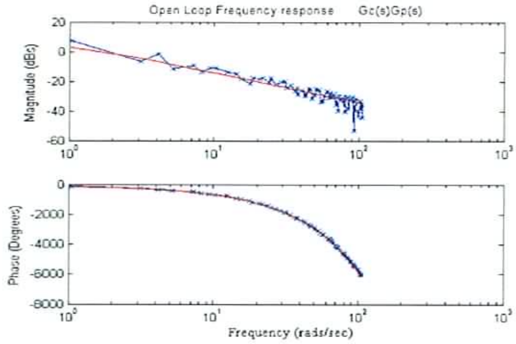

Figure D-13 - Open loop frequency response obtained for model 10 using selection method 5 for sequence length of 127 bits

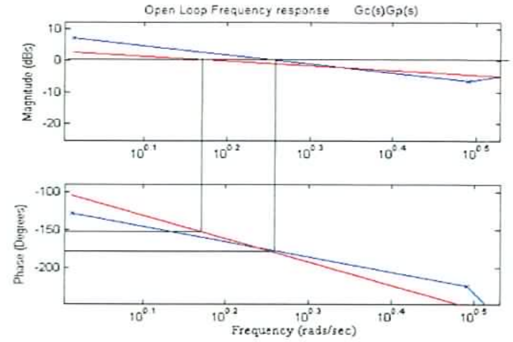

Figure D-14 - Close up of Figure D-13, estimated response represent by blue line, actual response represented by red line.

Figure D-13 is a plot of the open loop frequency response obtained for model 10 using selection method 5. On closer inspection of the plot (see Figure D-14) it may be seen that the frequency resolution around the 0dB crossing point is extremely poor. This leads to an inaccurate identification of this crossing point. From Figure D-14, it can be seen that the estimated crossing point (intersection of blue and black line on the magnitude plot) and the actual crossing point (intersection of red and black line on the magnitude plot) occur at different points. This error, in conjunction with the poor resolution in the phase response plot, leads to an inaccurate estimation of the system phase margin. Increasing the frequency response resolution by increasing the sequence length will lead to improved accuracy in the phase margin estimation.

The results displayed in yellow in Table D-3 highlight a number of entries in which the *% Error Lcmax & band* value is excessively high. As discussed in the previous section this high error is due to poor resolution in the frequency range around the point where the closed loop frequency response peak occurs. Figure D-15 and Figure D-16 illustrate this point.

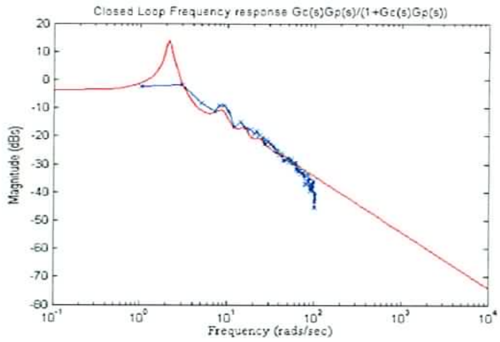

Figure D-15 - Closed loop frequency response plot obtained for model 9 using Selection method 5

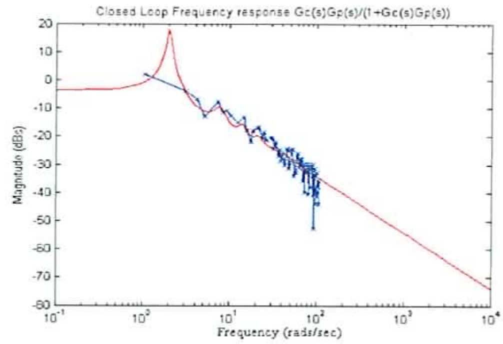

Figure D-16 - Closed loop frequency response plot obtained for model 10 using Selection method 5

Figure D-15 and Figure D-16 show the closed loop frequency response plot obtained for models 9 and 10 respectively using selection method 5. The red line represents the actual closed loop response and the blue line represents the estimated response. As discussed previously, increasing the PRBS sequence length and pulse period will increase the resolution around this point.

A number of table entries have been highlighted in blue. As discussed in the previous section, the high *% Error Lcmax & band* values associated with these results is emphasised due to the small value of the closed loop frequency response peak. Also, as previously discussed, each selection method in Table D-3 consistently provided a more accurate estimation of the *% Error Gm & Pm* values than for those of the *% Error Lcmax & band* values. One reason for this result was discussed in section D.1.1.

An additional point that may be worth noting is that, even though an increase in sequence length should provide increased resolution and as a result increased accuracy, the *% Error Gm & Pm* values and *% Error Lcmax & band* values for selection method 5 displayed in Table D-3 are actual worse than those obtained in Table D-2. In this case increasing the sequence length had the opposite effect to what was expected. This decrease in accuracy is as a result of the manner in which selection method 5 calculated the necessary PRBS pulse period. Selection methods 1 - 4 do not require knowledge of the sequence length, however, selection method 5 does. In this case increasing the sequence length had the effect of decreasing the PRBS pulse period (*period (secs)*, Table D-3) with the net effect of reducing the low frequency content of the PRBS test signal. This had the overall effect of reducing the resolution and thus accuracy of selection method 5 around the frequencies of interest.

D.1.3 Sequence Length = 511 bits:

The following results were obtained using a PRBS sequence length of 511 bits. For a detailed explanation of the results illustrated in Table D-4 and Figure D-17 to Figure D-19, refer to section D.1.1.

Table D-4 - Results obtained for FOLPD model in closed loop using a sequence length of 511 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	% Error second	Normalised error sec
	18.62	18.25	110.44	110.08	2.35	-3.55	-3.52	3.69	3.90	6.22	8.59	0.10	0.04	40.88	0.25	0.003
Method 1	12.73	12.57	100.23	100.15	1.32	-3.52	-3.52	5.45	5.60	2.86	4.19	0.20	0.07	71.54	0.20	0.003
Hang C.C. and Sin K.K.	9.47	9.38	90.10	90.23	1.09	-2.46	-2.50	6.04	6.14	3.33	4.42	0.30	0.11	112.42	0.37	0.005
	7.26	7.21	79.98	80.31	1.14	-0.10	-0.11	5.71	5.73	9.83	10.98	0.40	0.14	143.08	1.38	0.018
(Tu needed)	5.63	5.59	69.90	70.38	1.31	2.41	2.40	5.14	5.20	1.62	2.93	0.50	0.17	173.74	0.28	0.004
	4.34	4.32	59.86	60.46	1.52	4.97	4.97	4.67	4.73	1.28	2.80	0.60	0.20	204.40	0.26	0.003
	3.30	3.29	49.88	50.54	1.77	7.66	7.66	4.33	4.34	0.27	2.04	0.70	0.23	235.06	0.06	0.001
	2.44	2.43	39.95	40.61	2.04	10.55	10.57	3.97	4.00	0.88	2.92	0.80	0.26	265.72	0.23	0.003
	1.70	1.70	30.09	30.69	2.31	13.93	13.93	3.69	3.72	0.85	3.17	0.90	0.28	286.16	0.24	0.003
	1.07	1.07	20.30	20.76	2.59	18.09	18.17	3.45	3.48	1.30	3.89	1.00	0.31	316.82	0.40	0.005
	18.62	18.25	110.37	110.08	2.31	-3.52	-3.52	3.89	3.90	0.44	2.76	0.10	0.25	255.50	0.11	0.001
Method 2	12.73	12.57	100.23	100.15	1.32	-3.52	-3.52	5.56	5.60	0.81	2.13	0.20	0.25	255.50	0.20	0.003
Gunes R.A.	9.47	9.38	90.09	90.23	1.10	-2.46	-2.50	6.10	6.14	2.31	3.41	0.30	0.25	255.50	0.58	0.008
	7.26	7.21	79.98	80.31	1.15	-0.10	-0.11	5.71	5.73	9.93	11.08	0.40	0.25	255.50	2.48	0.033
(Tmin needed)	5.63	5.59	69.90	70.38	1.31	2.41	2.40	5.16	5.20	1.14	2.46	0.50	0.25	255.50	0.29	0.004
	4.34	4.32	59.86	60.45	1.52	4.98	4.97	4.72	4.73	0.37	1.89	0.60	0.25	255.50	0.09	0.001
	3.30	3.29	49.88	50.54	1.77	7.66	7.66	4.33	4.34	0.28	2.05	0.70	0.25	255.50	0.07	0.001
	2.44	2.43	39.95	40.61	2.04	10.54	10.57	3.98	4.00	0.65	2.72	0.80	0.25	255.50	0.17	0.002
	1.70	1.70	30.09	30.69	2.32	13.91	13.93	3.69	3.72	0.95	3.27	0.90	0.25	255.50	0.24	0.003
	1.07	1.07	20.30	20.76	2.59	17.95	18.17	3.44	3.48	2.20	4.79	1.00	0.25	255.50	0.55	0.007
	18.62	18.25	110.37	110.08	2.31	-3.52	-3.52	3.88	3.90	0.55	2.87	0.10	0.32	327.04	0.18	0.002
Method 3	12.73	12.57	100.23	100.15	1.32	-3.52	-3.52	5.59	5.60	0.25	1.57	0.20	0.22	224.84	0.06	0.001
Marzocca C. and Corsi F.	9.47	9.38	90.09	90.23	1.10	-2.46	-2.50	6.09	6.14	2.50	3.60	0.30	0.20	204.40	0.50	0.007
	7.26	7.21	79.95	80.31	1.14	-0.10	-0.11	5.70	5.73	9.84	10.99	0.40	0.22	224.84	2.17	0.029
(Bw needed)	5.63	5.59	69.90	70.38	1.31	2.41	2.40	5.17	5.20	0.96	2.27	0.50	0.24	245.28	0.23	0.003
	4.34	4.32	59.85	60.46	1.52	4.98	4.97	4.69	4.73	0.94	2.46	0.60	0.27	275.94	0.25	0.003
	3.30	3.29	49.85	50.54	1.77	7.66	7.66	4.32	4.34	0.36	2.14	0.70	0.29	286.38	0.11	0.001
	2.44	2.43	39.95	40.61	2.04	10.58	10.57	3.97	4.00	0.89	2.93	0.80	0.31	316.82	0.28	0.004
	1.70	1.70	30.09	30.69	2.31	13.89	13.93	3.69	3.72	1.13	3.44	0.90	0.34	347.48	0.39	0.005
	1.07	1.07	20.30	20.76	2.59	18.06	18.17	3.45	3.48	1.50	4.09	1.00	0.36	367.92	0.54	0.007
	18.62	18.25	110.37	110.08	2.31	-3.52	-3.52	3.89	3.90	0.22	2.53	0.10	0.54	551.88	0.12	0.002
Method 4	12.73	12.57	100.23	100.15	1.32	-3.52	-3.52	5.58	5.60	0.36	1.68	0.20	0.37	378.14	0.13	0.002
Yaacob S. and	9.47	9.38	90.09	90.23	1.10	-2.46	-2.50	6.11	6.14	2.09	3.19	0.30	0.34	347.48	0.71	0.009
Mohamed F.A.	7.26	7.21	79.98	80.31	1.15	-0.10	-0.11	5.72	5.73	9.47	10.61	0.40	0.37	378.14	3.50	0.046
	5.63	5.59	69.90	70.38	1.31	2.41	2.40	5.20	5.20	0.56	1.87	0.50	0.40	408.80	0.23	0.003
(Bw needed)	4.34	4.32	59.86	60.46	1.52	4.98	4.97	4.72	4.73	0.33	1.85	0.60	0.44	449.68	0.14	0.002
	3.30	3.29	49.88	50.54	1.77	7.66	7.66	4.33	4.34	0.26	2.03	0.70	0.48	490.56	0.13	0.002
	2.44	2.43	39.95	40.61	2.04	10.58	10.57	4.00	4.00	0.14	2.18	0.80	0.52	531.44	0.07	0.001
	1.70	1.70	30.09	30.69	2.32	13.93	13.93	3.71	3.72	0.26	2.58	0.90	0.56	572.32	0.15	0.002
	1.07	1.07	20.30	20.76	2.60	18.14	18.17	3.46	3.48	0.67	3.27	1.00	0.60	613.20	0.40	0.005
	18.62	18.25	112.69	110.08	4.38	-3.34	-3.52	3.43	3.90	17.16	21.54	0.10	0.06	62.34	1.05	0.014
Method 5	13.05	12.57	118.13	100.15	21.80	-2.76	-3.52	3.15	5.60	65.35	87.15	0.20	0.04	43.95	2.81	0.037
Landau I.D.	9.18	9.38	72.01	90.23	22.29	-0.35	-2.50	5.65	6.14	93.71	116.00	0.30	0.04	37.81	3.47	0.046
	6.91	7.21	54.01	80.31	36.87	2.19	-0.11	5.13	5.73	2101.15	2138.02	0.40	0.04	35.79	75.64	1.000
(Tr needed)	4.66	5.59	44.55	70.38	53.33	3.11	2.40	4.44	5.20	44.18	97.52	0.50	0.04	35.79	1.59	0.452
	4.03	4.32	53.21	60.46	18.74	6.71	4.97	4.32	4.73	43.57	62.30	0.60	0.04	37.81	1.61	0.458
	2.88	3.29	28.23	50.54	56.53	9.01	7.66	3.76	4.34	31.04	87.58	0.70	0.04	35.79	1.12	0.318
	1.99	2.43	21.26	40.61	65.72	12.36	10.57	3.76	4.00	22.92	88.64	0.80	0.04	35.79	0.83	0.235
	1.45	1.70	19.55	30.69	50.87	14.07	13.93	3.24	3.72	13.96	64.83	0.90	0.04	33.84	0.53	0.151
	0.87	1.07	13.51	20.76	53.92	19.40	18.17	2.59	3.48	32.30	86.22	1.00	0.04	33.84	1.23	0.349

From Figure D-17, it can be seen that for this particular PRBS sequence length, selection methods 1 - 4 provided relative similar *Overall abs % error* results. While selection method 1 performed slightly poorer than selection methods 2 - 4 for models with t_d/T_c ratios below 0.3, all 4 methods generated results with *Overall abs % error* below 10%. Selection method 5 provided relatively poor *Overall abs % error* results and has done so consistently (see Table D-2 and Table D-3).

As can be seen in Figure D-19, selection method 4 required the longest simulation time and selection method 5 required the shortest. The simulation times required for the other methods lies in between these two extremes.

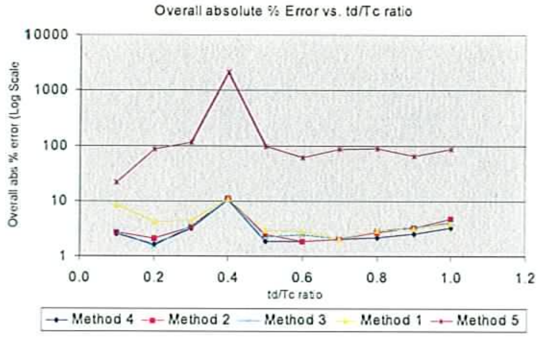

Figure D-17 - Overall percentage error for parameter selection methods

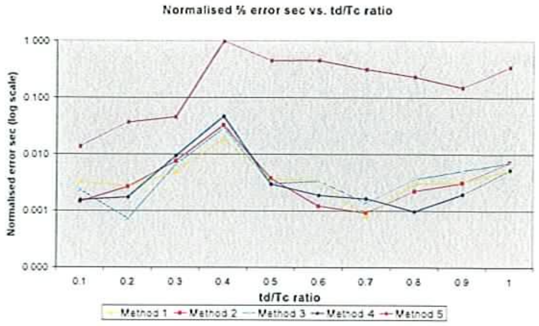

Figure D-18 - Simulation time weighted by (multiplied by) Percentage error for FOLPD models tested, sequence length of 511 bits

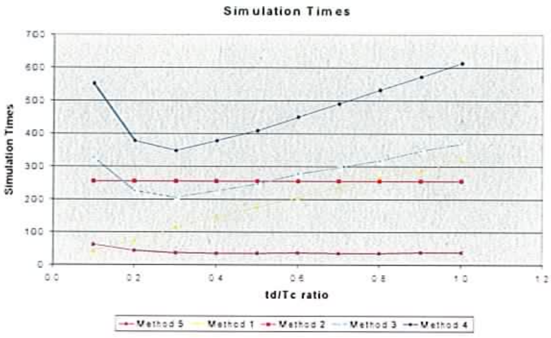

Figure D-19 - Simulation times for parameter selection methods

From the efficiency plot of Figure D-18, it can be seen that selection methods 1-4 generated similarly shaped efficiency plots while selection method 5 proved to be the most inefficient of all 5 methods. This result is consistent with the findings of sections D.1.1 and D.1.2.

The results highlighted in gold in Table D-4 illustrate the *% Error Gm & Pm* and *% Error Lcmax & band* values obtained using selection method 5. These are by far the worst set of results for any of the five selection methods. Figure D-20 and Figure D-21 illustrate why these results are so poor. For both the open and closed loop frequency response shown in these figures, the frequency resolution around the points of interest, namely the gain and phase margin frequencies and the closed loop peak frequency, was poor in comparison to the plots obtained for the other selection methods. As has already been discussed in section D.1.2, increasing the sequence length for this particular PRBS pulse selection method does not have the desired effect of increasing the frequency resolution of the response plots. The results highlighted in yellow are the simulation times necessary for selection method 5. It can be seen that method 5's simulation times remain consistently lower than any of the other 4 methods for each of the PRBS sequence lengths tested. This leads to a lack of low frequency content within the testing signal, which in turn leads to inaccurate estimations of the system's characteristics. This is one major drawback to using selection method 5.

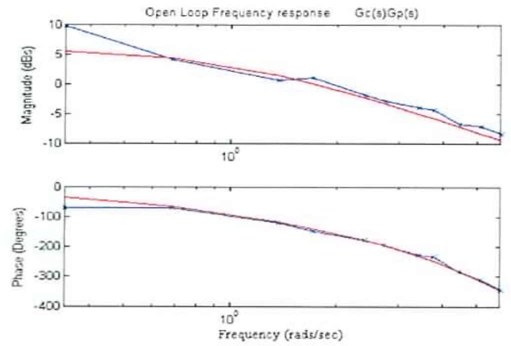

Figure D-20 - Open loop frequency response obtained for model 8 using method 5 for sequence length of 511 bits

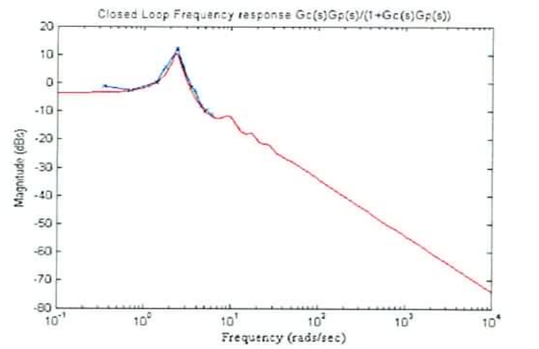

Figure D-21 - Closed loop frequency response plot obtained for model 8 using Selection method 5 for sequence length of 511 bits

As discussed in previous sections, the results highlighted in blue illustrate a number of examples where a large percentage error was obtained despite the fact that relative difference between the estimated and actual results is quite small.

D.1.4 Sequence Length = 1023 bits:

The following results were obtained using a PRBS sequence length of 1023 bits. See section D.1.1 for a detailed explanation of Table D-5 and Figure D-22 to Figure D-24.

Table D-5 - Results obtained for FOLPD model in closed loop using a sequence length of 1023 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	% Error second	Normalised error sec
	18.62	18.25	110.39	110.08	2.33	-3.53	-3.52	3.84	3.90	1.81	4.14	0.10	0.04	81.84	0.07	0.020
Method 1	12.73	12.57	100.23	100.15	1.32	-3.52	-3.52	5.53	5.60	1.36	2.68	0.20	0.07	143.22	0.09	0.026
Hang C.C. and Sin K.K	9.47	9.38	90.09	90.23	1.10	-2.45	-2.50	6.09	6.14	2.51	3.61	0.30	0.11	225.05	0.28	0.075
	7.26	7.21	79.98	80.31	1.15	-0.10	-0.11	5.70	5.73	9.86	11.01	0.40	0.14	286.44	1.38	0.375
(Tu needed)	5.63	5.59	69.90	70.38	1.31	2.41	2.40	5.20	5.20	0.50	1.81	0.50	0.17	347.82	0.09	0.023
	4.34	4.32	59.86	60.46	1.52	4.98	4.97	4.73	4.73	0.19	1.71	0.60	0.20	409.20	0.04	0.010
	3.30	3.29	49.88	50.54	1.77	7.65	7.66	4.33	4.34	0.35	2.12	0.70	0.23	470.58	0.08	0.022
	2.44	2.43	39.95	40.61	2.04	10.55	10.57	3.99	4.00	0.24	2.28	0.80	0.26	531.96	0.06	0.017
	1.70	1.70	30.09	30.69	2.32	13.93	13.93	3.71	3.72	0.35	2.66	0.90	0.28	572.88	0.10	0.026
	1.07	1.07	20.30	20.76	2.59	18.16	18.17	3.47	3.48	0.43	3.02	1.00	0.31	634.26	0.13	0.036
	18.62	18.25	110.37	110.08	2.31	-3.52	-3.52	3.88	3.90	0.53	2.84	0.10	0.25	511.50	0.13	0.036
Method 2	12.73	12.57	100.23	100.15	1.32	-3.52	-3.52	5.60	5.60	0.08	1.40	0.20	0.25	511.50	0.02	0.005
Gunes R.A.	9.47	9.38	90.09	90.23	1.10	-2.45	-2.50	6.12	6.14	2.00	3.10	0.30	0.25	511.50	0.50	0.136
	7.26	7.21	79.98	80.31	1.15	-0.10	-0.11	5.72	5.73	9.60	10.75	0.40	0.25	511.50	2.40	0.651
(Tmin needed)	5.63	5.59	69.90	70.38	1.31	2.41	2.40	5.18	5.20	0.79	2.10	0.50	0.25	511.50	0.20	0.053
	4.34	4.32	59.86	60.46	1.52	4.98	4.97	4.72	4.73	0.47	1.99	0.60	0.25	511.50	0.12	0.032
	3.30	3.29	49.88	50.54	1.77	7.66	7.66	4.32	4.34	0.38	2.16	0.70	0.25	511.50	0.10	0.026
	2.44	2.43	39.95	40.61	2.04	10.58	10.57	3.98	4.00	0.55	2.59	0.80	0.25	511.50	0.14	0.037
	1.70	1.70	30.09	30.69	2.32	13.92	13.93	3.71	3.72	0.34	2.66	0.90	0.25	511.50	0.09	0.023
	1.07	1.07	20.30	20.76	2.59	18.15	18.17	3.46	3.48	0.56	3.15	1.00	0.25	511.50	0.14	0.038
	18.62	18.25	110.37	110.08	2.31	-3.52	-3.52	3.88	3.90	0.64	2.95	0.10	0.32	654.72	0.21	0.056
Method 3	12.73	12.57	100.23	100.15	1.32	-3.52	-3.52	5.58	5.60	0.35	1.67	0.20	0.22	450.12	0.08	0.021
Marzocca C. and Corsi F.	9.47	9.38	90.09	90.23	1.10	-2.45	-2.50	6.11	6.14	2.10	3.20	0.30	0.20	409.20	0.42	0.114
	7.26	7.21	79.98	80.31	1.15	-0.10	-0.11	5.72	5.73	9.54	10.69	0.40	0.22	450.12	2.10	0.570
(Bw needed)	5.63	5.59	69.90	70.38	1.31	2.41	2.40	5.20	5.20	0.57	1.88	0.50	0.24	491.04	0.14	0.037
	4.34	4.32	59.86	60.46	1.52	4.98	4.97	4.73	4.73	0.22	1.74	0.60	0.27	552.42	0.06	0.016
	3.30	3.29	49.88	50.54	1.77	7.66	7.66	4.32	4.34	0.47	2.24	0.70	0.29	593.34	0.14	0.037
	2.44	2.43	39.95	40.61	2.04	10.58	10.57	3.98	4.00	0.50	2.54	0.80	0.31	634.26	0.15	0.042
	1.70	1.70	30.09	30.69	2.32	13.93	13.93	3.70	3.72	0.45	2.77	0.90	0.34	695.64	0.15	0.042
	1.07	1.07	20.30	20.76	2.60	18.17	18.17	3.46	3.48	0.48	3.08	1.00	0.36	736.56	0.17	0.047
	18.62	18.25	110.37	110.08	2.31	-3.52	-3.52	3.89	3.90	0.31	2.63	0.10	0.54	1104.60	0.17	0.046
Method 4	12.73	12.57	100.23	100.15	1.32	-3.52	-3.52	5.59	5.60	0.16	1.45	0.20	0.37	757.02	0.06	0.004
Yaacob S. and	9.47	9.38	90.09	90.23	1.10	-2.45	-2.50	6.12	6.14	1.90	3.00	0.30	0.34	695.64	0.65	0.006
Mohamed F.A.	7.26	7.21	79.98	80.31	1.15	-0.10	-0.11	5.73	5.73	9.50	10.65	0.40	0.37	757.02	3.52	0.035
	5.63	5.59	69.90	70.38	1.31	2.41	2.40	5.19	5.20	0.69	1.99	0.50	0.40	818.40	0.27	0.003
(Bw needed)	4.34	4.32	59.86	60.46	1.52	4.98	4.97	4.73	4.73	0.24	1.76	0.60	0.44	900.24	0.10	0.001
	3.30	3.29	49.88	50.54	1.77	7.66	7.66	4.32	4.34	0.35	2.12	0.70	0.48	982.08	0.17	0.002
	2.44	2.43	39.95	40.61	2.05	10.58	10.57	3.99	4.00	0.24	2.29	0.80	0.52	1063.90	0.13	0.001
	1.70	1.70	30.09	30.69	2.32	13.93	13.93	3.72	3.72	0.05	2.37	0.90	0.56	1145.80	0.03	0.000
	1.07	1.07	20.30	20.76	2.60	18.17	18.17	3.47	3.48	0.28	2.88	1.00	0.60	1227.60	0.17	0.002
	18.57	18.25	118.42	110.08	9.34	-3.47	-3.52	3.24	3.90	18.45	27.78	0.10	0.06	112.53	1.01	0.010
Method 5	11.57	12.57	82.38	100.15	25.69	-1.04	-3.52	4.25	5.60	94.51	120.20	0.20	0.04	79.79	3.69	0.037
Landau I.D.	9.61	9.38	123.15	90.23	38.98	-1.75	-2.50	5.77	6.14	36.01	74.98	0.30	0.03	67.52	1.19	0.012
	7.39	7.21	114.23	80.31	44.75	0.01	-0.11	5.78	5.73	106.16	150.91	0.40	0.03	65.47	3.40	0.034
(Tr needed)	5.86	5.59	108.86	70.38	59.57	2.37	2.40	4.60	5.20	8.77	68.34	0.50	0.03	65.47	0.28	0.003
	4.75	4.32	58.26	60.46	13.66	4.25	4.97	4.28	4.73	23.98	37.64	0.60	0.03	67.52	0.79	0.008
	3.60	3.29	47.47	50.54	15.63	6.93	7.66	4.10	4.34	15.11	30.74	0.70	0.03	67.52	0.50	0.005
	2.47	2.43	40.71	40.61	2.21	9.95	10.57	3.84	4.00	9.89	12.10	0.80	0.03	65.47	0.32	0.003
	1.77	1.70	32.73	30.69	11.23	13.68	13.93	3.25	3.72	14.36	25.60	0.90	0.03	69.56	0.49	0.005
	1.08	1.07	22.05	20.76	7.65	17.68	18.17	2.89	3.48	19.61	27.27	1.00	0.03	69.56	0.67	0.007

From Figure D-22, it can be seen that selection methods 1-4 generated almost identical *Overall abs % error* results. While the results obtained using selection method 5 have improved the *Overall abs % error* values generated, are still well above those generated by the other 4 selection methods. From Figure D-24, it can be seen that method 4 required the longest simulation times while method 5 required the shortest. As mentioned before the other three methods simulation times lie in between these two extremes.

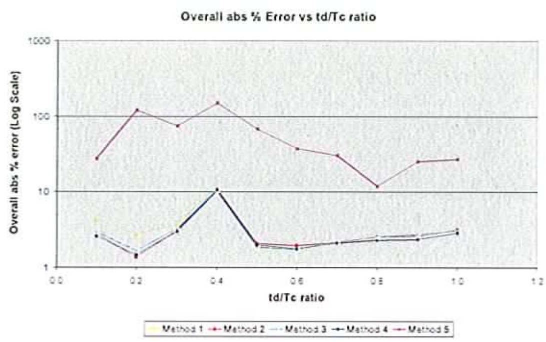

Figure D-22 - Overall percentage error for parameter selection methods

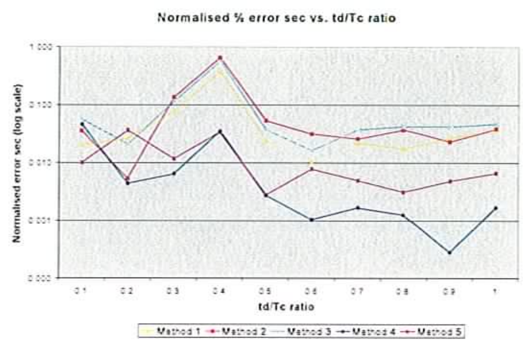

Figure D-23 - Simulation time weighted by (multiplied by) Percentage error for FOLPD models tested, sequence length of 1023 bits

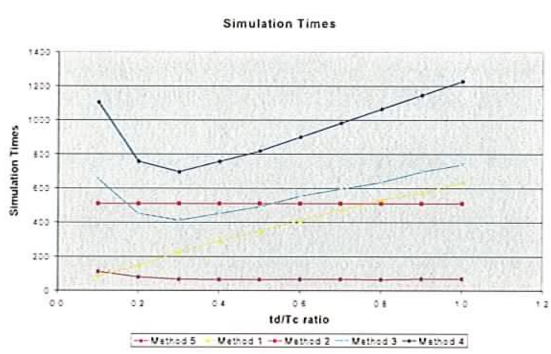

Figure D-24 - Simulation times for parameter selection methods

From the efficiency plot of Figure D-23 it can be seen that selection method 4 was the most efficient for this sequence length while selection method 5 was a close second. This result is dissimilar to the results obtained in previous sections.

It should be noted that the efficiency of selection method 5 appears to be increasing as the PRBS sequence length increases, while its simulation time remains significantly lower than the other selection methods. The argument may be made, therefore, that if one was to significantly increase the sequence length for this method, its accuracy should also increase significantly while its necessary simulation time should remain small, thus making this method more attractive than any of the other 4 methods. However, the results of previous sections do not reinforce this claim. If one compares the current simulation times of selection method 5 with those of selection method 4 (section D.1.1, Table D-2) it can be seen that they are broadly similar. However, the *Overall abs % error* results obtained for selection method 4 in section D.1.1 are still much better than those of selection method 5 displayed in Table D-5. Therefore, even though for the situation described, both methods had similar simulation times, their *Overall abs % error* results are entirely different, with selection method 4 proving more attractive. Also, as discussed in section D.1.2, increasing the PRBS sequence length did not increase the accuracy of selection method 5 as expected.

The results highlighted in blue in Table D-5 have been discussed in previous sections and illustrate the fact that the large *% Error Lcmax & band* results obtained are exaggerated by the small closed loop peak value of model 4.

D.2 Typical Process Models - Closed Loop:

This section contains results obtained during the testing of 6 models of various orders. These models are intended to represent a variety of typical processes and were used by Wang, Lee and Lin in [82] to evaluate the accuracy of an assortment of relay based approaches to system identification.

Table D-6 - Various Typical process models [82]

Model No.	Typical Process Models	Model No.	Typical Process Models
1	$\frac{1}{(2s+1)^2}e^{-2s}$	4	$\frac{1}{(s+1)(5s+1)^2}e^{-2.5s}$
2	$\frac{1}{(2s+1)^5}e^{-2s}$	5	$\frac{1}{(s+1)^8}$
3	$\frac{1}{(s+1)(s^2+s+1)}e^{-0.5s}$	6	$\frac{1}{(5s+1)}e^{-5s}$

These typical process model were tested in closed loop configuration as illustrated in the Simulink model displayed in Figure D-25.

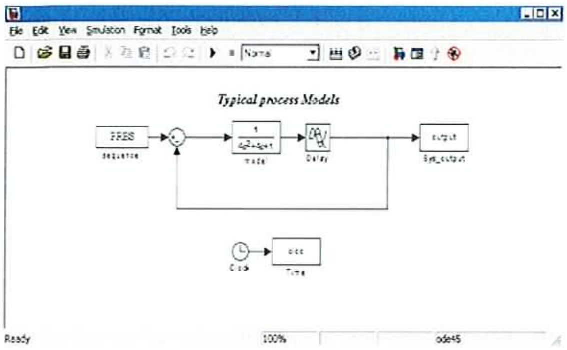

Figure D-25 - Simulink test set-up for Typical process models

In a similar manner to the testing procedure of the 10 FOPLD models discussed in section D.1, it can be seen from Figure D-25 that the PRBS testing signal was injected into the closed loop system on top of any existing set point. The input and controlled variable signal were then recorded and processed off-line.

D.2.1 Sequence Length = 63 bits:

The following results were obtained using a PRBS sequence length of 63 bits. Detailed explanations of the contents of Table D-7 and accompanying Figure D-26 to Figure D-28 can be found in section D.1.1.

Table D-7 - Results obtained for Typical processes in closed loop using a sequence length of 63 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	% Error second	Normalised error sec
	8.67	8.65	180.00	180.00	0.20	-2.21	-2.13	0.73	0.81	13.75	13.95	0.50	0.96	120.96	13.20	0.131
Method 1	6.06	6.03	180.00	180.00	0.51	0.82	1.31	0.36	0.39	45.46	45.99	0.06	2.22	279.72	100.97	1.000
Hang C.C. and Sin K.K.	6.91	6.79	180.00	180.00	1.69	-0.15	0.48	1.09	1.23	141.87	143.56	0.25	0.64	80.64	90.80	0.916
(Tu needed)	11.52	11.50	180.00	180.00	0.17	-4.07	-4.03	0.31	0.32	5.63	5.79	0.10	1.96	246.96	11.03	0.111
	5.53	5.50	180.00	180.00	0.57	1.92	2.19	0.52	0.58	21.98	22.54	0.00	1.52	191.52	33.41	0.337
	7.09	7.09	180.00	180.00	0.06	-1.13	-1.02	0.58	0.60	14.25	14.29	1.00	1.55	195.30	22.08	0.223
	8.69	8.65	180.00	180.00	0.41	-2.79	-2.13	0.60	0.81	57.11	57.53	0.50	0.50	63.00	28.56	0.288
Method 2	6.38	6.03	180.00	180.00	5.66	-0.53	1.31	0.20	0.39	189.26	194.94	0.06	0.50	63.00	94.63	0.955
Gurnee R.A.	6.84	6.79	180.00	180.00	0.69	0.09	0.48	1.20	1.23	84.71	85.40	0.25	0.50	63.00	42.35	0.428
(Tmin needed)	10.23	11.50	180.00	180.00	11.06	-12.36	-4.03	0.40	0.32	231.34	242.40	0.10	0.25	31.50	57.84	0.584
	6.48	5.50	180.00	180.00	17.76	0.23	2.19	0.40	0.58	120.64	138.43	0.00	0.25	31.50	30.16	0.304
	7.09	7.09	180.00	180.00	0.01	-1.16	-1.02	0.56	0.60	20.61	20.62	1.00	1.25	157.50	25.76	0.260
	8.66	8.65	180.00	180.00	0.14	-2.18	-2.13	0.77	0.81	7.04	7.17	0.50	1.55	195.30	10.91	0.110
Method 3	6.05	6.03	180.00	180.00	0.20	1.31	1.31	0.37	0.39	4.82	5.02	0.06	3.22	405.72	15.52	0.157
Verzocca C. and Corsi F.	6.83	6.79	180.00	180.00	0.48	0.46	0.48	1.17	1.23	8.79	9.27	0.25	1.02	128.52	8.96	0.090
(Bw needed)	11.51	11.50	180.00	180.00	0.10	-4.06	-4.03	0.31	0.32	3.98	4.09	0.10	3.87	487.62	15.42	0.156
	5.51	5.50	180.00	180.00	0.08	2.16	2.19	0.55	0.58	6.25	6.33	0.00	2.17	273.42	13.57	0.137
	7.09	7.09	180.00	180.00	0.01	-1.05	-1.02	0.57	0.60	8.17	8.18	1.00	2.11	265.86	17.24	0.174
(Bw needed)	8.66	8.65	180.00	180.00	0.12	-2.14	-2.13	0.81	0.81	0.58	0.70	0.50	2.58	325.08	1.50	0.015
Method 4	6.04	6.03	180.00	180.00	0.15	1.23	1.31	0.37	0.39	10.92	11.08	0.06	5.37	676.62	58.66	0.592
Yaacob S. and	6.82	6.79	180.00	180.00	0.35	0.46	0.48	1.23	1.23	4.34	4.69	0.25	1.70	214.20	7.38	0.075
Mohamed F.A.	11.51	11.50	180.00	180.00	0.08	-4.03	-4.03	0.31	0.32	3.44	3.52	0.10	6.45	812.70	22.21	0.224
	5.51	5.50	180.00	180.00	0.08	2.10	2.19	0.55	0.58	9.22	9.30	0.00	3.62	456.12	33.37	0.337
	7.09	7.09	180.00	180.00	0.03	-1.00	-1.02	0.57	0.60	6.85	6.88	1.00	3.51	442.26	24.03	0.243
	8.72	8.65	180.00	180.00	0.83	-2.04	-2.13	0.50	0.81	41.81	42.64	0.50	0.40	49.90	16.56	0.167
Method 5	6.52	6.03	180.00	180.00	8.06	1.15	1.31	0.24	0.39	51.14	59.18	0.06	0.83	104.96	42.60	0.430
Landau I.D.	8.59	6.79	180.00	180.00	26.42	-0.73	0.48	0.72	1.23	292.36	318.79	0.25	0.28	34.78	80.69	0.814
(Tr needed)	11.68	11.50	180.00	180.00	1.50	-3.94	-4.03	0.19	0.32	43.24	44.74	0.10	1.06	133.31	45.75	0.462
	5.96	5.50	180.00	180.00	8.26	2.07	2.19	0.36	0.58	43.80	52.06	0.00	0.56	70.06	24.35	0.246
	7.02	7.09	180.00	180.00	0.99	-2.96	-1.02	0.44	0.60	217.27	218.26	1.00	0.46	57.46	99.07	1.000

From Figure D-26, it can be seen that selection method 4 provided the lowest *Overall abs % error* values, followed closely by selection method 3. Selection methods 2 and 5 both performed poorly with average *Overall abs % error* values hovering around the 100% point. With respect to *Overall abs % error* values, selection method 1 performed poorly for models 1-3 but improves for models 4 - 6.

Examining Figure D-28, it can be seen that as for the FOLPD models of section D.1, selection method 4 required the longest simulation times, followed by method 3 then method 1. Selection method 2 required the shortest simulation times in the majority of cases although method 5 required a shorter simulation time for models 3 and 6.

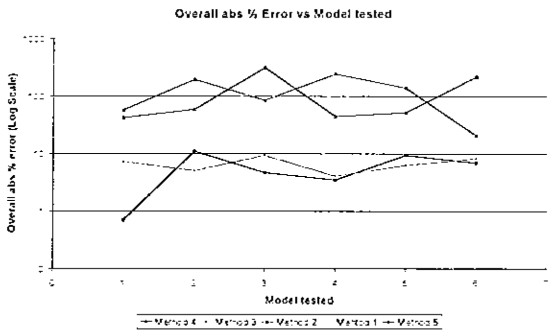

Figure D-26 - Overall percentage error for parameter selection methods

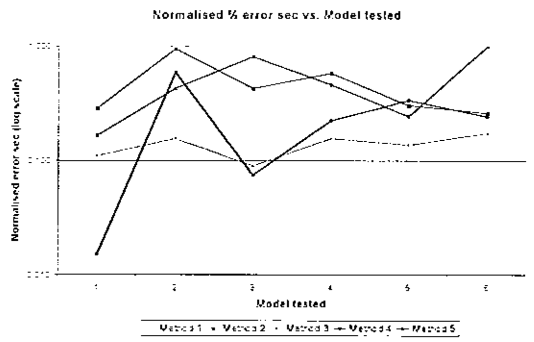

Figure D-27 - Simulation time weighted by (multiplied by) Percentage error for Typical process models tested, sequence length of 63bits

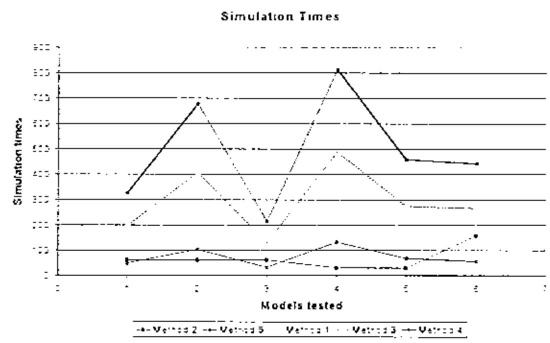

Figure D-28 - Simulation times for parameter selection methods

From the efficiency plot of Figure D-27 it can be seen that selection method 3 was the most efficient in the majority of cases with method 4 beating it for models 1 and 3. Selection methods 2 and 5 provided similar efficiency results, while method 1 performed quite poorly for models 2 and 3 then improved for models 4 - 6.

A number of interesting results were highlighted in Table D-7. For the result highlighted in gold a large *% Error Gm & Pm* value was obtained due to a lack of resolution in the low frequency range of the open loop frequency response plot. This caused an inaccurate estimation of the gain margin of the system to be obtained. This low frequency resolution problem has been discussed in connection with selection method 5 in previous sections. Figure D-29 and Figure D-30 help illustrate this problem.

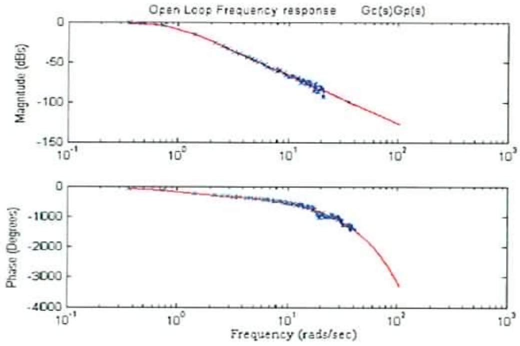

Figure D-29 - Open loop frequency response obtained for model 3 using method 5 for a sequence length of 63 bits

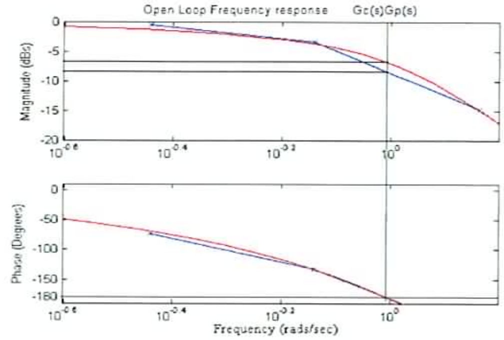

Figure D-30 - Close up of Figure D-36

Figure D-29 illustrates the estimate of the open loop frequency response of model 3 obtained using selection method 5 in blue. The red line indicates the actual open loop frequency response. On closer inspection (see Figure D-30) it can be seen how the low resolution in the magnitude plot leads to an overestimation of the gain margin of the system. Refer to section D.1.2 and D.1.3 for a more complete discussion of this problem.

The results highlighted in yellow indicate where an inaccurate estimate of the closed loop peak value generated a large *% Error Lcmax & band* value. Again this inaccuracy can be attributed to poor frequency resolution around the point of interest.

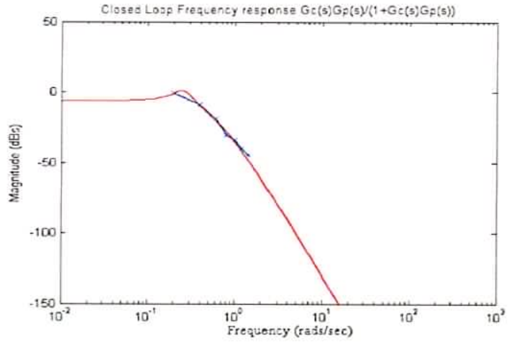

Figure D-31 - Closed loop frequency response plot obtained for model 2 using Selection method 2 for sequence length of 63 bits

From Figure D-31 it can be seen that the poor estimate of the closed loop peak value for model 2 obtained using selection method 2 is due to poor frequency resolution around the peak value. This problem may be remedied by increasing the PRBS sequence length.

The table entries highlighted in blue illustrate results that generated large *% Error Lcmax & band* values where the relative difference between the estimated value and actual value was quite small. This point has previously been discussed in section D.1.1.

D.2.2 Sequence Length = 127 bits:

The following results were obtained using a sequence length of 127 bits. For a detailed explanation of Table D-8 and Figure D-32 to Figure D-34 see section D.1.1.

Table D-8 - Results obtained for Typical processes in closed loop using a sequence length of 127 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	% Error second	Normalised error sec
	8.66	8.65	180.00	180.00	0.14	-2.18	-2.13	0.77	0.81	7.06	7.20	0.50	0.96	243.84	6.78	0.076
Method 1	6.04	6.03	180.00	180.00	0.19	1.29	1.31	0.35	0.39	4.65	4.84	0.06	2.22	563.88	10.31	0.116
Hang C.C. and Sin K.K.	6.83	6.79	180.00	180.00	0.50	0.49	0.45	1.16	1.23	8.46	8.96	0.25	0.64	162.56	5.41	0.061
(Tu needed)	11.51	11.50	180.00	180.00	0.10	-4.06	-4.03	0.30	0.32	6.10	6.19	0.10	1.96	497.84	11.95	0.135
	5.51	5.50	180.00	180.00	0.18	2.01	2.19	0.55	0.58	12.68	12.85	0.00	1.52	386.08	19.27	0.217
	7.09	7.09	180.00	180.00	0.03	-1.01	-1.02	0.57	0.60	4.76	4.79	1.00	1.55	393.70	7.38	0.083
	8.67	8.65	180.00	180.00	0.22	-2.13	-2.13	0.79	0.81	2.41	2.63	0.50	0.50	127.00	1.21	0.014
Method 2	6.06	6.03	180.00	180.00	0.50	-0.70	1.31	0.30	0.39	177.36	177.66	0.06	0.50	127.00	88.68	1.000
Gunee R.A.	6.81	6.79	180.00	180.00	0.26	0.39	0.48	1.19	1.23	21.60	21.86	0.25	0.50	127.00	10.80	0.140
(Tmin needed)	11.71	11.50	180.00	180.00	1.79	-4.22	-4.03	0.20	0.32	42.86	44.65	0.10	0.25	63.50	10.72	0.139
	5.88	5.50	180.00	180.00	6.95	1.43	2.19	0.40	0.58	66.29	73.24	0.00	0.25	63.50	16.57	0.215
	7.09	7.09	180.00	180.00	0.02	-1.04	-1.02	0.59	0.60	3.18	3.19	1.00	1.25	317.50	3.97	0.052
	8.66	8.65	180.00	180.00	0.13	-2.14	-2.13	0.80	0.81	2.09	2.21	0.50	1.55	393.70	3.23	0.042
Method 3	6.04	6.03	180.00	180.00	0.15	1.29	1.31	0.38	0.39	2.74	2.89	0.06	3.22	817.88	8.82	0.114
Marzocca C. and Corsi F.	6.82	6.79	180.00	180.00	0.38	0.49	0.48	1.21	1.23	3.04	3.42	0.25	1.02	259.08	3.10	0.040
(Bw needed)	11.51	11.50	180.00	180.00	0.08	-4.03	-4.03	0.32	0.32	0.24	0.32	0.10	3.87	982.98	0.94	0.012
	5.51	5.50	180.00	180.00	0.10	2.13	2.19	0.57	0.58	4.54	4.64	0.00	2.17	551.18	9.85	0.128
	7.09	7.09	180.00	180.00	0.04	-1.02	-1.02	0.59	0.60	2.59	2.62	1.00	2.11	535.94	5.46	0.071
(Bw needed)	8.66	8.65	180.00	180.00	0.12	-2.13	-2.13	0.81	0.81	0.60	0.72	0.50	2.58	655.32	1.55	0.020
Method 4	6.04	6.03	180.00	180.00	0.11	1.31	1.31	0.39	0.39	0.95	1.06	0.06	5.37	1364.00	5.10	0.066
Yaacob S. and	6.81	6.79	180.00	180.00	0.24	0.49	0.48	1.22	1.23	2.26	2.50	0.25	1.70	431.80	3.83	0.050
Mohamed F.A.	11.51	11.50	180.00	180.00	0.08	-4.03	-4.03	0.32	0.32	0.79	0.87	0.10	6.45	1638.30	5.11	0.066
	5.51	5.50	180.00	180.00	0.08	2.17	2.19	0.57	0.58	1.88	1.95	0.00	3.62	919.48	6.75	0.087
	7.09	7.09	180.00	180.00	0.04	-1.01	-1.02	0.59	0.60	2.26	2.31	1.00	3.51	891.54	7.95	0.103
	8.68	8.65	180.00	180.00	0.34	-2.39	-2.13	0.73	0.81	22.48	22.82	0.50	0.34	86.36	7.64	0.099
Method 5	6.14	6.03	180.00	180.00	1.76	0.27	1.31	0.28	0.39	108.00	109.76	0.06	0.71	181.36	77.11	1.000
Landau I.D.	6.85	6.79	180.00	180.00	0.75	0.60	0.48	1.04	1.23	80.88	81.63	0.25	0.24	60.20	19.17	0.297
(Tr needed)	11.54	11.50	180.00	180.00	0.29	-4.07	-4.03	0.27	0.32	15.76	16.05	0.10	0.91	230.38	14.30	0.221
	5.45	5.50	180.00	180.00	1.02	1.19	2.19	0.52	0.58	56.28	57.29	0.00	0.48	121.16	26.84	0.415
	7.05	7.09	180.00	180.00	0.57	-2.55	-1.02	0.51	0.60	165.29	165.86	1.00	0.39	99.31	64.63	1.000

From Figure D-32, it can be seen that selection method 4 provided the smallest *Overall abs % error* results in the majority of cases. Selection method 3 also performed reasonably well, providing the lowest *Overall abs % error* value for model 4. Selection method 1 also performed well returning an average *Overall abs % error* of below 10%. Selection methods 2 and 5 performed poorly in the majority of cases.

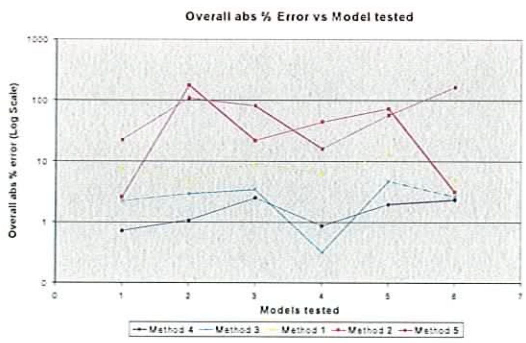

Figure D-32 - Overall percentage error for parameter selection methods

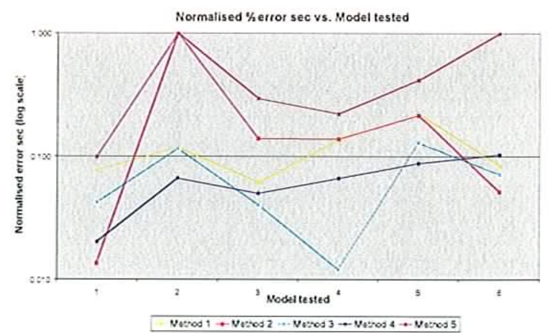

Figure D-33 - Simulation time weighted by (multiplied by) Percentage error for Typical process models tested, sequence length of 127 bits

From the efficiency plot of Figure D-33, it can be seen that selection methods 3 and 4 are the most efficient. Selection method 5 is the most inefficient of the five methods. Method 1 performs reasonably well while method 2 is somewhat erratic.

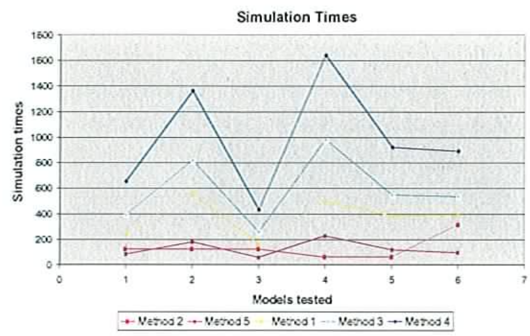

Figure D-34 - Simulation times for parameter selection methods

From Figure D-34 it can be seen that the best performing methods, methods 3 and 4, require the longest simulation times while the methods that return the poorer results require significantly less testing times.

The results in yellow in Table D-8 highlight values were a large *% Error Lcmax & band* value was obtained due to poor frequency resolution around the peak value of the closed loop frequency response. This problem has been identified and discussed in previous sections (refer to section D.1.2). Also highlighted (in blue), are results obtained that indicate a large *% Error Lcmax & band* value when the relative difference between the estimated and actual characteristic value is small. This issue has also been discussed in previous sections (refer to section D.1.1).

D.2.3 Sequence Length = 511 bits:

The following results were obtained using a PRBS sequence length of 511 bits. For a detailed explanation of Table D-9 and Figure D-35 to Figure D-37 refer to section D.1.1.

Table D-9 - Results obtained for Typical processes in closed loop using a sequence length of 511 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Simtime (secs)	% Error second	Normalised error sec
	8.66	8.65	180.00	180.00	0.12	-2.13	-2.13	0.81	0.81	0.44	0.55	0.50	0.96	981.12	0.42	0.044
Method 1	6.04	6.03	180.00	180.00	0.09	1.31	1.31	0.39	0.39	0.84	0.93	0.06	2.22	2268.60	1.86	0.197
Hang C.C. and Sin K.K.	6.81	6.79	180.00	180.00	0.26	0.50	0.48	1.23	1.23	5.22	5.48	0.25	0.64	654.06	3.34	0.353
(Tu needed)	11.51	11.50	180.00	180.00	0.08	-4.03	-4.03	0.32	0.32	0.08	0.16	0.10	1.96	2003.10	0.16	0.017
	5.51	5.50	180.00	180.00	0.07	2.19	2.19	0.57	0.58	1.05	1.12	0.00	1.52	1553.40	1.59	0.168
	7.09	7.09	180.00	180.00	0.05	-1.00	-1.02	0.59	0.60	2.43	2.47	1.00	1.55	1584.10	3.76	0.398
	8.66	8.65	180.00	180.00	0.12	-2.13	-2.13	0.81	0.81	0.37	0.49	0.50	0.50	511.00	0.18	0.019
Method 2	6.05	6.03	180.00	180.00	0.24	1.29	1.31	0.37	0.39	6.58	6.82	0.06	0.50	511.00	3.29	0.348
Guinee R.A.	6.81	6.79	180.00	180.00	0.26	0.51	0.48	1.23	1.23	5.57	5.83	0.25	0.50	511.00	2.78	0.294
(Tmin needed)	11.52	11.50	180.00	180.00	0.18	-4.03	-4.03	0.30	0.32	7.89	8.08	0.10	0.25	255.50	1.97	0.208
	5.52	5.50	180.00	180.00	0.36	1.92	2.19	0.54	0.58	18.89	19.28	0.00	0.25	255.50	4.72	0.499
	7.09	7.09	180.00	180.00	0.05	-1.00	-1.02	0.59	0.60	3.46	3.50	1.00	1.25	1277.50	4.32	0.457
	8.66	8.65	180.00	180.00	0.12	-2.13	-2.13	0.81	0.81	0.28	0.40	0.50	1.55	1584.10	0.44	0.046
Method 3	6.04	6.03	180.00	180.00	0.09	1.31	1.31	0.39	0.39	0.16	0.25	0.06	3.22	3290.60	0.51	0.054
Marzocca C. and Corsi F.	6.81	6.79	180.00	180.00	0.23	0.50	0.48	1.23	1.23	4.93	5.16	0.25	1.02	1042.40	5.03	0.531
(Bw needed)	11.51	11.50	180.00	180.00	0.07	-4.03	-4.03	0.32	0.32	1.39	1.46	0.10	3.87	3955.10	5.40	0.570
	5.51	5.50	180.00	180.00	0.06	2.19	2.19	0.58	0.58	0.44	0.51	0.00	2.17	2217.70	0.96	0.102
	7.09	7.09	180.00	180.00	0.05	-1.00	-1.02	0.59	0.60	2.76	2.80	1.00	2.11	2156.40	5.82	0.615
(Bw needed)	8.66	8.65	180.00	180.00	0.12	-2.13	-2.13	0.81	0.81	0.20	0.31	0.50	2.58	2636.60	0.51	0.054
Method 4	6.04	6.03	180.00	180.00	0.09	1.31	1.31	0.39	0.39	0.47	0.56	0.06	5.37	5488.10	2.51	0.265
Yaacob S. and	6.81	6.79	180.00	180.00	0.24	0.51	0.48	1.23	1.23	5.57	5.80	0.25	1.70	1737.40	9.47	1.000
Mohamed F.A.	11.51	11.50	180.00	180.00	0.07	-4.03	-4.03	0.32	0.32	1.39	1.46	0.10	6.45	6591.90	8.99	1.000
	5.50	5.50	180.00	180.00	0.06	2.19	2.19	0.58	0.58	0.52	0.58	0.00	3.62	3699.60	1.89	0.210
	7.09	7.09	180.00	180.00	0.05	-1.00	-1.02	0.60	0.60	2.56	2.61	1.00	3.51	3587.20	8.98	1.000
	8.79	8.65	158.69	180.00	13.45	-2.32	-2.13	0.79	0.81	11.03	24.48	0.50	0.26	269.61	2.91	0.363
Method 5	6.07	6.03	163.38	180.00	9.79	1.23	1.31	0.38	0.39	9.90	19.69	0.06	0.56	567.21	5.50	0.686
Landau I.D.	7.00	6.79	159.49	180.00	14.44	0.33	0.48	1.14	1.23	39.90	54.34	0.25	0.18	188.06	7.34	0.916
(Tr needed)	11.54	11.50	180.00	180.00	0.30	-4.06	-4.03	0.30	0.32	8.06	8.35	0.10	0.71	720.51	5.68	0.709
	5.52	5.50	180.00	180.00	0.29	1.78	2.19	0.56	0.58	21.61	21.90	0.00	0.37	379.16	8.02	1.000
	7.15	7.09	155.84	180.00	14.28	-1.13	-1.02	0.53	0.60	23.48	37.76	1.00	0.30	310.69	7.14	1.000

From Figure D-35, it can be seen that selection methods 3 and 4 provided the lowest *Overall abs % error* results in the majority of cases with method 1 providing a lower *Overall abs % error* result for model 4. As in previous sections, selection method 5 performed the worst and selection method 2 provided average results.

The efficiency plot of Figure D-36 indicates that for this particular set of models and PRBS sequence length, method 4 performed quite inefficiently. Selection methods 2 and 3 proved quite efficient while method 1 was the most efficient for model 4.

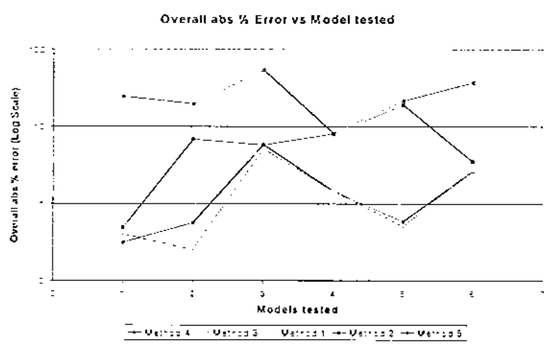

Figure D-35 - Overall percentage error for parameter selection methods

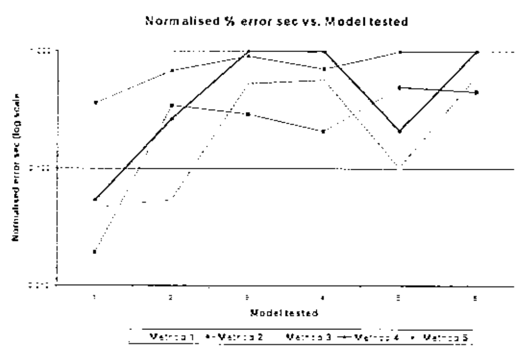

Figure D-36 - Simulation time weighted by (multiplied by) Percentage error for Typical process models tested, sequence length of 511 bits

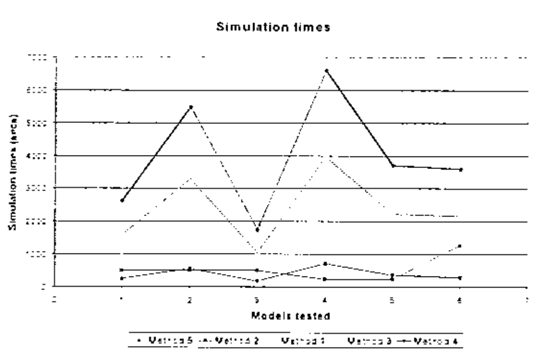

Figure D-37 - Simulation times for parameter selection methods

The simulation times plotted in Figure D-37 were similar to those of section D.2.2 differing only by a factor of 4 (3 in the case of selection method 5).

The results in blue in Table D-9 indicate instances where a large *% Error Lcmax & band* value was obtained when the relative difference between estimated and actual values was reasonably small. See section D.1.1 for a discussion on this type of result.

D.3 Typical Process Models with Generic PI Controller:

This section contains results obtained during the testing of the 6 typical process models discussed in section D.2, and generic PI controllers designed for these processes. The PI controllers were designed using a tuning rule formulated by Edgar *et al.* [87]. Table D-10 contains a list of the typical process models and their associated controllers.

Table D-10 - Typical processes & Generic controller

Model No.	Typical Process Models	Controllers
1	$\frac{1}{(2s+1)^2}e^{-2s}$	$1.5972\left(1+\frac{1}{9.618s}\right)$
2	$\frac{1}{(2s+1)^5}e^{-2s}$	$1.182\left(1+\frac{1}{22.21s}\right)$
3	$\frac{1}{(s+1)(s^2+s+1)}e^{-0.5s}$	$1.29\left(1+\frac{1}{6.36s}\right)$
4	$\frac{1}{(s+1)(5s+1)^2}e^{-2.5s}$	$2.22\left(1+\frac{1}{19.56s}\right)$
5	$\frac{1}{(s+1)^8}$	$1.11\left(1+\frac{1}{15.17s}\right)$
6	$\frac{1}{(5s+1)}e^{-5s}$	$1.33\left(1+\frac{1}{15.49s}\right)$

These typical process model and PI controllers were tested in closed loop configuration as illustrated in the Simulink model displayed in Figure D-38.

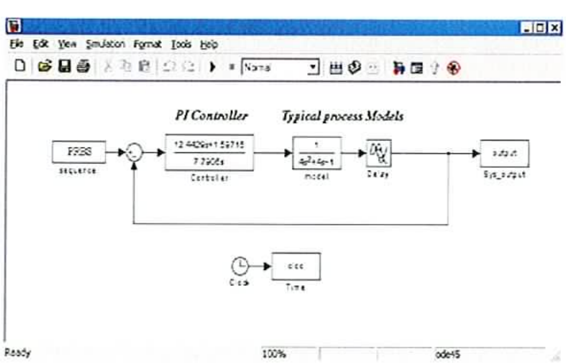

Figure D-38 - Simulink test set-up for Typical process models and PI controllers

From Figure D-38 it can be seen that the testing configuration for the typical process models and PI controllers is similar to that described in sections D.1 and D.2. Again the PRBS test signal was injected into the closed loop system on top of the existing set point.

D.3.1 Sequence Length = 63 bits:

The following results were obtained using a PRBS sequence length of 63 bits. For a comprehensive explanation of Table D-11 and Figure D-39 to Figure D-41 refer to section D.1.1.

Table D-11 - Results obtained for Typical processes & PI controller in closed loop using a sequence length of 63 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	% Error second	Normalised error sec
	3.38	3.37	36.85	36.86	0.31	7.84	8.40	0.75	0.78	10.20	10.51	0.50	1.06	133.56	1403.59	0.051
Method 1	3.63	3.61	56.50	56.67	0.84	6.77	6.97	0.33	0.33	3.65	4.49	0.06	2.40	302.40	1356.84	0.049
Hang C C	3.47	3.39	53.31	53.27	2.44	7.66	7.90	1.03	1.12	11.38	13.82	0.25	0.68	85.68	1184.31	0.043
and Sin K K	2.97	2.96	23.33	23.26	0.69	9.69	10.88	0.37	0.38	14.60	15.29	0.10	2.18	274.68	4199.89	0.152
(Tu needed)	3.69	3.67	63.10	63.46	1.15	6.53	6.63	0.49	0.49	2.22	3.36	0.00	1.64	206.64	695.15	0.025
	3.76	3.77	53.82	54.33	1.12	6.03	6.29	0.47	0.50	9.75	10.88	1.00	1.69	212.94	2315.91	0.084
	3.47	3.37	36.71	36.86	3.30	6.12	8.40	0.60	0.78	50.47	53.77	0.50	0.50	63.00	3387.49	0.123
Method 2	3.69	3.61	180.00	56.67	219.95	4.11	6.97	0.20	0.33	80.65	300.60	0.06	0.50	63.00	18937.59	0.685
Guinee RA	3.57	3.39	51.87	53.27	7.83	6.75	7.90	1.00	1.12	25.54	33.37	0.25	0.50	63.00	2102.08	0.076
(Tmn needed)	0.44	2.96	180.00	23.26	759.08	-1.47	10.88	0.40	0.38	118.47	877.55	0.10	0.25	31.50	27642.85	1.000
	4.50	3.67	180.00	63.46	206.36	3.01	6.63	0.40	0.49	73.20	279.57	0.00	0.25	31.50	8806.32	0.872
	3.75	3.77	53.29	54.33	2.33	6.01	6.29	0.48	0.50	8.74	11.07	1.00	1.25	157.50	1742.93	0.173
	3.38	3.37	36.78	36.86	0.43	8.07	8.40	0.74	0.78	8.62	9.06	0.50	1.61	202.86	1837.25	0.182
Method 3	3.61	3.61	56.54	56.67	0.36	6.94	6.97	0.32	0.33	4.19	4.55	0.06	3.77	475.02	2160.98	0.214
Marzocca C	3.43	3.39	52.85	53.27	1.84	7.43	7.90	1.07	1.12	10.55	12.39	0.25	1.12	141.12	1749.12	0.173
and Corsi F	2.97	2.96	23.27	23.26	0.30	10.01	10.88	0.36	0.38	12.58	12.88	0.10	3.30	415.80	5353.73	0.530
(Bw needed)	3.67	3.67	63.21	63.46	0.51	6.54	6.63	0.46	0.49	6.75	7.26	0.00	2.58	325.08	2359.75	0.234
	3.77	3.77	54.17	54.33	0.34	6.01	6.29	0.48	0.50	8.72	9.06	1.00	2.50	315.00	2852.47	0.283
	3.38	3.37	36.77	36.86	0.42	8.29	8.40	0.78	0.78	1.50	1.91	0.50	2.68	337.68	646.52	0.064
Method 4	3.61	3.61	56.58	56.67	0.30	6.95	6.97	0.33	0.33	1.42	1.71	0.06	6.28	791.28	1356.27	0.134
Yaacob S. and	3.41	3.39	52.96	53.27	0.98	7.90	7.90	1.13	1.12	0.57	1.55	0.25	1.86	234.36	363.97	0.036
Mohamed F.A	2.96	2.96	23.25	23.26	0.25	10.82	10.88	0.38	0.38	0.62	0.87	0.10	5.51	694.26	601.69	0.060
(Bw needed)	3.67	3.67	63.32	63.46	0.36	6.54	6.63	0.49	0.49	2.02	2.38	0.00	4.30	541.80	1292.10	0.128
	3.77	3.77	54.19	54.33	0.27	6.29	6.29	0.50	0.50	0.51	0.78	1.00	4.17	525.42	407.25	0.040
	3.40	3.37	39.12	36.86	6.87	7.72	8.40	0.49	0.78	44.78	51.65	0.50	0.40	50.90	2629.30	0.260
Method 5	3.70	3.61	51.91	56.67	10.96	3.60	6.97	0.28	0.33	63.79	74.74	0.06	1.07	135.07	10094.51	1.000
Landau I D	4.46	3.39	57.32	53.27	39.07	0.77	7.90	0.60	1.12	136.51	175.58	0.25	0.33	41.83	7344.86	0.758
(Tr needed)	2.86	2.96	26.90	23.26	18.85	9.96	10.88	0.36	0.38	12.42	31.27	0.10	0.82	103.32	3230.56	0.333
	3.96	3.67	63.75	63.46	8.54	0.95	6.63	0.50	0.49	88.55	97.09	0.00	0.79	99.79	9688.67	1.000
	3.68	3.77	55.38	54.33	4.35	6.04	6.29	0.48	0.50	7.46	11.81	1.00	0.62	78.12	922.59	1.000

From Figure D-39, it can be seen that selection method 4 out-performed every other selection method returning *Overall abs % error* values below 3% for each of the closed loop systems tested. Selection methods 1 and 3 performed reasonable well returning *Overall abs % error* values around the 10% mark, while methods 2 and 5 performed poorly for this particular PRBS sequence length.

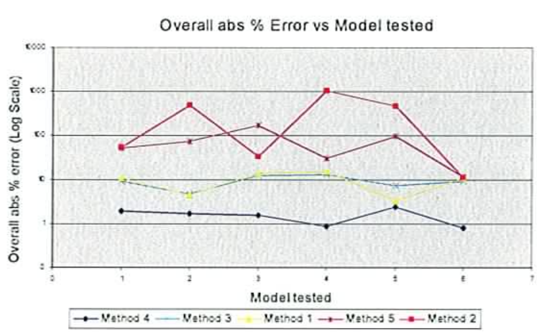

Figure D-39 - Overall percentage error for parameter selection methods

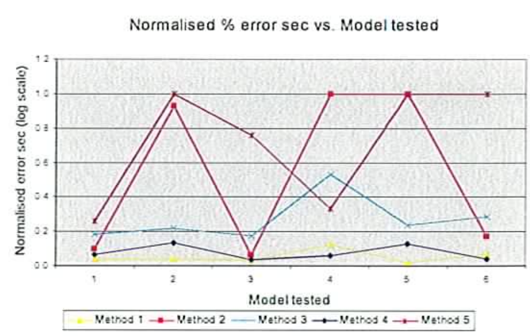

Figure D-40 - Simulation time weighted by (multiplied by) Percentage error for Typical process models & controller tested, sequence length of 63bits

From the efficiency plot of Figure D-40, it can be seen that methods 1 and 4 proved most efficient with method 3 also performing reasonably well. As was the case in previous sections, method 2 and 5 proved relatively inefficient.

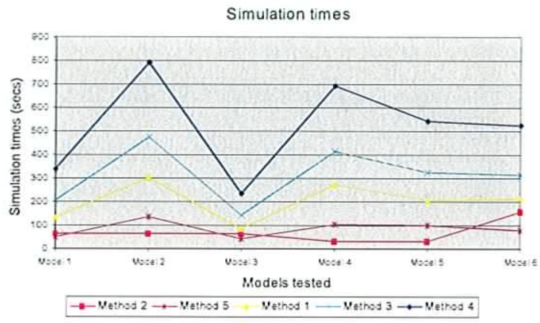

Figure D-41 - Simulation times for parameter selection methods

From the simulation times plot of Figure D-41, it can be seen that method 4 required the longest simulation times while both method 2 and 5 required significantly less time to run.

The results highlighted in gold in Table D-11 indicate cases where inaccurate estimates of the phase margin were obtained using selection method 2. These inaccuracies were as a result of an inability of selection method 2 to identify the 0dB crossing point of the open loop magnitude response plot. This is illustrated in Figure D-42 and Figure D-43.

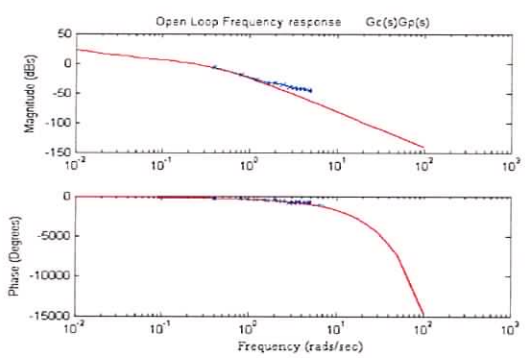

Figure D-42 - Open loop frequency response obtained for Typical process model 4 and associated controller using method 2 for a sequence length of 63 bits

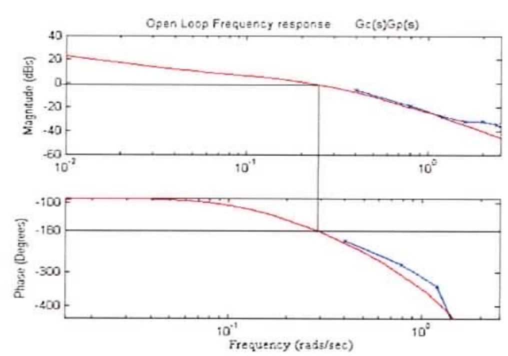

Figure D-43 - Close up of Figure D-42

From Figure D-43 it can be seen that the open loop frequency response estimate in blue failed to identify the point where the 0dB crossing point occurs (i.e. the intersection of the red and the black line on the magnitude response plot). This issue is discussed more thoroughly in section D.1.1. Increasing the PRBS pulse period and sequence length will improve the chances of identifying this crossing point.

The results highlighted in yellow in Table D-11 indicate cases where poor estimates of the closed loop frequency response peak values led to a large *% Error Lcmax & band* value. This issue has been discussed in section D.1.1 and is illustrated in Figure D-44.

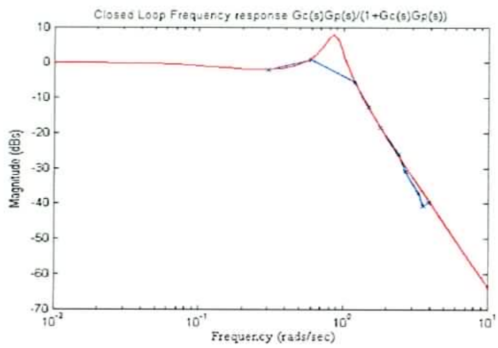

Figure D-44 - Closed loop frequency response plot obtained for Typical process model 3 and associated controller using Selection method 5 for sequence length of 63 bits

From Figure D-44 it can be seen that poor resolution around the point where the closed loop peak values occur, results in an inaccurate estimation of this peak value. This issue is discussed in more detail in section D.1.1.

D.3.2 Sequence Length = 127 bits:

The following results were obtained using a PRBS sequence length of 127 bits. Refer to section D.1.1 for a detailed explanation of Table D-12 and accompanying figures (Figure D-45 to Figure D-47).

Table D-12 - Results obtained for Typical processes & PI controller in closed loop using a sequence length of 127 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	% Error second	Normalised error sec
	3.38	3.37	36.77	36.66	0.42	8.14	8.40	0.75	0.78	7.38	7.60	0.50	1.06	269.24	2099.71	0.199
Method 1	3.61	3.61	56.57	56.67	0.35	6.66	6.97	0.33	0.33	1.65	2.00	0.06	2.40	609.60	1220.74	0.116
Hang C.C	3.42	3.39	52.94	53.27	1.42	7.79	7.90	1.09	1.12	3.93	5.35	0.25	0.68	172.72	923.71	0.087
and Sin K.K	2.97	2.96	23.24	23.26	0.29	10.64	10.68	0.36	0.38	6.68	6.97	0.10	2.18	553.72	3857.60	0.365
(Tu needed)	3.67	3.67	63.34	63.46	0.37	6.59	6.63	0.48	0.49	2.11	2.48	0.00	1.64	416.56	1031.02	0.098
	3.77	3.77	54.26	54.33	0.13	6.11	6.29	0.50	0.50	3.36	3.49	1.00	1.69	429.26	1496.96	0.142
	3.38	3.37	36.70	36.66	0.68	7.57	8.40	0.79	0.78	11.32	12.00	0.50	0.50	127.00	1523.42	0.144
Method 2	3.47	3.61	49.98	56.67	15.58	4.15	6.97	0.30	0.33	50.54	66.12	0.06	0.50	127.00	8397.45	0.795
Guinee R.A	3.43	3.39	52.93	53.27	1.82	7.43	7.90	1.09	1.12	8.82	10.64	0.25	0.50	127.00	1350.75	0.128
(Tmin needed)	2.67	2.96	23.59	23.26	11.08	5.74	10.88	0.20	0.38	95.16	106.24	0.10	0.25	63.50	6746.26	0.639
	3.97	3.67	54.47	63.46	22.49	4.27	6.63	0.40	0.49	54.90	77.39	0.00	0.25	63.50	4914.36	0.465
	3.76	3.77	54.15	54.33	0.39	5.96	6.29	0.51	0.50	8.17	8.56	1.00	1.25	317.50	2717.37	0.257
	3.38	3.37	36.77	36.66	0.41	8.31	8.40	0.77	0.78	2.56	2.97	0.50	1.61	408.94	1214.74	0.115
Method 3	3.61	3.61	56.58	56.67	0.22	6.90	6.97	0.33	0.33	1.65	1.87	0.06	3.77	957.58	1789.04	0.169
Marzocca C	3.40	3.39	52.94	53.27	0.99	7.78	7.90	1.10	1.12	2.90	3.89	0.25	1.12	284.48	1105.94	0.105
and Corsi F	2.96	2.96	23.24	23.26	0.29	10.66	10.68	0.37	0.38	1.59	1.88	0.10	3.30	838.20	1573.45	0.149
(Bw needed)	3.67	3.67	63.34	63.46	0.26	6.54	6.63	0.48	0.49	3.47	3.73	0.00	2.58	655.32	2447.07	0.232
	3.77	3.77	54.22	54.33	0.24	6.29	6.29	0.49	0.50	1.07	1.31	1.00	2.50	635.00	834.67	0.079
	3.38	3.37	36.77	36.66	0.41	8.40	8.40	0.78	0.78	0.63	1.04	0.50	2.68	680.72	710.82	0.067
Method 4	3.61	3.61	56.59	56.67	0.21	6.96	6.97	0.33	0.33	0.58	0.79	0.06	6.28	1595.10	1256.77	0.119
Yaacob S. and	3.40	3.39	52.96	53.27	0.91	7.89	7.90	1.12	1.12	0.38	1.28	0.25	1.86	472.44	605.46	0.057
Mohamed F.A	2.96	2.96	23.23	23.26	0.28	10.85	10.88	0.38	0.38	1.07	1.35	0.10	5.51	1399.50	1894.43	0.179
(Bw needed)	3.67	3.67	63.33	63.46	0.29	6.63	6.63	0.48	0.49	1.45	1.74	0.00	4.30	1092.20	1904.14	0.180
	3.77	3.77	54.25	54.33	0.20	6.27	6.29	0.50	0.50	0.68	0.89	1.00	4.17	1059.20	942.28	0.089
	3.33	3.37	34.20	36.66	8.33	7.83	8.40	0.71	0.78	15.36	23.69	0.50	0.35	88.14	2087.95	0.198
Method 5	3.58	3.61	53.56	56.67	6.20	5.52	6.97	0.27	0.33	39.10	45.29	0.06	0.92	233.17	10560.43	1.000
Landau I.D	3.90	3.39	54.02	53.27	16.27	3.25	7.90	1.05	1.12	65.54	81.81	0.25	0.28	72.14	5901.56	0.706
(Tr needed)	2.98	2.96	23.26	23.26	0.74	8.68	10.88	0.28	0.38	46.11	46.84	0.10	0.70	178.56	8364.37	1.000
	3.69	3.67	60.55	63.46	5.09	6.63	6.63	0.51	0.49	4.33	9.42	0.00	0.68	172.21	1621.96	0.256
	3.81	3.77	53.38	54.33	2.79	5.12	6.29	0.37	0.50	44.16	46.94	1.00	0.53	135.13	6343.58	1.000

From Figure D-45, it can be seen that selection method 4 returned the lowest *Overall abs % error* values for each of the closed loop systems tested. Methods 1 and 3 performed well, returning *Overall abs % error* values below 10%, while methods 2 and 5 performed the poorest. These results are similar to those obtained in section D.3.1.

From Figure D-46, it can be seen that selections methods 1, 3 and 4 proved relatively efficient, while methods 2 and 5 performed poorly in comparison.

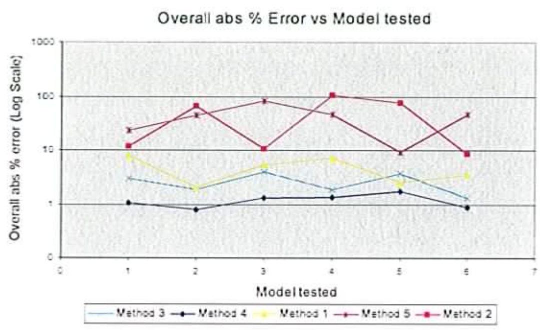

Figure D-45 - Overall percentage error for parameter selection methods

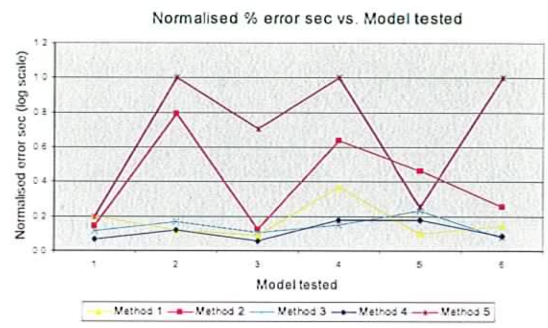

Figure D-46 - Simulation time weighted by (multiplied by) Percentage error for Typical process models & controller tested, sequence length of 127 bits

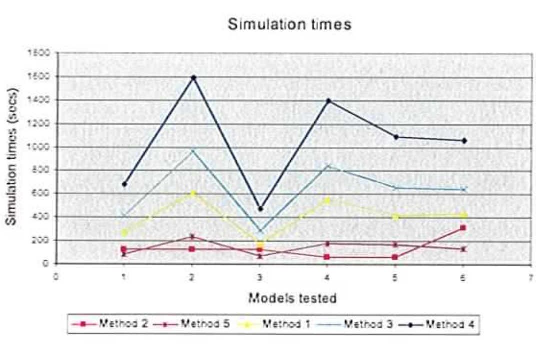

Figure D-47 - Simulation times for parameter selection methods

The simulation plot of Figure D-47 has an identical shape to that of Figure D-41. Again, the selection methods that returned the lowest *Overall abs % error* values required the longest simulation times and those methods that performed poorest required the shortest simulation times.

The results highlighted in blue in Table D-12 identify entries where large *% Error Lcmax & band* values were returned when the relative difference between the estimated and actual closed loop frequency domain characteristics is relatively small. This issue has been discussed in previous sections.

D.3.3 Sequence Length = 511 bits:

The following results were obtained using a PRBS sequence length of 511 bits. See section D.1.1 for a detailed explanation of Table D-13 and Figure D-48 to Figure D-50.

Table D-13 - Results obtained for Typical processes & PI controller in closed loop using a sequence length of 511 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	% Error second	Normalised error sec
	3.38	3.37	36.77	36.66	0.41	8.40	8.40	0.78	0.78	0.39	0.60	0.50	1.06	1083.30	870.40	0.077
Method 1	3.61	3.61	56.59	56.67	0.19	6.97	6.97	0.33	0.33	0.97	1.16	0.06	2.40	2452.60	2845.62	0.252
Hang C.C.	3.40	3.39	52.95	53.27	0.64	7.88	7.90	1.12	1.12	0.31	1.15	0.25	0.68	694.96	799.76	0.071
and Sin K.K.	2.96	2.96	23.23	23.26	0.28	10.87	10.88	0.38	0.38	0.67	0.96	0.10	2.18	2228.00	2128.56	0.188
(Tu needed)	3.67	3.67	63.34	63.46	0.26	6.62	6.63	0.48	0.49	2.26	2.53	0.00	1.64	1676.10	4232.39	0.375
	3.77	3.77	54.26	54.33	0.19	6.29	6.29	0.50	0.50	0.47	0.67	1.00	1.69	1727.20	1154.00	0.102
	3.38	3.37	36.77	36.66	0.41	8.38	8.40	0.76	0.78	2.54	2.96	0.50	0.50	511.00	1510.10	0.134
Method 2	3.62	3.61	56.58	56.67	0.39	6.91	6.97	0.32	0.33	4.02	4.42	0.06	0.50	511.00	2256.36	0.200
Gunee R.A.	3.40	3.39	52.96	53.27	0.68	7.90	7.90	1.11	1.12	1.24	2.12	0.25	0.50	511.00	1082.60	0.096
(Tmn needed)	2.97	2.96	23.25	23.26	0.45	10.40	10.88	0.34	0.38	13.85	14.30	0.10	0.25	255.50	3653.15	0.323
	3.68	3.67	63.24	63.46	0.61	6.44	6.63	0.49	0.49	3.25	3.86	0.00	0.25	255.50	985.71	0.087
	3.77	3.77	54.26	54.33	0.19	6.28	6.29	0.50	0.50	0.42	0.62	1.00	1.25	1277.50	786.41	0.070
	3.38	3.37	36.76	36.66	0.42	8.40	8.40	0.78	0.78	0.17	0.58	0.50	1.61	1645.40	958.53	0.085
Method 3	3.61	3.61	56.59	56.67	0.19	6.97	6.97	0.33	0.33	0.88	1.07	0.06	3.77	3852.90	4122.73	0.365
Marzocca C.	3.40	3.39	52.95	53.27	0.63	7.90	7.90	1.12	1.12	0.06	0.68	0.25	1.12	1144.60	1011.84	0.090
and Corsi F.	2.96	2.96	23.23	23.26	0.29	10.88	10.88	0.38	0.38	0.05	0.34	0.10	3.30	3372.60	1138.16	0.101
(Bw needed)	3.67	3.67	63.34	63.46	0.26	6.63	6.63	0.49	0.49	0.84	1.10	0.00	2.58	2636.80	2905.63	0.257
	3.77	3.77	54.26	54.33	0.19	6.29	6.29	0.50	0.50	0.37	0.56	1.00	2.50	2555.00	1437.59	0.127
	3.38	3.37	36.76	36.66	0.42	8.40	8.40	0.78	0.78	0.01	0.42	0.50	2.68	2739.00	1160.44	0.103
Method 4	3.61	3.61	56.59	56.67	0.19	6.97	6.97	0.33	0.33	0.92	1.11	0.06	6.28	6418.20	7132.78	0.632
Yaacob S. and	3.40	3.39	52.95	53.27	0.81	7.90	7.90	1.12	1.12	0.34	1.16	0.25	1.86	1900.90	2200.46	0.196
Mohamed F.A.	2.96	2.96	23.23	23.26	0.29	10.88	10.88	0.38	0.38	0.18	0.47	0.10	5.51	5631.20	2619.42	0.232
(Bw needed)	3.67	3.67	63.34	63.46	0.25	6.63	6.63	0.49	0.49	0.84	1.10	0.00	4.30	4394.60	4816.80	0.427
	3.77	3.77	54.26	54.33	0.19	6.29	6.29	0.50	0.50	0.26	0.45	1.00	4.17	4261.70	1927.14	0.171
	3.38	3.37	36.77	36.66	0.42	8.31	8.40	0.77	0.78	1.78	2.20	0.50	0.27	275.94	607.25	0.054
Method 5	3.64	3.61	55.73	56.67	2.55	6.13	6.97	0.33	0.33	12.92	15.48	0.06	0.71	729.71	11292.64	1.000
Landau I.D.	3.44	3.39	54.10	53.27	3.07	7.69	7.90	1.11	1.12	3.26	6.33	0.25	0.22	225.66	1429.41	0.169
(Tr needed)	2.93	2.96	22.94	23.26	2.39	10.70	10.88	0.34	0.38	12.79	15.19	0.10	0.55	558.01	8473.39	1.000
	3.66	3.67	60.76	63.46	4.43	6.65	6.63	0.49	0.49	0.45	4.88	0.00	0.53	539.62	2632.10	1.000
	3.77	3.77	54.03	54.33	0.74	6.24	6.29	0.51	0.50	2.04	2.78	1.00	0.41	422.09	1171.90	1.000

From Figure D-48, it can be seen that selection methods 3 and 4 returned relatively similar *Overall abs % error* values. Selection method 1 also performs well returning *Overall abs % error* values below 3%. Methods 2 and 5 also performed reasonably well returning *Overall abs % error* values below 16%.

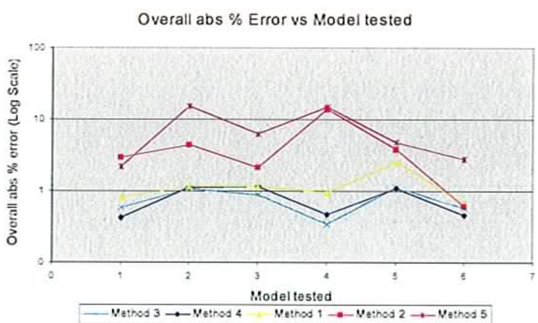

Figure D-48 - Overall percentage error for parameter selection methods

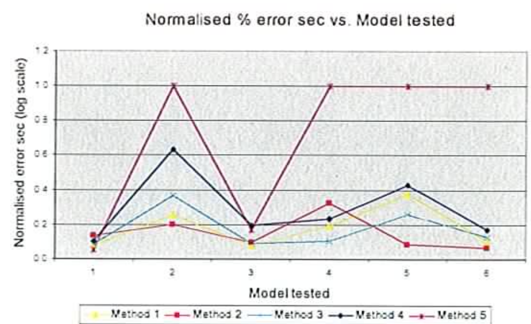

Figure D-49 - Simulation time weighted by (multiplied by) Percentage error for Typical process models & controller tested, sequence length of 511 bits

From the efficiency plot of Figure D-49, it can be seen that selection methods 1, 2, 3 and 4 were relatively efficient with method 5 proving most inefficient.

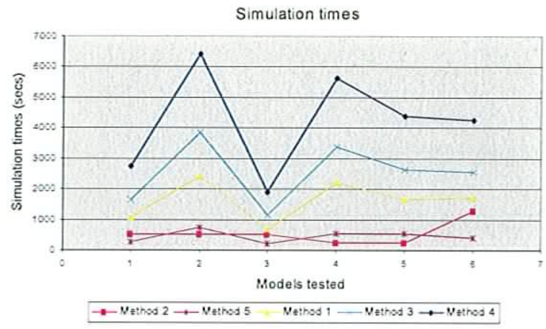

Figure D-50 - Simulation times for parameter selection methods

As previously discussed, the most accurate selection methods required the longest simulation times while the less accurate methods required significantly less time.

The entries in Table D-13 highlighted in blue represented results where large *% Error Lcmax & band* values were obtained when the relative difference between the estimated and actual values was small. This matter has been examined in previous sections.

D.3.4 Sequence Length = 1023 bits:

The following results were obtained using a PRBS sequence length of 1023 bits. Refer to section D.1.1 for a comprehensive explanation of Table D-14 and Figure D-51 to Figure D-53.

Table D-14 - Results obtained for Typical processes & PI controller in closed loop using a sequence length of 1023 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	% Error second	Normalised error sec
	3.38	3.37	36.76	36.66	0.42	8.40	8.40	0.78	0.78	0.30	0.72	0.50	1.06	2168.60	1563.94	0.120
Method 1	3.61	3.61	56.59	56.67	0.19	6.97	6.97	0.33	0.33	0.66	1.05	0.06	2.40	4910.40	5148.58	0.394
Hang C.C.	3.40	3.39	52.96	53.27	0.82	7.90	7.90	1.12	1.12	0.03	0.66	0.25	0.68	1391.30	1190.41	0.091
and Sin K.K	2.96	2.96	23.23	23.26	0.29	10.88	10.88	0.38	0.38	0.10	0.39	0.10	2.18	4460.30	1734.37	0.133
(Tu needed)	3.67	3.67	63.34	63.46	0.26	6.63	6.63	0.48	0.49	1.47	1.73	0.00	1.64	3355.40	5803.98	0.444
	3.77	3.77	54.26	54.33	0.19	6.29	6.29	0.50	0.50	0.33	0.52	1.00	1.69	3457.70	1781.85	0.136
	3.38	3.37	36.77	36.66	0.41	8.38	8.40	0.77	0.78	1.01	1.43	0.50	0.50	1023.00	1459.45	0.112
Method 2	3.61	3.61	56.58	56.67	0.24	6.91	6.97	0.33	0.33	1.30	1.54	0.06	0.50	1023.00	1578.79	0.121
Guinee R.A.	3.40	3.39	52.96	53.27	0.83	7.90	7.90	1.12	1.12	0.21	1.03	0.25	0.50	1023.00	1055.94	0.081
(Tmin needed)	2.97	2.96	23.24	23.26	0.30	10.35	10.88	0.37	0.38	7.90	8.20	0.10	0.25	511.50	4196.13	0.321
	3.67	3.67	63.29	63.46	0.43	6.43	6.63	0.49	0.49	3.34	3.77	0.00	0.25	511.50	1928.45	0.148
	3.77	3.77	54.26	54.33	0.19	6.29	6.29	0.50	0.50	0.26	0.45	1.00	1.25	2557.50	1139.57	0.087
	3.38	3.37	36.76	36.66	0.42	8.40	8.40	0.78	0.78	0.30	0.71	0.50	1.61	3294.10	2350.36	0.180
Method 3	3.61	3.61	56.59	56.67	0.19	6.97	6.97	0.33	0.33	0.78	0.97	0.06	3.77	7713.40	7448.51	0.570
Marzocca C.	3.40	3.39	52.96	53.27	0.82	7.90	7.90	1.12	1.12	0.15	0.97	0.25	1.12	2291.50	2215.55	0.170
and Corsi F.	2.96	2.96	23.23	23.26	0.29	10.88	10.88	0.38	0.38	0.11	0.40	0.10	3.30	6751.60	2696.22	0.206
(Bw needed)	3.67	3.67	63.34	63.46	0.25	6.63	6.63	0.49	0.49	0.95	1.21	0.00	2.58	5278.70	6374.60	0.488
	3.77	3.77	54.26	54.33	0.19	6.29	6.29	0.50	0.50	0.26	0.44	1.00	2.50	5115.00	2268.44	0.174
	3.38	3.37	36.76	36.66	0.41	8.40	8.40	0.78	0.78	0.22	0.64	0.50	2.68	5483.30	3502.44	0.268
Method 4	3.61	3.61	56.59	56.67	0.19	6.97	6.97	0.33	0.33	0.83	1.02	0.06	6.28	12849.00	13069.66	1.000
Yaacob S. and	3.40	3.39	52.96	53.27	0.82	7.90	7.90	1.12	1.12	0.27	1.09	0.25	1.66	3806.60	4129.49	0.389
Mohamed F.A	2.96	2.96	23.23	23.26	0.29	10.88	10.88	0.38	0.38	0.07	0.35	0.10	5.51	11273.00	3982.32	0.375
(Bw needed)	3.67	3.67	63.34	63.46	0.25	6.63	6.63	0.49	0.49	0.95	1.21	0.00	4.30	8797.60	10624.29	1.000
	3.77	3.77	54.26	54.33	0.19	6.29	6.29	0.50	0.50	0.48	0.66	1.00	4.17	8531.60	5663.81	0.734
	3.39	3.37	35.90	36.66	3.22	8.45	8.40	0.78	0.78	1.07	4.29	0.50	0.24	497.18	2131.28	0.276
Method 5	3.61	3.61	55.87	56.67	1.51	6.92	6.97	0.33	0.33	2.10	3.60	0.06	0.64	1315.60	4738.66	0.614
Landau I.D.	3.30	3.39	48.07	53.27	12.55	8.19	7.90	1.11	1.12	4.40	16.96	0.25	0.20	407.15	6903.48	0.894
(Tr needed)	2.96	2.96	23.45	23.26	0.86	10.62	10.88	0.37	0.38	3.66	4.72	0.10	0.49	1006.60	4755.07	0.616
	3.71	3.67	62.67	63.46	2.49	6.60	6.63	0.47	0.49	5.45	7.94	0.00	0.48	971.65	7720.21	1.000
	3.77	3.77	54.57	54.33	0.50	6.22	6.29	0.50	0.50	2.08	2.58	1.00	0.37	761.11	1966.61	1.000

From Figure D-51, it can be seen that selection methods 1, 3 and 4 all provided extremely accurate results. Although selection method 5 performed poorly in previous results for this particular instance it returned *Overall abs % error* results below 10% in a number of cases.

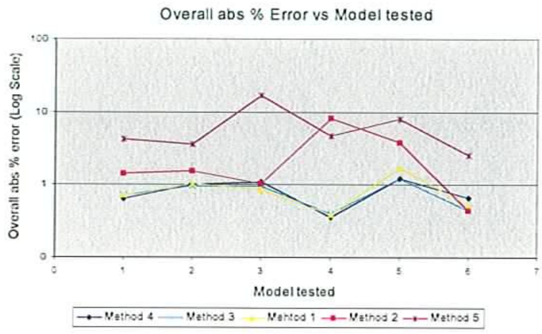

Figure D-51 - Overall percentage error for parameter selection methods

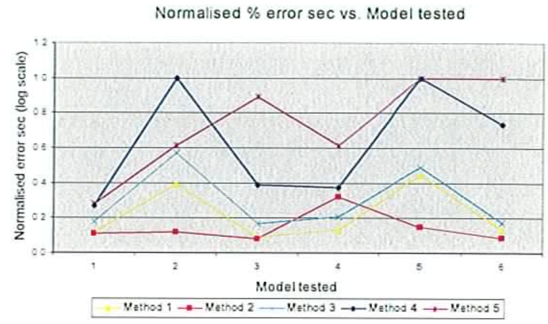

Figure D-52 - Simulation time weighted by (multiplied by) Percentage error for Typical process models & controller tested, sequence length of 1023 bits

From the efficiency plots of Figure D-52, it can be seen that methods 1, 2 and 3 performed efficiently, while methods 4 and 5 did not perform as well.

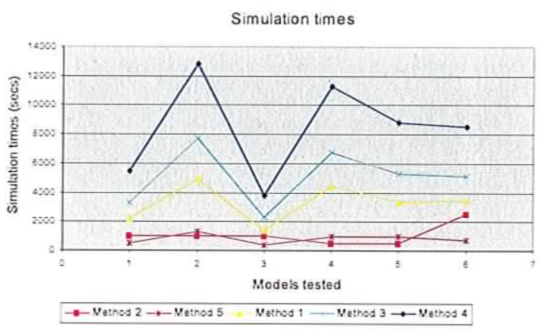

Figure D-53 - Simulation times for parameter selection methods

The simulation time plots of Figure D-53 were as expected with method 4 requiring the longest simulation times and methods 2 and 5 requiring the shortest times. The other two methods lie in between these two extremes.

The blue entry in Table D-14 highlights a result where a large *% Error Lcmax & band* value was obtained when the relative difference between the estimated and actual closed loop characteristics values was small. This matter has been discussed in previous sections.

D.4 Accuracy in the Presence of Noise (Method 4):

For each of the typical process models and PI controllers, the accuracy of the PRBS evaluation approach in the presence of Gaussian white noise was tested. A band limited white noise block was inserted into the Simulink model, as illustrated in Figure D-54, and the power of the noise output was varied from 10% to 50% of the closed loop system's controlled variable signal. This noise signal was then added to the controlled variable, as shown in Figure D-54, in an effort to mimic the effects of measurement noise in a real system.

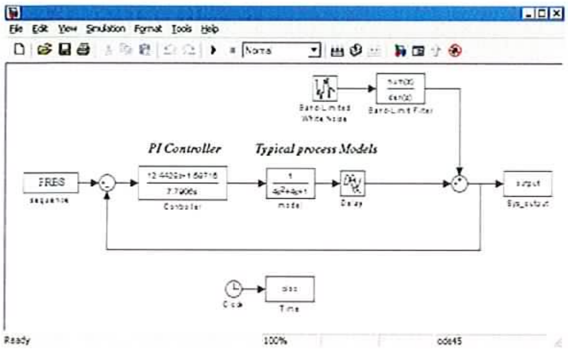

Figure D-54 - Simulink test set-up for Typical process models and PI controllers in the presence of noise

A sampling frequency of 100 Hz was chosen, resulting in a Nyquist frequency of 50 Hz (i.e. Nyquist = Sampling frequency/2). The noise signal was band limited to the 0.8 times the Nyquist frequency of the closed loop system (40Hz) using a 12^{th} order Butterworth filter. The noise power parameter of the noise block was set using equation (D.4.1)

$$\boxed{\text{Noise power} = \left(\left(\text{rms Amplitude of noise}\right) \times \sqrt{\text{sampling period}}\right)^2} \quad \text{(D.4.1)}$$

Where the

$$\boxed{\text{rms Amplitude of noise} = \text{rms Amplitude of controlled variable} \times (10\%,\ 20\% \ldots\ 50\%)}$$

On the basis of results obtained in the previous sections, it was decided to use selection method 4 to evaluate the accuracy of the PRBS evaluation approach. Therefore the following results were obtained using selection method 4 only and varying PRBS sequence lengths. Table D-10 contains a list of the typical process models and controllers tested in this section.

D.4.1 Typical Process Models & Controllers, Sequence Length = 63 bits:

The following results were obtained using a PRBS sequence length of 63 bits and selection method 4. A detailed explanation of Table D-15 can be found in section D.1.1.

Table D-15 - Results obtained for Typical processes & PI controller in closed loop for varying noise levels, sequence length = 63 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	Noise Level
Model 1	3.38	3.37	36.77	36.86	0.42	8.29	8.40	0.78	0.78	1.50	1.91	0.50	2.68	337.68	
Model 2	3.61	3.61	56.58	56.67	0.30	6.95	6.97	0.33	0.33	1.42	1.71	0.06	6.28	791.28	
Model 3	3.41	3.39	52.95	53.27	0.98	7.90	7.90	1.13	1.12	0.57	1.55	0.25	1.86	234.36	0%
Model 4	2.96	2.96	23.25	23.26	0.25	10.82	10.88	0.38	0.38	0.62	0.87	0.10	5.51	694.26	
Model 5	3.67	3.67	63.32	63.46	0.36	6.54	6.63	0.49	0.49	2.02	2.38	0.00	4.30	541.80	
Model 6	3.77	3.77	54.19	54.33	0.27	6.29	6.29	0.50	0.50	0.51	0.78	1.00	4.17	525.42	
Model 1	3.44	3.37	36.94	36.86	2.37	8.21	8.40	0.74	0.78	6.81	9.17	0.50	2.68	337.68	
Model 2	3.63	3.61	54.69	56.67	4.11	6.96	6.97	0.33	0.33	1.17	5.28	0.06	6.28	791.28	
Model 3	3.33	3.39	53.69	53.27	2.90	8.08	7.90	1.13	1.12	2.60	5.70	0.25	1.86	234.36	10%
Model 4	3.03	2.96	22.99	23.26	3.59	10.60	10.88	0.38	0.38	0.76	4.34	0.10	5.51	694.26	
Model 5	3.68	3.67	62.76	63.46	1.36	6.33	6.63	0.49	0.49	5.06	6.42	0.00	4.30	541.80	
Model 6	3.71	3.77	54.73	54.33	2.36	6.36	6.29	0.50	0.50	1.50	3.86	1.00	4.17	525.42	
Model 1	3.46	3.37	37.00	36.86	3.01	8.18	8.40	0.74	0.78	7.19	10.20	0.50	2.68	337.68	
Model 2	3.64	3.61	54.12	56.67	5.33	6.97	6.97	0.33	0.33	1.07	6.39	0.06	6.28	791.28	
Model 3	3.30	3.39	54.14	53.27	4.23	8.15	7.90	1.13	1.12	3.72	7.94	0.25	1.86	234.36	20%
Model 4	3.06	2.96	22.88	23.26	4.99	10.60	10.88	0.38	0.38	0.81	5.80	0.10	5.51	694.26	
Model 5	3.68	3.67	62.54	63.46	1.74	6.25	6.63	0.49	0.49	6.33	8.07	0.00	4.30	541.80	
Model 6	3.68	3.77	55.03	54.33	3.61	6.39	6.29	0.50	0.50	1.97	5.58	1.00	4.17	525.42	
Model 1	3.47	3.37	37.04	36.86	3.39	8.16	8.40	0.74	0.78	7.49	10.87	0.50	2.68	337.68	
Model 2	3.64	3.61	53.75	56.67	6.15	6.93	6.97	0.33	0.33	1.13	7.28	0.06	6.28	791.28	
Model 3	3.28	3.39	54.30	53.27	5.17	8.21	7.90	1.13	1.12	4.42	9.59	0.25	1.86	234.36	30%
Model 4	3.08	2.96	22.60	23.26	6.08	10.79	10.88	0.36	0.38	5.56	11.64	0.10	5.51	694.26	
Model 5	3.68	3.67	62.37	63.46	2.03	6.19	6.63	0.49	0.49	7.31	9.34	0.00	4.30	541.80	
Model 6	3.66	3.77	55.30	54.33	4.66	6.41	6.29	0.50	0.50	2.32	6.98	1.00	4.17	525.42	
Model 1	3.47	3.37	37.08	36.86	3.63	8.13	8.40	0.74	0.78	7.74	11.36	0.50	2.68	337.68	
Model 2	3.65	3.61	53.48	56.67	6.78	6.98	6.97	0.33	0.33	1.20	7.98	0.06	6.28	791.28	
Model 3	3.26	3.39	54.42	53.27	5.93	8.25	7.90	1.13	1.12	5.00	10.93	0.25	1.86	234.36	40%
Model 4	3.10	2.96	22.73	23.26	7.00	10.79	10.88	0.36	0.38	5.60	12.60	0.10	5.51	694.26	
Model 5	3.68	3.67	62.24	63.46	2.27	6.16	6.63	0.49	0.49	7.66	9.93	0.00	4.30	541.80	
Model 6	3.64	3.77	55.57	54.33	5.62	6.43	6.29	0.50	0.50	2.62	8.23	1.00	4.17	525.42	
Model 1	3.48	3.37	37.10	36.86	3.79	8.12	8.40	0.74	0.78	7.96	11.74	0.50	2.68	337.68	
Model 2	3.65	3.61	53.26	56.67	7.29	6.93	6.97	0.33	0.33	1.26	8.55	0.06	6.28	791.28	
Model 3	3.25	3.39	54.52	53.27	6.58	8.29	7.90	1.13	1.12	5.52	12.09	0.25	1.86	234.36	50%
Model 4	3.11	2.96	22.67	23.26	7.81	10.78	10.88	0.36	0.38	5.63	13.44	0.10	5.51	694.26	
Model 5	3.68	3.67	62.12	63.46	2.47	6.14	6.63	0.49	0.49	7.94	10.41	0.00	4.30	541.80	
Model 6	3.63	3.77	55.83	54.33	6.52	6.44	6.29	0.50	0.50	2.88	9.40	1.00	4.17	525.42	

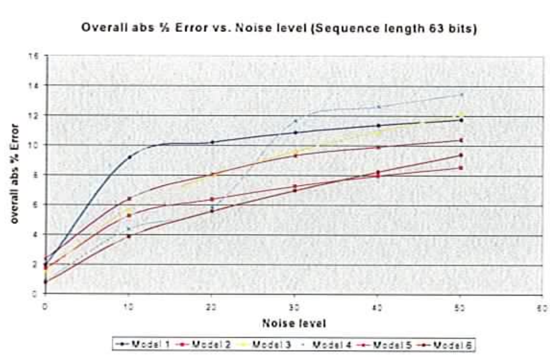

Figure D-55 - Percentage error for varying noise levels, sequence length of 63 bits

From Figure D-55, it can be seen that as the noise power increases from 10% to 50% of the controlled variable the *Overall abs % error* values for each of the closed loop systems also increases. This is to be expected.

It is worth noting from Figure D-55 that even for a very high noise power level (i.e. 50%) the *Overall abs % error* values obtained are still quite low. This illustrated how effectively the PRBS based approach deals with system measurement noise.

D.4.2 Typical Process Models & Controllers, Sequence Length = 127 bits:

The following results were obtained using a PRBS sequence length of 127 bits and selection method 4. Refer to section D.1.1 for a detailed explanation of Table D-15.

Table D-16 - Results obtained for Typical processes & PI controller in closed loop for varying noise levels, sequence length = 127 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	Noise Level
Model 1	3.38	3.37	36.77	36.66	0.42	8.29	8.40	0.78	0.78	1.50	1.91	0.50	2.68	337.68	
Model 2	3.61	3.61	56.58	56.67	0.30	6.95	6.97	0.33	0.33	1.42	1.71	0.06	6.28	791.28	
Model 3	3.41	3.39	52.95	53.27	0.98	7.90	7.90	1.13	1.12	0.57	1.55	0.25	1.66	234.36	0%
Model 4	2.96	2.96	23.25	23.26	0.25	10.62	10.68	0.38	0.38	0.62	0.87	0.10	5.51	694.26	
Model 5	3.67	3.67	63.32	63.46	0.36	6.54	6.63	0.49	0.49	2.02	2.38	0.00	4.30	541.60	
Model 6	3.77	3.77	54.19	54.33	0.27	6.29	6.29	0.50	0.50	0.51	0.78	1.00	4.17	525.42	
Model 1	3.35	3.37	36.43	36.66	1.66	8.55	8.40	0.78	0.78	2.39	4.25	0.50	2.68	680.72	
Model 2	3.55	3.61	56.73	56.67	1.61	7.07	6.97	0.33	0.33	1.67	3.28	0.06	6.28	1595.10	
Model 3	3.46	3.39	53.46	53.27	2.33	7.93	7.90	1.12	1.12	0.62	2.95	0.25	1.66	472.44	10%
Model 4	2.98	2.96	23.16	23.26	1.21	11.07	10.68	0.37	0.38	4.65	6.06	0.10	5.51	1399.50	
Model 5	3.64	3.67	64.43	63.45	2.43	6.68	6.63	0.48	0.49	2.18	4.61	0.00	4.30	1092.20	
Model 6	3.76	3.77	54.66	54.33	0.92	6.18	6.29	0.51	0.50	3.71	4.63	1.00	4.17	1059.20	
Model 1	3.34	3.37	36.33	36.66	2.53	8.61	8.40	0.76	0.78	5.52	8.06	0.50	2.68	680.72	
Model 2	3.53	3.61	56.78	56.67	2.33	7.14	6.97	0.33	0.33	2.65	4.98	0.06	6.28	1595.10	
Model 3	3.49	3.39	53.48	53.27	3.28	7.96	7.90	1.12	1.12	0.86	4.13	0.25	1.66	472.44	20%
Model 4	2.99	2.96	23.09	23.26	1.76	11.16	10.68	0.37	0.38	5.68	7.44	0.10	5.51	1399.50	
Model 5	3.62	3.67	64.62	63.46	3.46	6.77	6.63	0.47	0.49	5.88	9.34	0.00	4.30	1092.20	
Model 6	3.75	3.77	54.88	54.33	1.44	6.18	6.29	0.51	0.50	3.75	5.19	1.00	4.17	1059.20	
Model 1	3.33	3.37	36.26	36.66	3.04	8.70	8.40	0.76	0.78	6.56	9.60	0.50	2.68	680.72	
Model 2	3.51	3.61	64.05	56.67	15.65	7.19	6.97	0.34	0.33	5.79	21.44	0.06	6.28	1595.10	
Model 3	3.52	3.39	53.46	53.27	4.02	7.96	7.90	1.12	1.12	1.05	5.07	0.25	1.66	472.44	30%
Model 4	3.00	2.96	23.03	23.26	2.24	11.23	10.68	0.37	0.38	6.31	8.55	0.10	5.51	1399.50	
Model 5	3.61	3.67	65.11	63.46	4.23	6.84	6.63	0.47	0.49	6.90	11.13	0.00	4.30	1092.20	
Model 6	3.75	3.77	55.05	54.33	1.84	6.23	6.29	0.51	0.50	2.98	4.82	1.00	4.17	1059.20	
Model 1	3.32	3.37	36.21	36.66	3.45	8.78	8.40	0.76	0.78	7.43	10.88	0.50	2.68	680.72	
Model 2	3.50	3.61	64.35	56.67	16.59	7.23	6.97	0.34	0.33	6.42	23.01	0.06	6.28	1595.10	
Model 3	3.54	3.39	53.43	53.27	4.67	7.98	7.90	1.12	1.12	1.21	5.88	0.25	1.66	472.44	40%
Model 4	3.00	2.96	22.99	23.26	2.54	11.29	10.68	0.37	0.38	6.84	9.39	0.10	5.51	1399.50	
Model 5	3.60	3.67	65.35	63.46	4.87	6.90	6.63	0.47	0.49	7.76	12.63	0.00	4.30	1092.20	
Model 6	3.75	3.77	55.21	54.33	2.18	6.27	6.29	0.51	0.50	2.34	4.51	1.00	4.17	1059.20	
Model 1	3.31	3.37	36.17	36.66	3.82	8.84	8.40	0.76	0.78	8.19	12.01	0.50	2.68	680.72	
Model 2	3.48	3.61	64.57	56.67	17.35	7.27	6.97	0.34	0.33	6.97	24.32	0.06	6.28	1595.10	
Model 3	3.56	3.39	53.40	53.27	5.24	7.99	7.90	1.12	1.12	1.36	6.61	0.25	1.66	472.44	50%
Model 4	3.00	2.96	22.97	23.26	2.61	11.34	10.68	0.37	0.38	7.31	10.12	0.10	5.51	1399.50	
Model 5	3.59	3.67	65.55	63.46	5.43	6.95	6.63	0.47	0.49	8.51	13.95	0.00	4.30	1092.20	
Model 6	3.75	3.77	55.36	54.33	2.47	6.31	6.29	0.51	0.50	2.30	4.77	1.00	4.17	1059.20	

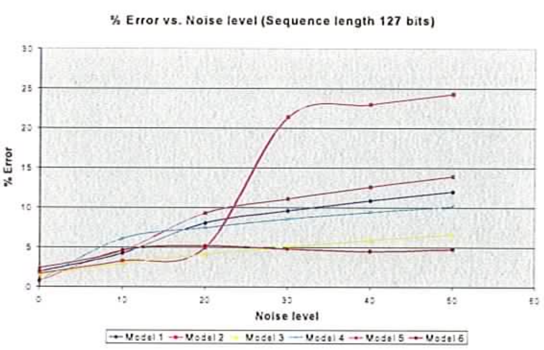

Figure D-56 - Percentage error for varying noise levels, sequence length of 127 bits

Comparing Figure D-56 to Figure D-55 it can be seen that increasing the PRBS sequence length from 63 bits to 127 bits had an adverse effect on the *Overall abs % error* values obtained. It can be seen that the *Overall abs % error* values actually increased when the PRBS sequence length was increased.

Increasing the PRBS sequence length results in an increase in the simulation times necessary to carry out the PRBS test. This means that the effects of the simulated measurement noise affect the system for a longer period of time. This is one possible explanation for the increase in the *Overall abs % error* results for an increase in PRBS sequence length. Filtering the controlled variable output prior to the signal processing stage and calculation of the required system characteristics may help attenuate the effects of the noise.

D.4.3 Typical Process Models & Controllers, Sequence Length = 511 bits:

The following results were obtained using a PRBS sequence length of 511 bits and selection method 4. See section D.1.1 for a detailed explanation of Table D-15.

Table D-17 - Results obtained for Typical processes & PI controller in closed loop for varying noise levels, sequence length = 511 bits

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	td/Tc ratio	period (secs)	Sim time (secs)	Noise Level
Model 1	3.38	3.37	36.77	36.66	0.42	8.29	8.40	0.78	0.78	1.50	1.91	0.50	2.68	337.68	
Model 2	3.61	3.61	56.58	56.67	0.30	6.95	6.97	0.33	0.33	1.42	1.71	0.06	6.28	791.28	
Model 3	3.41	3.39	52.95	53.27	0.98	7.90	7.90	1.13	1.12	0.57	1.55	0.25	1.66	234.36	0%
Model 4	2.96	2.96	23.25	23.26	0.25	10.82	10.68	0.38	0.38	0.62	0.87	0.10	5.51	694.26	
Model 5	3.67	3.67	63.32	63.45	0.36	6.54	6.63	0.49	0.49	2.02	2.38	0.00	4.30	541.60	
Model 6	3.77	3.77	54.19	54.33	0.27	6.29	6.29	0.50	0.50	0.51	0.78	1.00	4.17	525.42	
Model 1	3.40	3.37	37.48	36.66	2.58	8.64	8.40	0.78	0.78	3.39	5.97	0.50	2.68	2739.00	
Model 2	3.61	3.61	58.20	56.67	2.79	7.03	6.97	0.33	0.33	1.09	3.88	0.06	6.28	6418.20	
Model 3	3.39	3.39	54.10	53.27	1.73	8.03	7.90	1.12	1.12	1.84	3.58	0.25	1.66	1900.90	10%
Model 4	2.97	2.96	23.82	23.26	2.82	10.98	10.68	0.37	0.38	2.24	5.06	0.10	5.51	5631.20	
Model 5	3.72	3.67	65.65	63.46	5.07	6.80	6.63	0.49	0.49	3.34	8.40	0.00	4.30	4394.60	
Model 6	3.64	3.77	54.01	54.33	3.90	6.46	6.29	0.50	0.50	3.48	7.39	1.00	4.17	4261.70	
Model 1	3.41	3.37	37.75	36.66	3.48	8.74	8.40	0.77	0.78	5.25	8.72	0.50	2.68	2739.00	
Model 2	3.61	3.61	58.51	56.67	3.32	7.06	6.97	0.33	0.33	1.52	4.84	0.06	6.28	6418.20	
Model 3	3.38	3.39	55.19	53.27	3.92	8.08	7.90	1.09	1.12	4.86	8.77	0.25	1.66	1900.90	20%
Model 4	2.98	2.96	23.99	23.26	3.75	11.03	10.68	0.37	0.38	2.75	6.49	0.10	5.51	5631.20	
Model 5	3.74	3.67	66.68	63.45	6.89	6.87	6.63	0.49	0.49	4.39	11.28	0.00	4.30	4394.60	
Model 6	3.60	3.77	53.78	54.33	5.48	6.53	6.29	0.50	0.50	4.72	10.19	1.00	4.17	4261.70	
Model 1	3.41	3.37	37.96	36.66	4.12	8.82	8.40	0.77	0.78	6.21	10.33	0.50	2.68	2739.00	
Model 2	3.61	3.61	58.84	56.67	3.90	7.08	6.97	0.33	0.33	2.49	6.39	0.06	6.28	6418.20	
Model 3	3.38	3.39	55.12	53.27	3.90	8.12	7.90	1.09	1.12	5.36	9.26	0.25	1.66	1900.90	30%
Model 4	2.98	2.96	24.07	23.26	4.30	11.07	10.68	0.37	0.38	4.89	9.19	0.10	5.51	5631.20	
Model 5	3.75	3.67	67.14	63.46	8.03	6.93	6.63	0.49	0.49	5.25	13.28	0.00	4.30	4394.60	
Model 6	3.57	3.77	53.59	54.33	6.66	6.59	6.29	0.50	0.50	5.67	12.33	1.00	4.17	4261.70	
Model 1	3.41	3.37	38.11	36.66	4.65	8.89	8.40	0.77	0.78	7.01	11.66	0.50	2.68	2739.00	
Model 2	3.61	3.61	59.12	56.67	4.38	7.10	6.97	0.33	0.33	2.76	7.14	0.06	6.28	6418.20	
Model 3	3.37	3.39	55.04	53.27	3.83	8.16	7.90	1.09	1.12	5.65	9.69	0.25	1.66	1900.90	40%
Model 4	2.99	2.96	24.11	23.26	4.61	11.11	10.68	0.37	0.38	5.21	9.82	0.10	5.51	5631.20	
Model 5	3.76	3.67	67.17	63.45	8.43	7.00	6.63	0.49	0.49	6.39	14.82	0.00	4.30	4394.60	
Model 6	3.54	3.77	53.42	54.33	7.66	6.64	6.29	0.50	0.50	6.47	14.13	1.00	4.17	4261.70	
Model 1	3.42	3.37	38.26	36.66	5.10	8.95	8.40	0.77	0.78	7.72	12.82	0.50	2.68	2739.00	
Model 2	3.61	3.61	59.36	56.67	4.79	7.12	6.97	0.33	0.33	3.01	7.80	0.06	6.28	6418.20	
Model 3	3.37	3.39	58.54	53.27	10.48	8.19	7.90	1.09	1.12	6.31	16.79	0.25	1.66	1900.90	50%
Model 4	2.99	2.96	24.12	23.26	4.79	11.14	10.68	0.37	0.38	5.49	10.29	0.10	5.51	5631.20	
Model 5	3.77	3.67	67.18	63.46	8.75	7.07	6.63	0.49	0.49	7.42	16.17	0.00	4.30	4394.60	
Model 6	3.52	3.77	60.56	54.33	18.05	6.69	6.29	0.50	0.50	7.18	25.22	1.00	4.17	4261.70	

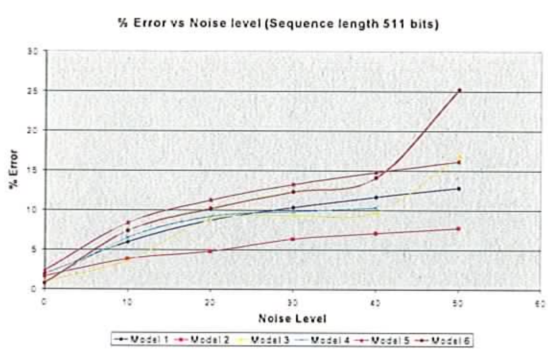

Figure D-57 - Percentage error for varying noise levels, sequence length of 511 bits

As was the case in the previous section, increasing the PRBS sequence length further causes the *Overall abs % error* values obtained to increase also. Figure D-57 illustrates this point by displaying the *Overall abs % error* values obtained for each of the selection methods for a PRBS sequence length of 511 bits.

From the results obtained in sections D.4.1 and D.4.2, it would appear to be advantageous to keep the PRBS sequence length as short as possible, thus keeping the simulation time necessary short.

D.5 Effects of Ignoring First PRBS Period:

Figure D-58 to Figure D-63 were obtained during the testing of typical process model 5 and its associated controller in closed loop. PRBS pulse period selection method 4 and a PRBS sequence length of 127 bits was used during the testing procedure. The system's actual open and closed loop frequency response plots are illustrated in red in Figure D-58 to Figure D-63 and the blue line represents the estimated response.

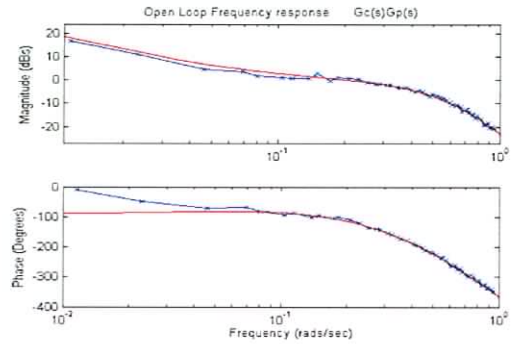

Figure D-58 - Open loop frequency response obtained using only one PRBS period.

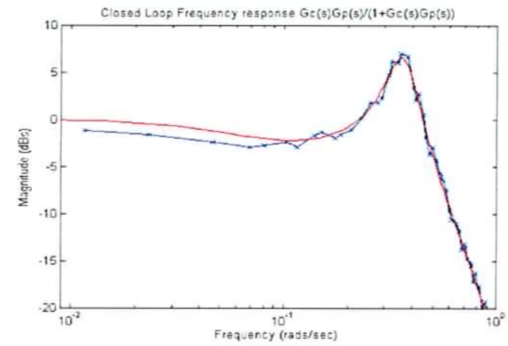

Figure D-59 - Closed loop frequency response obtained using only one PRBS period.

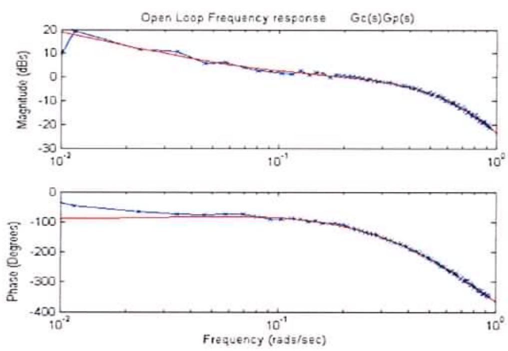

Figure D-60 - Open loop frequency response obtained using two PRBS periods.

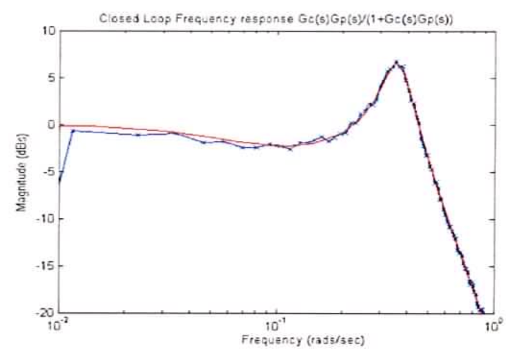

Figure D-61 - Closed loop frequency response obtained using two PRBS periods.

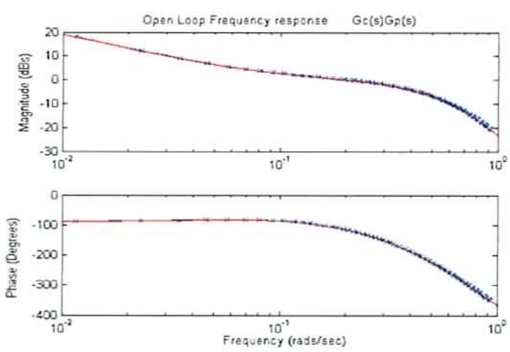

Figure D-62 - Open loop frequency response obtained when first PRBS period is ignored.

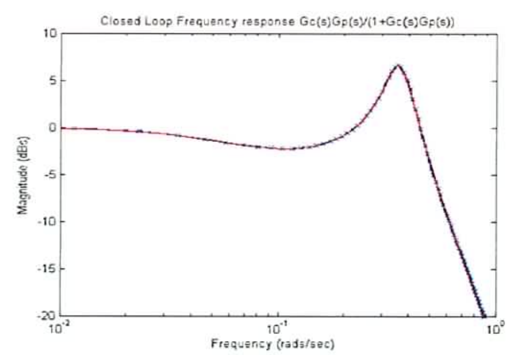

Figure D-63 - Closed loop frequency response obtained when first PRBS period is ignored.

According to [74], the total period of the PRBS sequence must be repeated at least once during the testing process and the data associated with the initial PRBS period should be ignored. Therefore, the total length of the PRBS test signal in bits, $L_{total} = n \times L$, where n is the number of repetitions and L is the length of the PRBS test signal (i.e. $L=2^N-1$, refer to Appendix A, section A.2). Figure D-58 and Figure D-59 illustrate the frequency response plots obtained when only one period of the PRBS signal was used in the testing procedure, i.e. n=1. Figure D-60 and Figure D-61 illustrate the open and closed loop frequency response plots obtained when two periods of the PRBS sequence were used in the system test. In this case, the data generated due to both sequences was processed in order to generate the response plots shown. Figure D-62 and Figure D-63 display the frequency response plots obtained when two periods of the PRBS sequences were used and the data associated with the initial PRBS period was ignored. Only the data associated with the second period was processed. Comparing the response plots obtained, it can be seen that the most accurate frequency response plots were obtained when two periods of the PRBS test signal were used and the data resulting from the initial PRBS period was ignored.

E: Relay Results:

E.1 FOLPD Models - Closed Loop:

The following results were obtained during the testing of 10 simulated first order lag plus delay (FOLPD) models listed in Table D.1 of Appendix D. The testing procedure involved the insertion of a relay and anti-aliasing filter into the closed loop system as illustrated in Figure E-1.

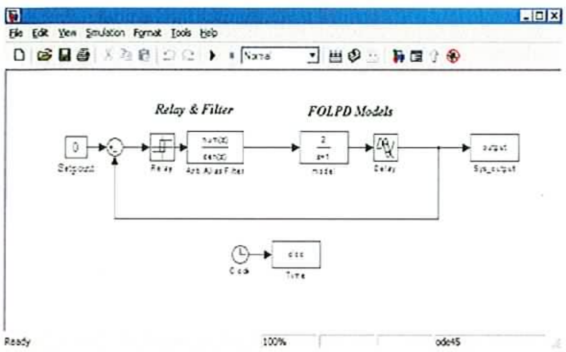

Figure E-1 - Simulink test set-up for Relay based testing of FOLPD models

From Figure E-1, it can be seen that, for the relay based system testing, the set point was set to zero and the relay, along with an anti-aliasing filter, was inserted before the FOLPD model in closed loop. The following figures will help explain why the anti-aliasing filter is necessary.

The output of an ideal relay is a square wave. A square wave consists of a fundamental frequency component and an infinite number of odd harmonics of this fundamental frequency. According to the Nyquist sampling theorem, in order to prevent aliasing, the sampling frequency must be greater than twice the highest frequency component of any of the signals to be measured [98]. Because of the infinite harmonics of the relay output, the Nyquist sampling theorem is not adhered to. The following figures are intended to highlight the importance of preventing aliasing of the relay output signal. Figure E-2 to Figure E-9 illustrate the results obtained with no anti-aliasing filter compared to those obtained with an anti-aliasing filter employed. The filter employed was a 12th order Butterworth filter with a cut off frequency of 0.8 times the Nyquist frequency (40 Hz).

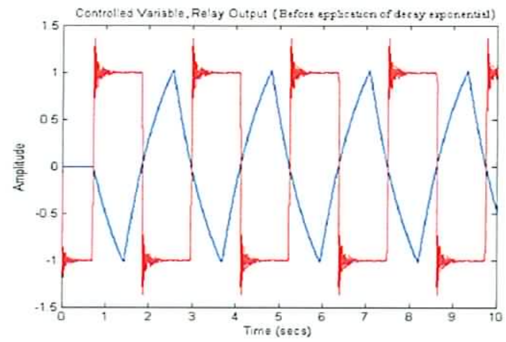

Figure E-2 - Controlled variable (blue) and relay output (red) for FOLPD model 7 with anti-aliasing filter

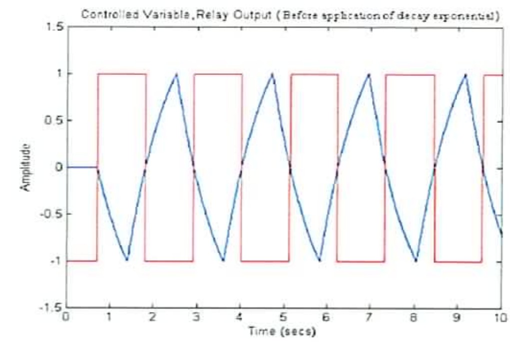

Figure E-3 - Controlled variable (blue) and relay output (red) for FOLPD model 7 with no anti-aliasing filter

Figure E-3 is a plot of the relay output in red and the controlled variable output in blue (before the decay exponential, discussed in Chapter 3, is applied), versus time, when no anti-aliasing filter is used. As can be seen, the relay output is a perfect square wave. When the anti-aliasing filter is introduced the corners of the square wave become slightly oscillatory. This effect is known as the Gibbs phenomenon and occurs when a square wave is reconstructed from a finite number of harmonics [99].

The following figures illustrate the effects on the frequency response plots of employing the anti-aliasing filter. A decay exponential was applied to the time domain signals before they were processed to generate the frequency response plots of Figure E-4 to Figure E-9.

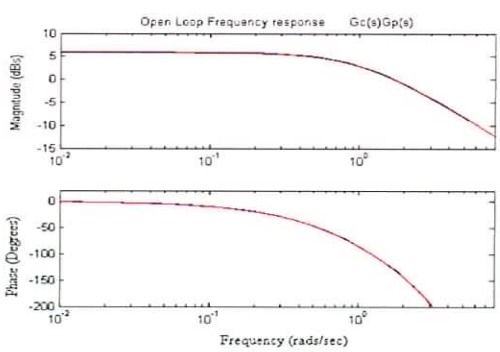

Figure E-4 - Estimated open loop frequency response (blue) and actual frequency response (red) for FOLPD model 7 with anti-aliasing filter

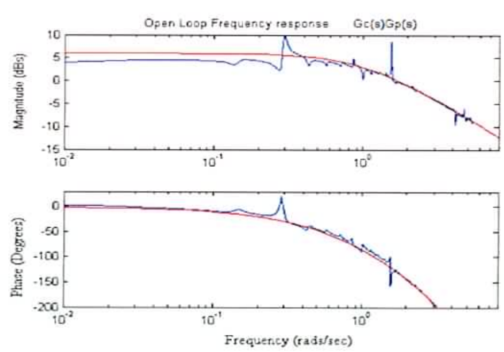

Figure E-5 - Estimated open loop frequency response (blue) and actual frequency response (red) for FOLPD model 7 with no anti-aliasing filter

From Figure E-4 and Figure E-5, it can be seen that when the filter is employed a much smoother estimation of the open loop frequency response plot is obtained. It is very important to avoid the 'spikiness' seen in Figure E-5 as these spike may lead to an inaccurate estimation of the gain and phase margins of the system.

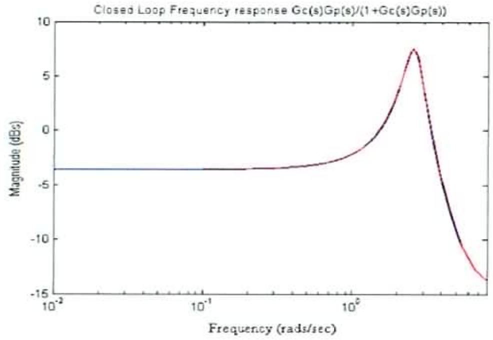

Figure E-6 - Estimated closed loop frequency response (blue) and actual frequency response (red) for FOLPD model 7 with anti-aliasing filter

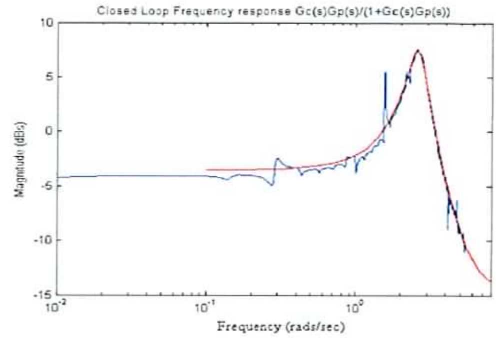

Figure E-7 - Estimated closed loop frequency response (blue) and actual frequency response (red) for FOLPD model 7 with no anti-aliasing filter

Figure E-6 and Figure E-7 illustrate the closed loop frequency response estimations obtained with and without the anti-aliasing filter employed. It can be seen that once the filter is employed the response becomes smoother and thus eliminates the possibility of identifying one of the aliasing spikes as the peak in the closed loop frequency response.

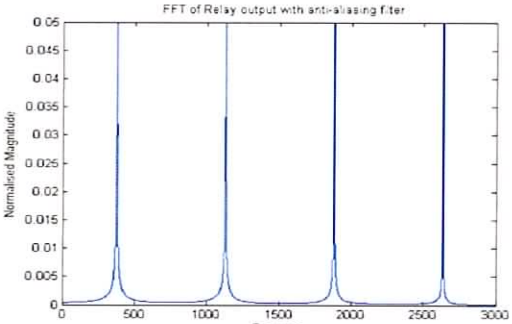

Figure E-8 - Magnitude of FFT of relay output when anti-aliasing filter is employed

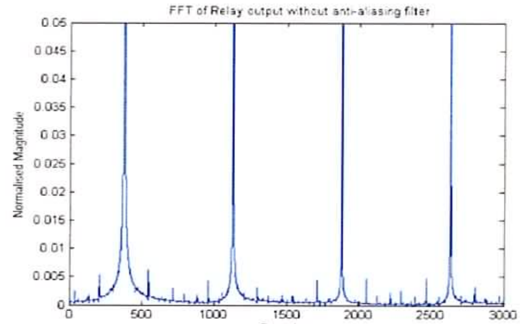

Figure E-9 - Magnitude of FFT of relay output when no anti-aliasing filter is employed

From Figure E-9, it can be seen that the magnitude of the FFT of the relay output contains a number of noise like spikes due to aliasing. It is these spikes that cause the inaccuracies seen in the estimations of the frequency response plots of Figure E-5 and Figure E-7. From Figure E-8, it can be seen that employing the anti-aliasing filter removes the spikes from the relay output magnitude plot resulting in the smoother frequency response plots seen in Figure E-4 and Figure E-6.

The table contents of Table E-1 are similar those of Appendix D and a comprehensive explanation of the table entries is given in section D.1.1. There is one additional column in the Relay results, namely *No. periods (secs)*. This column refers to the number of output oscillations taken during the testing procedure.

In Table E-1, each column corresponds to a set of system characteristics. For each FOLPD model there were six sets of characteristics obtained. For example, the first six rows of Table E-1 correspond to the results obtained in testing FOLPD model 1 for a varying number of output oscillations taken, the second six rows of Table E-1 correspond to the results obtained in testing FOLPD model 2 for a varying number of output oscillations taken, etc.

For each FOLPD model tested, the number of output oscillations taken varied from 20, to 250 oscillations (approximately). A number of interesting results in Table E-1 have been highlighted and will be discussed.

Table E-1 - Results obtained for FOLPD Models in closed loop for varying number of oscillations taken

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	No. periods (secs)	Sim time (secs)
Model 1	18.73	18.25	153.95	110.08	42.51	-5.67	-3.52	4.35	3.90	72.48	114.99	20.00	12.99
	18.19	18.25	122.61	110.08	11.68	-4.39	-3.52	4.06	3.90	28.83	40.51	49.00	32.49
	18.01	**18.25**	**115.69**	**110.08**	**6.38**	-3.93	-3.52	4.06	3.90	15.66	22.25	98.00	64.99
	17.95	18.25	113.61	110.08	4.82	**-3.80**	**-3.52**	**4.00**	**3.90**	**10.52**	15.34	148.00	97.49
	17.92	18.25	112.62	110.08	4.09	-3.76	-3.52	4.01	3.90	9.63	13.72	197.00	129.99
	17.91	18.25	112.04	110.08	3.66	-3.72	-3.52	4.06	3.90	9.84	13.50	246.00	162.49
Model 2	13.31	12.57	124.04	100.15	29.72	-4.65	-3.52	5.71	5.60	39.52	69.23	20.00	24.19
	12.75	**12.57**	**108.13**	**100.15**	**9.37**	-4.05	-3.52	5.71	5.60	16.93	26.30	50.00	60.49
	12.56	12.57	103.81	100.15	3.75	**-3.79**	**-3.52**	**5.66**	**5.60**	**8.61**	12.36	101.00	120.99
	12.50	12.57	102.33	100.15	2.77	-3.70	-3.52	5.68	5.60	6.27	9.05	151.00	181.49
	12.46	12.57	101.68	100.15	2.37	-3.65	-3.52	5.69	5.60	5.23	7.60	202.00	241.99
	12.45	12.57	101.29	100.15	2.13	-3.63	-3.52	5.67	5.60	4.24	6.37	252.00	302.49
Model 3	10.26	9.38	110.77	90.23	32.14	-3.81	-2.50	6.02	6.14	54.21	86.35	20.00	33.39
	9.66	**9.38**	**97.37**	**90.23**	**10.86**	-2.94	-2.50	6.10	6.14	18.51	29.37	50.00	83.49
	9.45	9.38	93.46	90.23	4.38	**-2.65**	**-2.50**	**6.10**	**6.14**	**6.75**	11.13	99.00	166.99
	9.39	9.38	92.16	90.23	2.23	-2.55	-2.50	6.12	6.14	2.31	4.54	149.00	250.49
	9.35	9.38	91.59	90.23	1.78	-2.50	-2.50	6.11	6.14	0.53	2.31	199.00	333.99
	9.33	9.38	91.15	90.23	1.51	-2.47	-2.50	6.13	6.14	1.37	2.88	248.00	417.49
Model 4	8.16	7.21	99.69	80.31	37.59	-1.73	-0.11	5.63	5.73	1496.05	1533.64	20.00	42.39
	7.54	7.21	87.21	80.31	13.12	-0.70	-0.11	5.69	5.73	542.66	555.78	50.00	105.99
	7.33	**7.21**	**83.46**	**80.31**	**5.55**	-0.34	-0.11	5.69	5.73	214.39	219.94	99.00	211.99
	7.26	7.21	82.19	80.31	3.00	-0.22	-0.11	5.69	5.73	101.69	104.68	149.00	317.99
	7.22	7.21	81.59	80.31	1.77	-0.16	-0.11	5.69	5.73	45.92	47.69	199.00	423.99
	7.20	7.21	81.26	80.31	1.31	**-0.12**	**-0.11**	**5.69**	**5.73**	**13.45**	14.76	249.00	529.99
Model 5	6.58	5.59	69.68	70.38	45.11	0.45	2.40	5.09	5.20	83.54	128.65	20.00	51.79
	5.94	5.59	77.32	70.38	16.17	1.67	2.40	5.14	5.20	31.77	47.94	50.00	129.49
	5.73	**5.59**	**73.51**	**70.38**	**6.97**	2.10	2.40	5.17	5.20	13.43	20.40	100.00	258.99
	5.66	5.59	72.27	70.38	3.94	**2.25**	**2.40**	**5.18**	**5.20**	**7.21**	11.15	151.00	388.49
	5.63	5.59	71.70	70.38	2.49	2.32	2.40	5.17	5.20	4.36	6.85	201.00	517.99
	5.60	5.59	71.33	70.38	1.59	2.36	2.40	5.17	5.20	2.42	4.01	251.00	647.49
Model 6	5.35	4.32	80.29	60.46	56.65	2.55	4.97	4.70	4.73	49.59	106.23	20.00	60.19
	4.70	4.32	67.66	60.46	20.67	4.04	4.97	4.68	4.73	20.09	40.76	50.00	150.49
	4.48	**4.32**	**63.74**	**60.46**	**9.15**	**4.56**	**4.97**	**4.70**	**4.73**	**9.04**	18.19	100.00	300.99
	4.41	4.32	62.44	60.46	5.32	4.75	4.97	4.70	4.73	5.24	10.57	150.00	451.49
	4.37	4.32	61.83	60.46	3.46	4.84	4.97	4.71	4.73	3.36	6.82	201.00	601.99
	4.35	4.32	61.47	60.46	2.38	4.89	4.97	4.71	4.73	2.22	4.60	251.00	752.49
Model 7	4.36	3.29	71.32	50.54	73.64	4.64	7.66	4.25	4.34	41.41	115.05	20.00	67.99
	3.69	3.29	58.15	50.54	27.24	6.44	7.66	4.29	4.34	16.98	44.22	50.00	169.99
	3.46	3.29	54.04	50.54	12.31	**7.11**	**7.66**	**4.31**	**4.34**	**7.91**	20.22	100.00	339.99
	3.39	**3.29**	**52.68**	**50.54**	**7.34**	7.33	7.66	4.31	4.34	4.78	12.12	150.00	509.99
	3.35	3.29	52.00	50.54	4.88	7.45	7.66	4.32	4.34	3.20	8.08	201.00	679.99
	3.33	3.29	51.62	50.54	3.45	7.52	7.66	4.31	4.34	2.43	5.89	251.00	849.99
Model 8	3.53	2.43	62.57	40.61	99.52	6.74	10.57	3.92	4.00	38.34	137.86	20.00	75.39
	2.84	2.43	48.70	40.61	37.15	8.97	10.57	3.97	4.00	16.00	53.15	50.00	188.49
	2.62	2.43	44.36	40.61	17.03	**9.82**	**10.57**	**3.98**	**4.00**	**7.59**	24.62	100.00	376.99
	2.54	**2.43**	**42.95**	**40.61**	**10.41**	10.12	10.57	3.98	4.00	4.92	15.33	150.00	565.49
	2.50	2.43	42.24	40.61	7.09	10.27	10.57	3.98	4.00	3.34	10.43	199.00	753.99
	2.48	2.43	41.80	40.61	5.06	10.36	10.57	3.98	4.00	2.56	7.62	249.00	942.49
Model 9	2.81	1.70	53.69	30.69	140.88	8.69	13.93	3.68	3.72	37.16	178.04	20.00	83.59
	2.12	1.70	39.25	30.69	53.12	11.75	13.93	3.70	3.72	16.23	69.35	50.00	208.99
	1.89	1.70	34.68	30.69	24.62	**12.87**	**13.93**	**3.70**	**3.72**	**8.20**	32.82	100.00	417.99
	1.82	1.70	33.16	30.69	15.14	13.27	13.93	3.70	3.72	5.31	20.45	150.00	626.99
	1.78	**1.70**	**32.41**	**30.69**	**10.40**	13.48	13.93	3.71	3.72	3.63	14.04	200.00	835.99
	1.76	1.70	31.96	30.69	7.59	13.60	13.93	3.70	3.72	2.78	10.37	251.00	1045.00
Model 10	2.21	1.07	45.16	20.76	224.39	11.22	18.17	3.46	3.48	38.83	263.22	20.00	90.79
	1.51	1.07	29.90	20.76	85.21	14.95	18.17	3.46	3.48	18.34	103.54	50.00	226.99
	1.27	1.07	25.04	20.76	39.85	**16.53**	**18.17**	**3.46**	**3.48**	**9.61**	49.47	100.00	453.99
	1.20	1.07	23.44	20.76	24.78	17.12	18.17	3.46	3.48	6.37	31.14	150.00	680.99
	1.16	1.07	22.63	20.76	17.24	17.43	18.17	3.47	3.48	4.44	21.68	200.00	907.99
	1.13	**1.07**	**22.15**	**20.76**	**12.74**	17.63	18.17	3.47	3.48	3.42	16.16	250.00	1135.00

From the results of Table E-1 the following figures were obtained. Figure E-10 compares the *Overall abs % error* values obtained versus *No. periods (secs)* for each of the FOLPD models tested (each model is represented by a different colour).

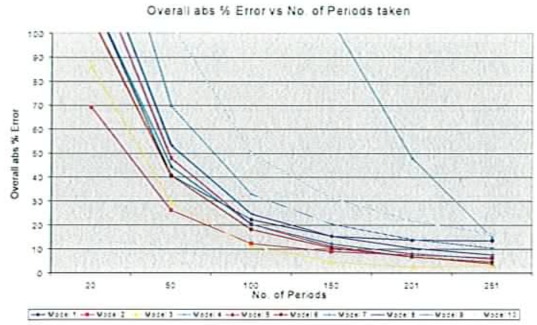

Figure E-10 - Overall percentage error obtained for varying the number of output oscillations recorded

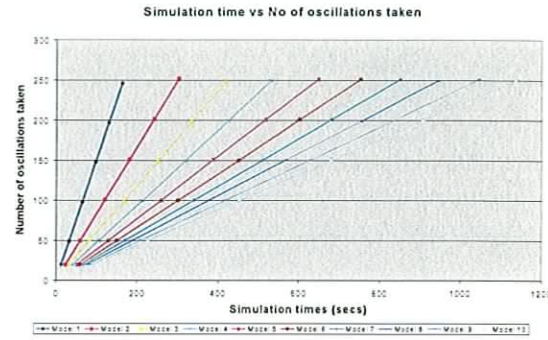

Figure E-11 - Simulation times for varying number of oscillations taken

From Figure E-10, it can be seen that as the number of output periods taken increases the *Overall abs % error* values obtained for each model decreases. However, the drawback in increasing the number of periods taken is that the necessary simulation times increase. This is illustrated in Figure E-11. At this point it may be worthwhile examining the effects of increasing the number of output periods taken. Figure E-12 to Figure E-29 examine the effects of increasing the number of output periods taken on the time domain signals, open loop frequency response plots and closed loop frequency response plots for FOLPD model 4. The details regarding the relay identification technique employed are discussed in Chapter 3, section 3.3.1.2.

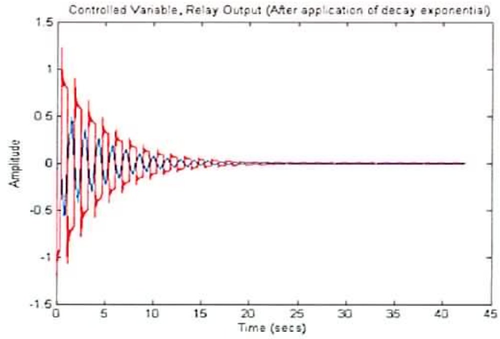

Figure E-12 - Relay output (red) and controlled variable output (blue) for 20 oscillations taken after the application of the decay exponential

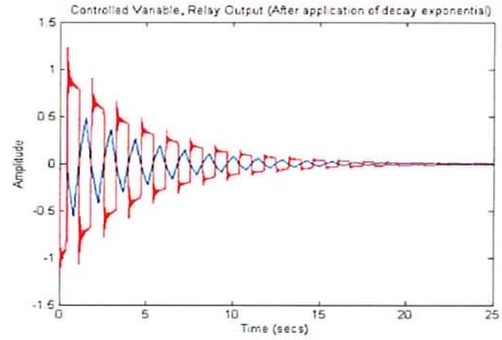

Figure E-13 - Close up of Figure E-12

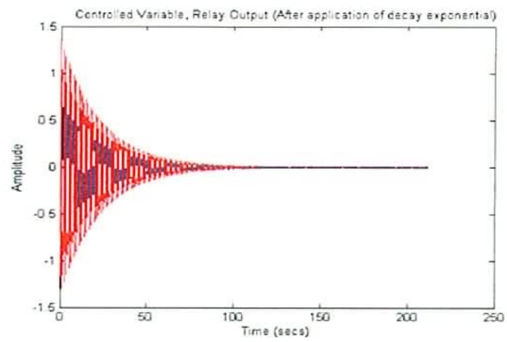

Figure E-14 - Relay output (red) and controlled variable output (blue) for 100 oscillations taken after the application of the decay exponential

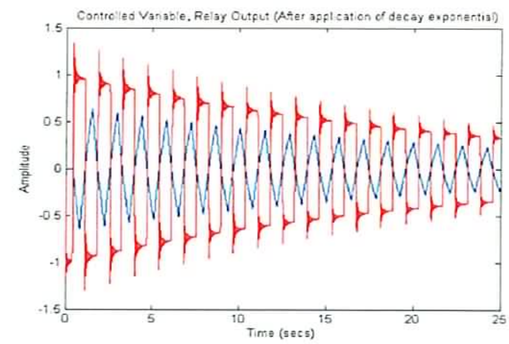

Figure E-15 - Close up of Figure E-14

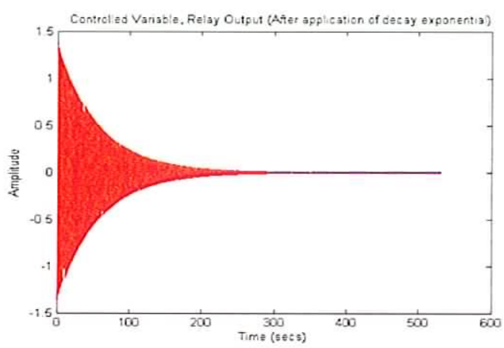

Figure E-16 - Relay output (red) and controlled variable output (blue) for 250 oscillations taken after the application of the decay exponential

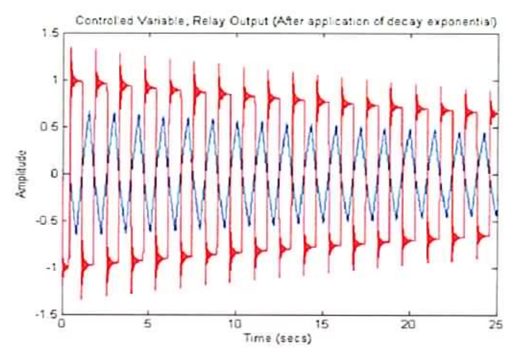

Figure E-17 - Close up of Figure E-16

Comparing Figure E-13, Figure E-15, and Figure E-17, it can be seen that as the number of oscillations taken increases, the rate of decay of the applied exponential decreases. Thus the time domain response plots are seen to 'die off' to zero at a slower rate. For more information regarding the reasons why this exponential is applied to the relay output and controlled variable output, refer to section 3.3.1.2 of Chapter 2 and section E.5 (of this Appendix).

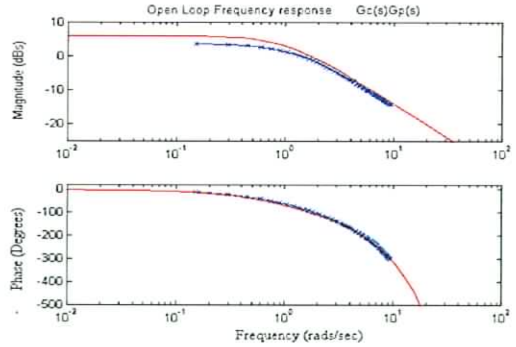

Figure E-18 - Estimated open loop frequency response (blue) and actual response (red) for 20 oscillations taken

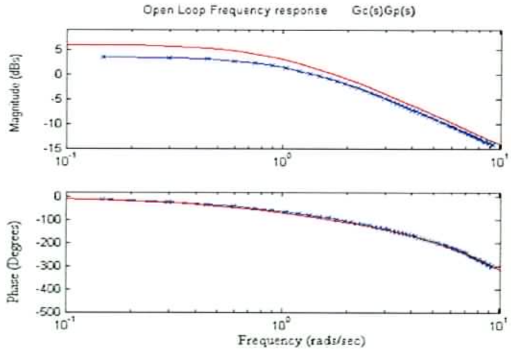

Figure E-19 - Close up of Figure E-18

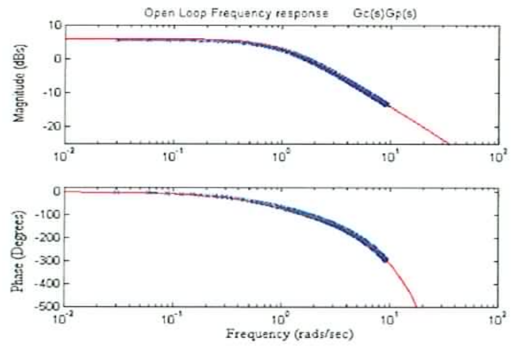

Figure E-20 - Estimated open loop frequency response (blue) and actual response (red) for 100 oscillations taken

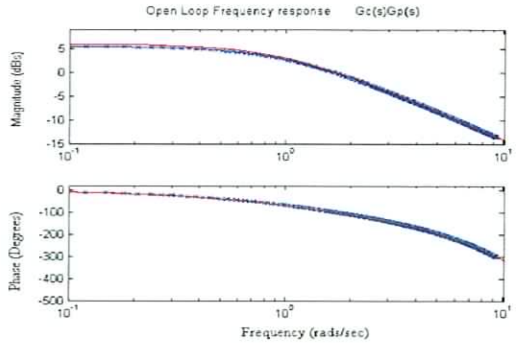

Figure E-21 - Close up of Figure E-20

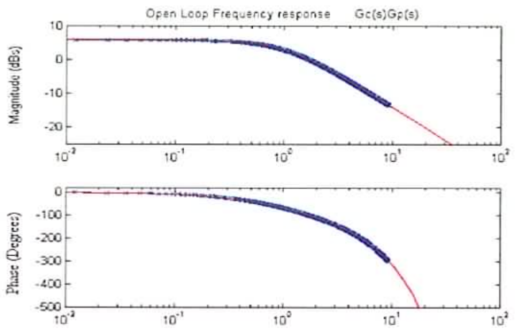

Figure E-22 - Estimated open loop frequency response (blue) and actual response (red) for 250 oscillations taken

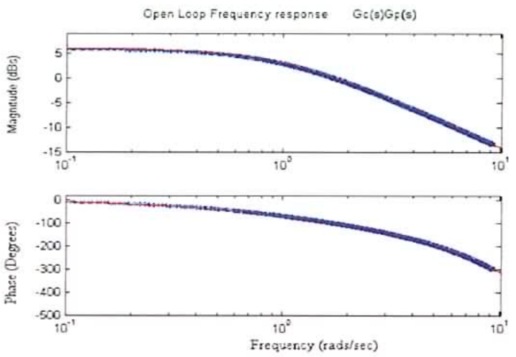

Figure E-23 - Close up of Figure E-22

Figure E-18 to Figure E-23 examine the effects of increasing the number of output oscillations taken on the estimated open loop frequency response. As can be seen from Figure E-19, Figure E-21 and Figure E-23, as the number of oscillations taken increases, both the accuracy and frequency resolution of the estimated open loop frequency response plots also increases. When only 20 oscillations were taken (Figure E-19) the simulation time was short but the estimated open loop frequency response plot significantly underestimated the actual response. As the number of oscillations taken increased the accuracy along with the necessary simulation times also increased.

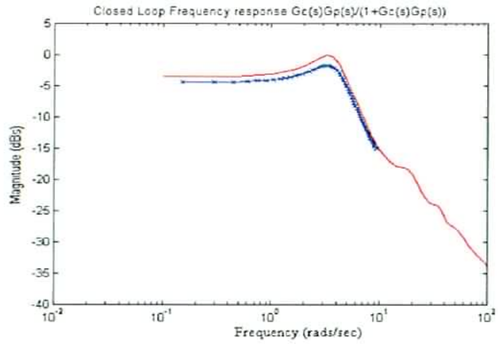

Figure E-24 - Estimated closed loop frequency response (blue) and actual response (red) for 20 oscillations taken

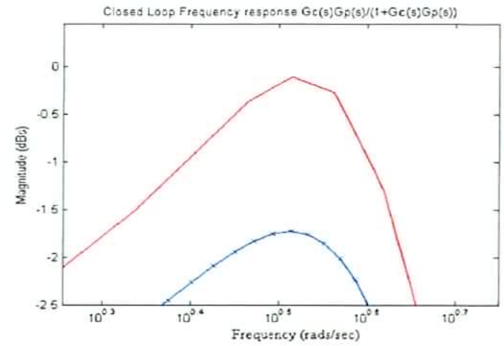

Figure E-25 - Close up of Figure E-24

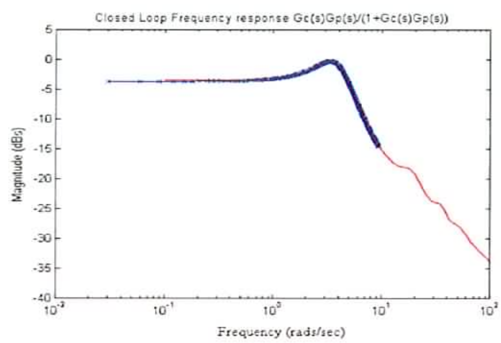

Figure E-26 - Estimated closed loop frequency response (blue) and actual response (red) for 100 oscillations taken

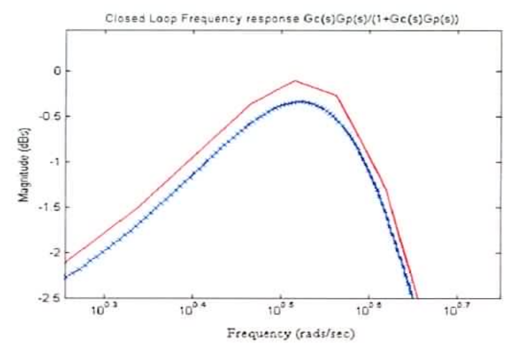

Figure E-27 - Close up of Figure E-26

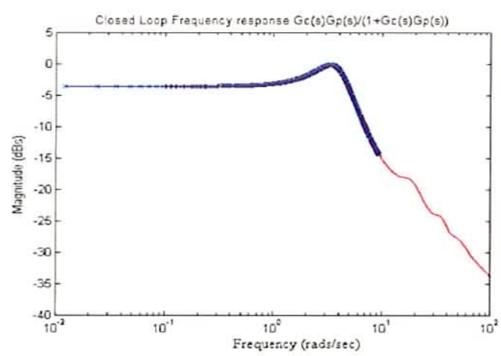

Figure E-28 - Estimated closed loop frequency response (blue) and actual response (red) for 250 oscillations taken

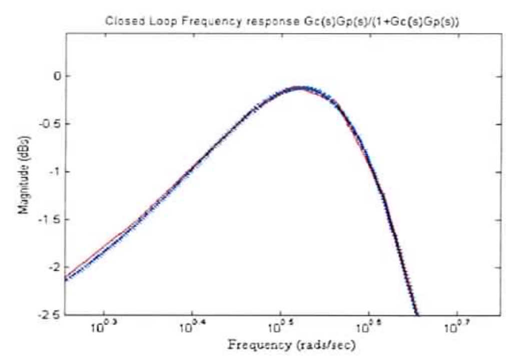

Figure E-29 - Close up of Figure E-28

Figure E-24 to Figure E-29 illustrate the effects of increasing the number of output oscillations taken on the estimated closed loop frequency response plot. Figure E-25, Figure E-27 and Figure E-29 illustrate how increasing the number of oscillations recorded increases both the accuracy and frequency resolution of the estimated closed loop frequency response plots. Table E-2 highlights how the effect of increasing the number of oscillations taken increases the necessary simulation times. It can also be seen from this table, that as the simulation time increases the rate of decay of the applied exponential or 'Decay factor' α decreases.

Table E-2 - Comparison of simulation times and decay factors for FOLPD model 4 for various numbers of oscillations taken

Number of Oscillations	20	100	250
Simulation time	42.29 secs	211.99 secs	529.99 secs
Decay factor, α	0.2173	0.0434	0.0174

The results of Table E-1 highlighted in gold and yellow illustrate an interesting trend in the *% Error Gm & Pm* and *% Error Lcmax & band* values. It can be seen that for the FOLPD models tested with t_d/T_c ratios below 0.6 (i.e. models 1-5, refer to Table D-1, Appendix D) the number of oscillations needed to obtain a *% Error Gm & Pm* of below or approximately 10% was less than the number of oscillations needed to achieve the same percentage error value for the *% Error Lcmax & band* values. However, as the t_d/T_c ratio became greater than 0.6, this situation was reversed.

E.2 Typical Process Models - Closed Loop:

This section contains results obtained during the testing of 6 models of various orders discussed in section D.2 of Appendix D. These models are intended to represent a variety of typical processes.

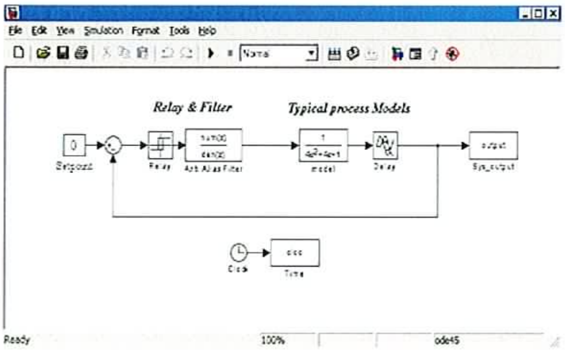

Figure E-30 - Simulink test set-up for Relay based testing of Typical process models

Figure 4-5 illustrates the set up used in the testing of the 6 typical process models of Table D-6, Appendix D. As can be seen, the test set up is similar to that of the FOLPD model tests discussed in section E.1, with the relay and anti-aliasing filter inserted into the closed loop.

Table E-3 - Results obtained for Typical processes in closed loop for varying number of oscillations taken

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	No. periods (secs)	Sim time (secs)
Model 1	9.88	8.65	180.00	180.00	14.26	-3.94	-2.13	0.79	0.81	88.53	102.78	20.00	287.79
	9.14	8.65	180.00	180.00	5.64	-2.87	-2.13	0.80	0.81	35.96	41.60	50.00	719.49
	8.89	8.65	180.00	180.00	2.74	-2.49	-2.13	0.81	0.81	17.84	20.58	100.00	1439.00
	8.80	8.65	180.00	180.00	1.77	-2.37	-2.13	0.81	0.81	11.73	13.49	150.00	2158.50
	8.76	8.65	180.00	180.00	1.28	-2.30	-2.13	0.81	0.81	8.64	9.92	200.00	2878.00
	8.74	8.65	180.00	180.00	0.99	-2.26	-2.13	0.81	0.81	6.79	7.78	250.00	3597.50
Model 2	7.40	6.03	180.00	180.00	22.69	-1.10	1.31	0.38	0.39	186.51	209.20	20.00	663.59
	6.58	6.03	180.00	180.00	9.11	0.30	1.31	0.39	0.39	78.05	87.15	50.00	1659.00
	6.31	6.03	180.00	180.00	4.54	0.80	1.31	0.39	0.39	39.39	43.93	100.00	3318.00
	6.22	6.03	180.00	180.00	3.01	0.97	1.31	0.39	0.39	26.21	29.23	150.00	4977.00
	6.17	6.03	180.00	180.00	2.25	1.06	1.31	0.39	0.39	19.56	21.81	200.00	6636.00
	6.14	6.03	180.00	180.00	1.79	1.11	1.31	0.39	0.39	15.55	17.34	250.00	8295.00
Model 3	8.40	6.79	180.00	180.00	23.56	-1.92	0.48	1.21	1.23	498.58	522.15	20.00	192.79
	7.43	6.79	180.00	180.00	9.34	-0.51	0.48	1.23	1.23	206.14	215.48	50.00	481.99
	7.10	6.79	180.00	180.00	4.51	0.00	0.48	1.23	1.23	100.01	104.52	100.00	963.99
	6.99	6.79	180.00	180.00	2.89	0.18	0.48	1.23	1.23	64.05	66.95	150.00	1446.00
	6.94	6.79	180.00	180.00	2.08	0.26	0.48	1.23	1.23	45.63	47.71	200.00	1928.00
	6.90	6.79	180.00	180.00	1.60	0.32	0.48	1.23	1.23	34.46	36.05	251.00	2410.00
Model 4	12.80	11.50	180.00	180.00	11.31	-5.61	-4.03	0.32	0.32	41.21	52.52	20.00	590.59
	12.02	11.50	180.00	180.00	4.53	-4.68	-4.03	0.32	0.32	17.88	22.41	50.00	1476.50
	11.76	11.50	180.00	180.00	2.23	-4.35	-4.03	0.32	0.32	8.49	10.72	100.00	2953.00
	11.67	11.50	180.00	180.00	1.46	-4.24	-4.03	0.32	0.32	5.76	7.23	150.00	4429.50
	11.63	11.50	180.00	180.00	1.07	-4.19	-4.03	0.32	0.32	4.40	5.47	200.00	5906.00
	11.60	11.50	180.00	180.00	0.84	-4.15	-4.03	0.32	0.32	3.32	4.17	250.00	7382.50
Model 5	6.89	5.50	180.00	180.00	25.26	-0.41	2.19	0.57	0.58	120.94	146.20	21.00	454.19
	6.06	5.50	180.00	180.00	10.08	1.10	2.19	0.58	0.58	50.43	60.50	51.00	1135.50
	5.78	5.50	180.00	180.00	4.98	1.64	2.19	0.58	0.58	25.71	30.69	101.00	2271.00
	5.68	5.50	180.00	180.00	3.28	1.83	2.19	0.58	0.58	16.97	20.24	151.00	3406.50
	5.63	5.50	180.00	180.00	2.42	1.92	2.19	0.58	0.58	12.52	14.94	201.00	4542.00
	5.61	5.50	180.00	180.00	1.91	1.98	2.19	0.58	0.58	10.04	11.95	251.00	5677.50
Model 6	8.27	7.09	180.00	180.00	16.60	-2.99	-1.02	0.59	0.60	195.08	211.68	20.00	447.99
	7.56	7.09	180.00	180.00	6.60	-1.81	-1.02	0.59	0.60	78.84	85.45	50.00	1120.00
	7.32	7.09	180.00	180.00	3.26	-1.41	-1.02	0.59	0.60	38.84	42.10	100.00	2240.00
	7.24	7.09	180.00	180.00	2.15	-1.27	-1.02	0.59	0.60	25.30	27.45	150.00	3360.00
	7.20	7.09	180.00	180.00	1.59	-1.20	-1.02	0.59	0.60	18.47	20.06	200.00	4480.00
	7.18	7.09	180.00	180.00	1.26	-1.16	-1.02	0.60	0.60	14.17	15.43	250.00	5600.00

Using the results displayed in Table E-3, Figure E-31 and Figure E-32 were obtained. From these figures, it can be seen that increasing the number of output periods recorded increased the accuracy of the results obtained but also increased the simulation times necessary to achieve these results.

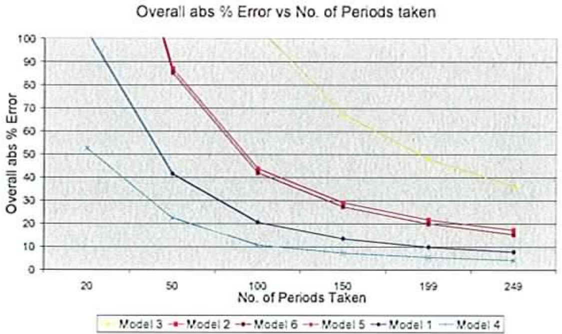

Figure E-31 - Overall percentage error obtained for varying the number of output oscillations recorded

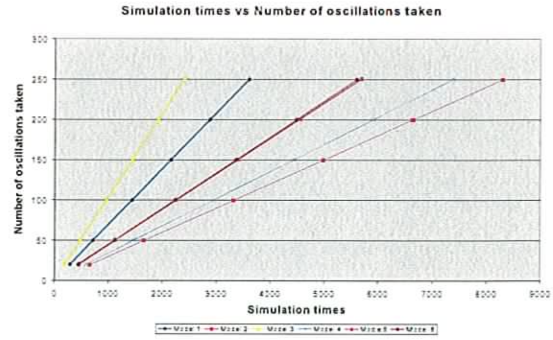

Figure E-32 - Simulation times for varying number of oscillations taken

The results highlighted in blue in Table E-3 indicate entries where a large *% Error Lcmax & band* were obtained when the absolute difference between the estimated and actual characteristic was comparatively small. This issue was discussed in section D.1.1

of Appendix D. It is also worth noting that the *Overall abs % error* values obtained were more heavily weighted by the *% Error Lcmax & band* values than by the *% Error Gm & Pm* value for each of these typical process models tested.

E.3 Typical Process Models with Generic PI Controller:

Table E-4 contains results obtained during the testing of the 6 typical process models and their associated PI controllers discussed in section D.3 of Appendix D. These models and controllers are displayed in Table D-10, of Appendix D.

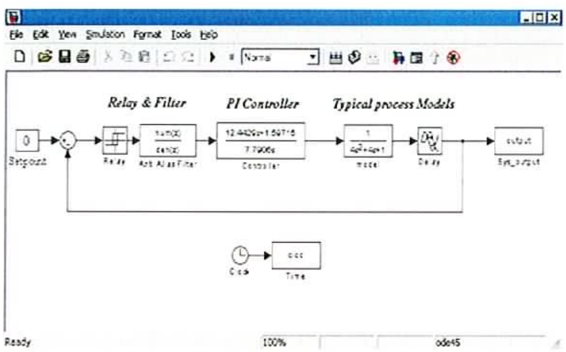

Figure E-33 - Simulink test set-up for Relay based testing of Typical process models and PI controllers

Figure E-33 illustrates the set up used in the testing of the 6 typical process models and PI controllers of Table D-10, Appendix D. As can be seen the test set up is similar to that of the FOLPD model tests previously discussed in section E.1.

The results of Table E-4 and subsequent figures, Figure E-34 and Figure E-35, were obtained for varying numbers of output oscillations recorded.

Table E-4 - Results obtained for Typical processes & PI controller in closed loop for varying number of oscillations taken

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	period (secs)	Sim time (secs)
	4.49	3.37	51.00	36.86	71.59	5.40	8.40	0.79	0.78	36.52	108.11	20.00	318.99
Model 1	3.81	3.37	42.38	36.86	28.08	7.12	8.40	0.78	0.78	15.50	43.59	50.00	797.49
	3.59	3.37	39.55	36.86	13.62	7.77	8.40	0.78	0.78	7.79	21.41	100.00	1595.00
	3.51	3.37	38.61	36.86	8.80	7.99	8.40	0.78	0.78	4.98	13.77	150.00	2392.50
	3.47	3.37	38.13	36.86	6.39	8.10	8.40	0.78	0.78	3.86	10.25	200.00	3190.00
	3.45	3.37	37.85	36.86	4.94	8.17	8.40	0.78	0.78	3.05	8.00	250.00	3987.50
	4.83	3.61	74.72	56.67	65.68	3.92	6.97	0.33	0.33	44.22	109.90	20.00	719.19
Model 2	4.09	3.61	64.23	56.67	26.83	5.67	6.97	0.34	0.33	19.41	46.24	50.00	1798.00
	3.85	3.61	60.48	56.67	13.40	6.31	6.97	0.33	0.33	9.68	23.08	100.00	3596.00
	3.77	3.61	59.21	56.67	8.89	6.53	6.97	0.33	0.33	6.39	15.27	150.00	5394.00
	3.72	3.61	58.57	56.67	6.62	6.64	6.97	0.33	0.33	4.89	11.51	200.00	7192.00
	3.70	3.61	58.19	56.67	5.27	6.71	6.97	0.33	0.33	3.81	9.08	250.00	8990.00
	4.81	3.39	71.28	53.27	75.58	4.42	7.90	1.12	1.12	44.26	119.84	20.00	207.19
Model 3	3.95	3.39	60.96	53.27	30.87	6.39	7.90	1.13	1.12	19.38	50.25	50.00	517.99
	3.66	3.39	57.14	53.27	15.18	7.15	7.90	1.12	1.12	9.66	24.83	100.00	1036.00
	3.56	3.39	55.83	53.27	9.87	7.41	7.90	1.12	1.12	6.16	16.03	150.00	1554.00
	3.52	3.39	55.17	53.27	7.20	7.54	7.90	1.12	1.12	4.63	11.83	200.00	2072.00
	3.49	3.39	54.77	53.27	5.59	7.63	7.90	1.12	1.12	3.50	9.09	251.00	2590.00
	4.17	2.96	34.41	23.26	88.94	7.39	10.88	0.38	0.38	32.75	121.69	20.00	664.99
Model 4	3.44	2.96	27.60	23.26	35.09	9.36	10.88	0.38	0.38	14.64	49.73	50.00	1662.50
	3.20	2.96	25.39	23.26	17.24	10.11	10.88	0.38	0.38	7.26	24.50	100.00	3325.00
	3.12	2.96	24.65	23.26	11.30	10.37	10.88	0.38	0.38	4.73	16.02	150.00	4987.50
	3.07	2.96	24.29	23.26	8.33	10.50	10.88	0.38	0.38	3.64	11.98	200.00	6650.00
	3.05	2.96	24.07	23.26	6.56	10.58	10.88	0.38	0.38	2.81	9.37	250.00	8312.50
	4.90	3.67	81.15	63.46	61.58	3.57	6.63	0.49	0.49	46.36	107.95	21.00	490.19
Model 5	4.16	3.67	71.09	63.46	25.43	5.33	6.63	0.49	0.49	19.71	45.14	51.00	1225.50
	3.91	3.67	67.36	63.46	12.75	5.97	6.63	0.49	0.49	10.06	22.81	101.00	2451.00
	3.83	3.67	66.07	63.46	8.46	6.20	6.63	0.49	0.49	6.71	15.17	151.00	3676.50
	3.79	3.67	65.42	63.46	6.30	6.31	6.63	0.49	0.49	5.02	11.32	201.00	4902.00
	3.76	3.67	65.02	63.46	4.98	6.38	6.63	0.49	0.49	3.99	8.97	251.00	6127.50
	4.80	3.77	68.77	54.33	54.13	3.66	6.29	0.51	0.50	42.94	97.07	20.00	494.59
Model 6	4.18	3.77	60.13	54.33	21.60	5.19	6.29	0.50	0.50	17.60	39.21	50.00	1236.50
	3.97	3.77	57.21	54.33	10.70	5.74	6.29	0.50	0.50	8.91	19.60	100.00	2473.00
	3.90	3.77	56.24	54.33	7.05	5.92	6.29	0.50	0.50	5.91	12.96	150.00	3709.50
	3.87	3.77	55.75	54.33	5.23	6.02	6.29	0.50	0.50	4.40	9.63	200.00	4946.00
	3.84	3.77	55.46	54.33	4.14	6.08	6.29	0.50	0.50	3.49	7.63	250.00	6182.50

From Figure E-34, it can be seen that as the number of output oscillations recorded increases, the *Overall abs % error* values obtained decrease. Figure E-35 illustrates how the necessary simulation times vary with the number of output oscillations recorded.

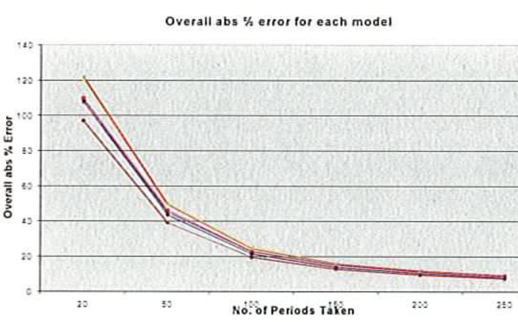

Figure E-34 - Overall percentage error obtained for varying the number of output oscillations recorded

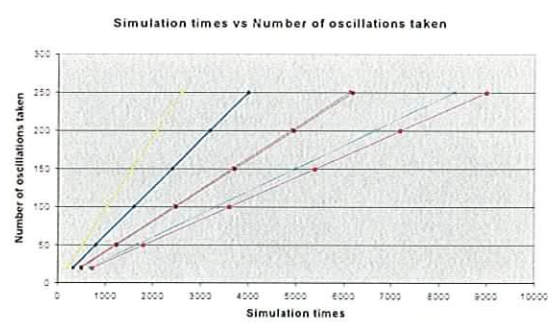

Figure E-35 - Simulation times for varying number of oscillations taken

Comparing Figure E-34 with Figure E-32 of the previous section, it can be seen that when the PI controllers are introduced into the closed loop system, the *Overall abs % error* values for each of the systems tested decreases at a faster rate than when the controllers are not implemented. It is also obvious from Figure E-34 that the *Overall abs % error* response curves obtained for each of the closed loop systems take on a similar shape. This is to be expected as the tuning rule applied to the typical process models causes each of the models to exhibit similar time domain characteristics which will be reflected in the frequency domain response plots and thus in the results obtained.

E.4 Accuracy in the Presence of Noise:

For each of the typical process models and PI controllers, the accuracy of the relay based evaluation approach in the presence of Gaussian white noise was tested. The testing model can be seen in Figure E-36 and details regarding the specification of the noise power are discussed in Appendix D, section D.4.

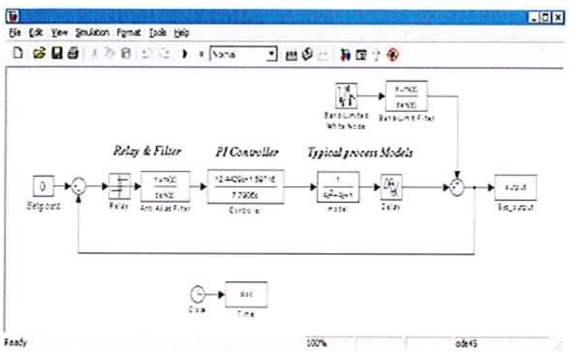

Figure E-36 - Simulink Relay based test set-up for Typical process models and PI controllers in the presence of noise

In Figure E-36, the output of the noise block is band-limited using a 12th order Butterworth filter with a cut off frequency of 40 Hz. The power of the noise block was varied from 10% to 50% of the controlled variable output signals power levels and the results of Table E-5 were obtained.

For each of the closed loop systems tested, a total of 250 output oscillations were recorded and processed in order to evaluate the required closed loop characteristics displayed in Table E-5 and Table E-6.

E.4.1 No Hysteresis:

The results of Table E-5 were obtained without employing hysteresis in the relay feedback system of Figure E-36. The noise power levels were varied from 10% to 50% of the controlled variable output power for each of the typical process models and PI controllers.

Table E-5 - Results obtained for Typical processes & PI controller in closed loop for varying noise levels, No Hysteresis

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	No. periods (secs)	Sim time (secs)	Noise Level
Model 1	3.45	3.37	37.85	36.86	4.92	8.17	8.40	0.78	0.78	-3.01	7.94	250.00	4002.50	0%
Model 2	3.70	3.61	58.19	56.67	5.26	6.71	6.97	0.33	0.33	-3.69	9.11	250.00	9005.00	
Model 3	3.49	3.39	54.76	53.27	5.55	7.63	7.90	1.12	1.12	-3.40	8.96	251.00	2605.00	
Model 4	3.05	2.96	24.06	23.26	6.52	10.59	10.88	0.38	0.38	-2.84	9.36	250.00	8332.50	
Model 5	3.76	3.67	65.03	63.46	4.99	6.38	6.63	0.49	0.49	-3.76	8.98	251.00	6140.00	
Model 6	3.84	3.77	55.45	54.33	4.13	6.08	6.29	0.50	0.50	-3.29	7.57	250.00	6197.50	
Model 1	3.44	3.37	38.59	36.86	6.84	8.37	8.40	0.78	0.78	1.02	7.85	250.00	3987.50	10%
Model 2	3.70	3.61	55.71	56.67	4.35	6.66	6.97	0.33	0.33	4.66	9.01	250.00	8990.00	
Model 3	8.13	3.39	69.84	53.27	170.64	8.99	7.90	0.26	1.12	90.27	260.90	250.00	2590.00	
Model 4	-19.85	2.96	180.00	23.26	1445.30	1.56	10.88	0.02	0.38	181.07	1626.40	250.00	8312.50	
Model 5	3.76	3.67	88.69	63.46	42.14	6.55	6.63	0.49	0.49	1.66	43.80	250.00	6127.50	
Model 6	3.85	3.77	58.16	54.33	9.12	6.33	6.29	0.49	0.50	2.62	11.73	250.00	6182.50	
Model 1	3.44	3.37	36.98	36.86	2.38	8.54	8.40	0.78	0.78	2.31	4.69	250.00	3987.50	20%
Model 2	3.70	3.61	55.11	56.67	5.45	6.65	6.97	0.33	0.33	5.04	10.49	250.00	8990.00	
Model 3	-19.46	3.39	89.75	53.27	742.12	8.65	7.90	0.35	1.12	78.70	820.81	250.00	2590.00	
Model 4	-9.69	2.96	102.74	23.26	769.24	11.23	10.88	0.38	0.38	4.21	773.45	250.00	8312.50	
Model 5	3.75	3.67	95.95	63.46	53.57	6.64	6.63	0.49	0.49	0.56	54.13	250.00	6127.50	
Model 6	3.84	3.77	75.34	54.33	40.72	6.48	6.29	0.49	0.50	5.14	45.85	250.00	6182.50	
Model 1	3.44	3.37	75.02	36.86	105.55	9.14	8.40	0.78	0.78	9.29	114.85	250.00	3987.50	30%
Model 2	3.71	3.61	74.12	56.67	33.51	6.64	6.97	0.33	0.33	4.92	38.43	250.00	8990.00	
Model 3	-11.50	3.39	85.35	53.27	499.09	9.47	7.90	0.25	1.12	97.31	596.40	250.00	2590.00	
Model 4	3.02	2.96	73.59	23.26	218.38	11.40	10.88	0.38	0.38	6.01	224.40	250.00	8312.50	
Model 5	-28.79	3.67	88.79	63.46	924.71	6.65	6.63	0.49	0.49	0.98	925.69	250.00	6127.50	
Model 6	3.05	3.77	97.33	54.33	98.24	6.56	6.29	0.49	0.50	6.47	104.72	250.00	6182.50	
Model 1	-4.37	3.37	116.81	36.86	446.36	9.88	8.40	0.17	0.78	96.25	542.60	250.00	3987.50	40%
Model 2	-47.53	3.61	72.91	56.67	1446.20	6.76	6.97	0.33	0.33	4.68	1450.90	250.00	8990.00	
Model 3	-17.08	3.39	67.96	53.27	631.11	11.90	7.90	0.24	1.12	129.39	760.50	250.00	2590.00	
Model 4	-16.22	2.96	105.14	23.26	1000.40	11.49	10.88	0.38	0.38	6.84	1007.20	250.00	8312.50	
Model 5	-7.36	3.67	91.47	63.46	344.89	6.78	6.63	0.49	0.49	2.94	347.83	250.00	6127.50	
Model 6	2.82	3.77	78.66	54.33	69.91	7.31	6.29	0.12	0.50	92.03	161.94	250.00	6182.50	
Model 1	-5.56	3.37	73.66	36.86	364.83	9.46	8.40	0.78	0.78	13.28	378.11	250.00	3987.50	50%
Model 2	-28.62	3.61	180.00	56.67	1111.10	3.13	6.97	0.08	0.33	132.48	1243.50	250.00	8990.00	
Model 3	-6.49	3.39	86.03	53.27	352.96	9.35	7.90	1.13	1.12	18.74	371.70	250.00	2590.00	
Model 4	-20.96	2.96	173.77	23.26	1455.50	7.66	10.88	0.07	0.38	111.10	1566.60	250.00	8312.50	
Model 5	-11.22	3.67	15.42	63.46	481.50	6.87	6.63	0.49	0.49	4.23	485.73	250.00	6127.50	
Model 6	-7.85	3.77	145.41	54.33	476.07	22.14	6.29	0.82	0.50	315.94	792.01	250.00	6182.50	

Figure E-37 is a plot of the *Overall abs % error* obtained for each of the six closed loop systems versus the noise levels tested.

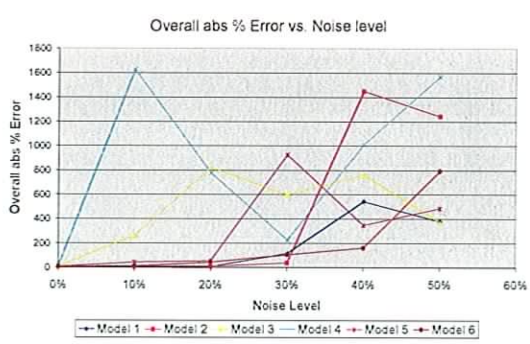

Figure E-37 - Percentage error for varying noise levels with no hysteresis employed

As can be seen from Figure E-37, the results obtained for each of the closed loop systems are both erratic and inaccurate when no hysteresis is employed. These poor estimations are due to spikes in the frequency response plots, which make it difficult to identify important peaks and crossing points.

A number of results of Table E-5 have been highlighted in red. As can be seen, for the particular set of results highlighted, the large *Overall abs % error* can be attributed mainly to very poor estimates of the system gain margin, thus resulting in very large *% Error Gm & Pm* values. As can be seen, for each of the cases highlighted, a negative estimate of the gain margin (*GM est*) was returned. These negative values were as a result of noise spikes masking the actual 0dB crossing point necessary to calculate accurate estimates of the system gain margin.

As can clearly be seen from the results displayed in Figure E-37, employing no hysteresis in the presence of noise returns extremely poor estimations of closed loop system's frequency domain characteristics.

E.4.2 Hysteresis of Twice the Noise Band:

The results of Table E-6 were obtained employing hysteresis of twice the noise band in the relay feedback system of Figure E-36.

Table E-6 - Results obtained for Typical processes & PI controller in closed loop for varying noise levels, Hysteresis set at 2 times the noise band

	GM est (dBs)	Gm act (dBs)	PM est (degrees)	Pm act (degrees)	% Error Gm & Pm	Lcmax est (dBs)	Lcmax act (dBs)	Band est (rads)	Band act (rads)	% Error Lcmax & band	Overall abs % Error	Sim time (secs)	Noise Level
Model 1	3.45	3.37	37.85	36.86	4.94	8.17	8.40	0.78	0.78	-3.05	8.00	3987.50	0%
Model 2	3.70	3.61	58.19	56.67	5.27	6.71	6.97	0.33	0.33	-3.73	9.08	8990.00	
Model 3	3.49	3.39	54.77	53.27	5.59	7.63	7.90	1.12	1.12	-3.50	9.09	2590.00	
Model 4	3.05	2.96	24.07	23.26	6.56	10.58	10.88	0.38	0.38	-2.81	9.37	8312.50	
Model 5	3.76	3.67	65.02	63.46	4.98	6.38	6.63	0.49	0.49	-3.77	8.97	6127.50	
Model 6	3.84	3.77	55.46	54.33	4.14	6.08	6.29	0.50	0.50	-3.26	7.63	6182.50	
Model 1	3.41	3.37	37.32	36.86	2.51	8.55	8.40	0.79	0.78	2.35	4.86	5000.00	10%
Model 2	3.73	3.61	57.00	56.67	4.01	6.81	6.97	0.32	0.33	6.07	10.08	10930.00	
Model 3	3.32	3.39	60.32	53.27	15.34	7.82	7.90	1.13	1.12	1.49	16.83	3057.50	
Model 4	2.87	2.96	24.10	23.26	6.61	10.78	10.88	0.37	0.38	2.81	9.41	10367.00	
Model 5	3.58	3.67	60.61	63.46	7.01	6.52	6.63	0.47	0.49	4.39	11.40	7405.00	
Model 6	3.76	3.77	53.15	54.33	2.25	6.58	6.29	0.47	0.50	10.29	12.54	8172.50	
Model 1	3.16	3.37	55.90	36.86	57.96	9.12	8.40	0.70	0.78	18.47	76.43	6117.50	20%
Model 2	3.49	3.61	56.71	56.67	3.19	7.29	6.97	0.32	0.33	8.24	11.42	12107.00	
Model 3	2.51	3.39	54.07	53.27	27.58	10.18	7.90	1.11	1.12	30.39	57.96	3665.00	
Model 4	2.56	2.96	22.72	23.26	15.69	11.48	10.88	0.37	0.38	7.30	22.99	12302.00	
Model 5	3.44	3.67	61.50	63.46	9.30	7.93	6.63	0.47	0.49	22.11	31.41	8195.00	
Model 6	4.79	3.77	54.71	54.33	27.90	7.99	6.29	0.47	0.50	32.93	60.83	10797.00	
Model 1	2.84	3.37	38.81	36.86	21.02	12.33	8.40	0.71	0.78	56.48	77.50	7577.50	30%
Model 2	3.08	3.61	56.75	56.67	14.88	9.38	6.97	0.31	0.33	41.36	56.23	15737.00	
Model 3	1.71	3.39	56.18	53.27	54.98	12.00	7.90	0.96	1.12	66.21	121.19	4142.50	
Model 4	2.34	2.96	21.41	23.26	29.00	13.35	10.88	0.37	0.38	24.68	53.68	14412.00	
Model 5	2.46	3.67	61.52	63.46	35.95	16.30	6.63	0.47	0.49	148.47	184.42	10937.00	
Model 6	3.06	3.77	55.34	54.33	20.75	8.59	6.29	0.47	0.50	43.44	64.19	14442.00	
Model 1	3.98	3.37	45.24	36.86	40.88	14.21	8.40	0.52	0.78	102.86	143.74	8492.50	40%
Model 2	2.95	3.61	63.83	56.67	30.87	11.76	6.97	0.31	0.33	75.54	106.41	23102.00	
Model 3	-0.27	3.39	82.47	53.27	162.64	16.69	7.90	1.04	1.12	118.45	281.08	6152.50	
Model 4	1.91	2.96	28.27	23.26	56.95	13.98	10.88	0.36	0.38	32.50	89.45	16842.00	
Model 5	1.18	3.67	67.01	63.46	73.35	14.87	6.63	0.45	0.49	131.75	205.10	16827.00	
Model 6	4.95	3.77	66.70	54.33	54.11	13.12	6.29	0.48	0.50	113.66	167.77	19172.00	
Model 1	1.79	3.37	27.44	36.86	72.41	11.88	8.40	0.67	0.78	55.38	127.79	10782.00	50%
Model 2	-3.17	3.61	62.64	56.67	198.35	25.12	6.97	0.37	0.33	272.44	470.80	32157.00	
Model 3	0.15	3.39	67.12	53.27	121.46	38.16	7.90	0.80	1.12	411.71	533.17	8442.50	
Model 4	1.90	2.96	26.93	23.26	51.45	14.72	10.88	0.38	0.38	35.73	87.17	19715.00	
Model 5	3.73	3.67	79.17	63.46	26.31	19.65	6.63	0.44	0.49	206.79	233.10	23462.00	
Model 6	2.19	3.77	66.55	54.33	64.36	21.52	6.29	0.44	0.50	255.21	319.57	24522.00	

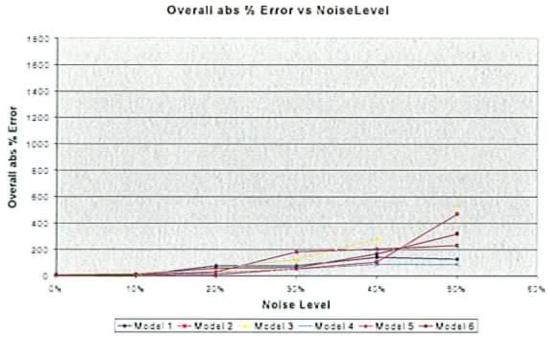

Figure E-38 - Percentage error for varying noise levels with hysteresis of twice the noise band employed

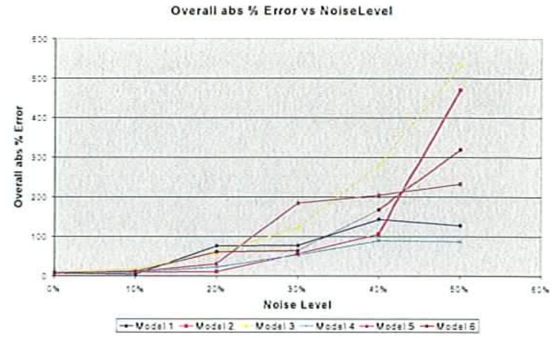

Figure E-39 - Close up of Figure E-38

Comparing Figure E-38 to Figure E-37, it can be seen that after employing a hysteresis of twice the noise band, the results obtained for each of the closed loop systems become less erratic and more accurate. However, from the results highlighted in red, it can be seen that in some cases inaccurate estimates of the 0dB crossing point, and thus system gain margin, were still being obtained, even after the implementation of anti-noise measures, in the form of the introduction of relay hysteresis. Also, it can be seen that for a noise power level of 50%, the results obtained are quite inaccurate thus making this evaluation method unsuitable for systems with noise levels of this type of magnitude. Also comparing the relay based evaluation results in the presence of noise to those obtained using the PRBS based approach, Appendix D, section D.4, it can be seen that the PRBS based approach is a much more suitable technique to employ in closed loop systems with high noise levels.

E.5 Effects of Decay Exponential:

The following figures illustrate the effects of applying a decay exponential to the relay output and controlled variable output before determining the open and closed loop system's frequency responses. For each of the figures displayed, the red line represents the actual frequency response while the blue line represents the estimated response. These response plots illustrate typical results with respect to the effects of the application of the decay exponential.

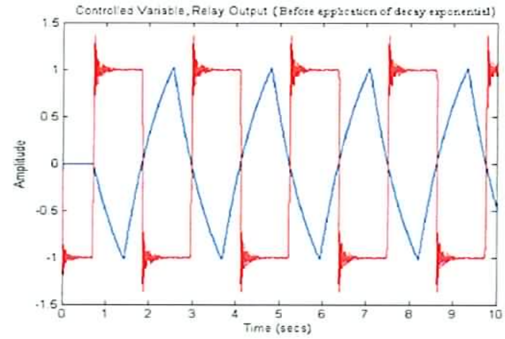

Figure E-40 - Relay output (red) versus Controlled variable output (blue), before application of decay exponential

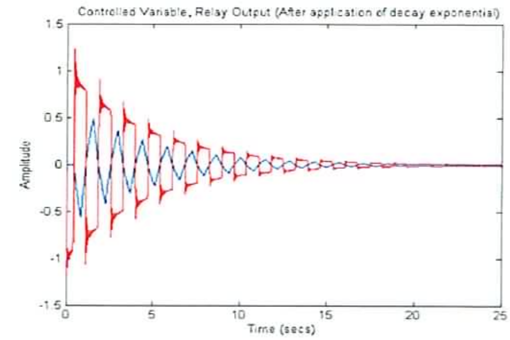

Figure E-41 - Relay output (red) versus Controlled variable output (blue), after application of decay exponential

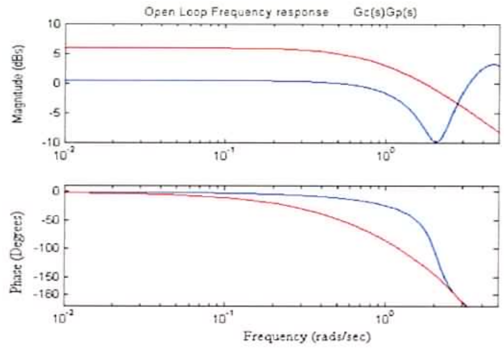

Figure E-42 - Open loop frequency response obtained for FOLPD model 7 with no decay exponential employed

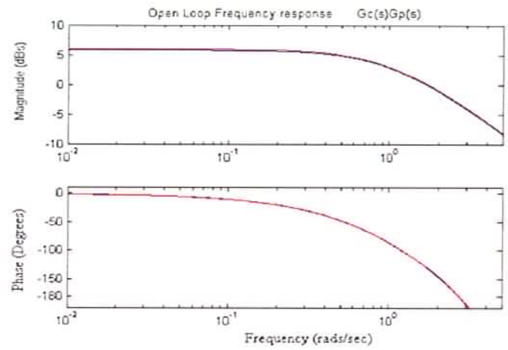

Figure E-43 - Open loop frequency response obtained for FOLPD model 7 with decay exponential employed

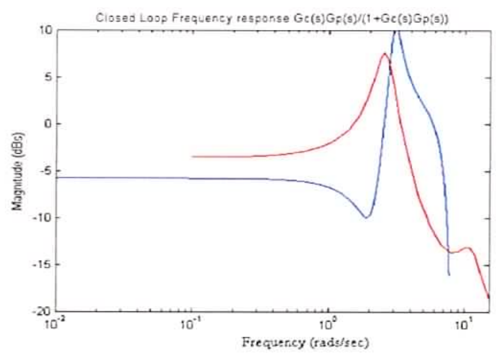

Figure E-44 - Closed loop frequency response obtained for FOLPD model 7 with no decay exponential employed

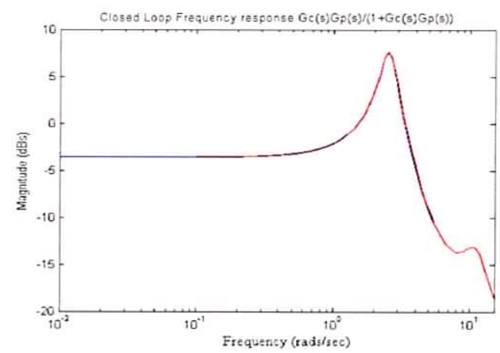

Figure E-45 - Closed loop frequency response obtained for FOLPD model 7 with decay exponential employed

It can be seen from Figure E-42 to Figure E-45, that in order to obtain an accurate estimation of both the open and closed loop frequency responses it is necessary to apply the decay exponential to the time domain signals. A thorough explanation as to why this is necessary may be found in [82].

F: Alternate Tuning Rules Suggestions:

Taking two of the typical process models and their associated PI controllers discussed in Appendix D and E, namely process models 2 and 4 shown in Table F-1, the evaluation tool developed and discussed in Chapter 3 was applied and the resulting suggested tuning rules generated by the PI(D) database were recorded and evaluated. The justification for the choice of these two particular models is discussed in section 4.4 of Chapter 4 of this thesis.

Table F-1 - Typical process models evaluated

Model No.	Typical process model	PI Controller
2	$\frac{1}{(2s+1)^5}e^{-2s}$	$1.182\left(1+\frac{1}{22.21s}\right)$
4	$\frac{1}{(s+1)(5s+1)^2}e^{-2.5s}$	$2.22\left(1+\frac{1}{19.56s}\right)$

The objective in evaluating the suggested tuning rules was to determine whether improved control could be achieved, where suggested, thus validating the evaluation strategy developed in Chapter 3. The results from the simulations carried out on models 2 and 4, and their associated controllers, for both the PRBS and relay based evaluation procedures, are displayed in sections F.1 and F.2 respectively.

F.1 PRBS Approach Results:

For the results of Table F-2 to Table F-5, the PRBS sequence length was kept fixed at 63 bits and PRBS pulse period selection method 4 was used. The tuning rules displayed in each of the tables were extracted from the PI(D) database based on their ability to achieve three separate objectives, namely minimising settling time, minimising rise time and minimising overshoot. For example, considering the results of Table F-2, the three tuning rules under the objective of *minimising settling time*, namely the rule by Rovira *et al.*, Smith & Corripio and also the rule from Schneider, were extracted because according to the results generated in the database, these rules, when applied to the actual process model, they would achieve the shortest settling times of any of the rules contained within the database. Similarly, the three tuning rules listed in Table F-2 under the objective of *minimising rise time* were chosen based on the assertion that, when applied to the actual process model, would achieve the shortest rise time of any of the rules contained within the database. After identifying the rules that would best achieve the individual objectives, the estimates of their associated settling time, rise time and percentage overshoot, generated by the database, were recorded. The rules were then applied to the actual process model in order to 1.) Determine how accurate the estimates, generated by the database, of the closed loop time domain characteristics actual were and 2.) Determine if, where suggested, improved performance could be achieved over the existing controller by application of the suggested tuning rules.

The entries of Table F-2 can be explained as follows:

- *Suggested PI Tuning rules* are the rules chosen from the PI database, based on the results generated, that best meet the specified objective (i.e. minimise settling time, minimise rise time or minimise overshoot).
- *Kc* is the controller gain calculated based on the estimated process model and the selected tuning rule formula.
- *Ti* is the controller integral time constant calculated based on the estimated process model and the selected tuning rule formula.
- *Est Ts (secs)* is the estimate of the closed loop system's settling time, generated by the PI database, based on the estimated process model and selected tuning rule controller parameters.
- *Est Tr (secs)* is the estimate of the closed loop system's rise time, calculated by the PI database, based on the estimated process model and selected tuning rule controller parameters.
- *Est over (%)* is the estimated closed loop system percentage overshoot, generated by the PI database, based on the estimated process model and selected tuning rule controller parameters.
- *Km*, *Tm* and *td* are the estimated process model FOLPD model parameters (i.e. static gain, model time constant and time delay respectively)
- *Act Ts (secs)* is the actual closed loop settling time calculated when the selected tuning rule was applied to the actual typical process model (i.e. typical process model 2 or 4)
- *Act Tr (secs)* is the actual closed loop rise time calculated when the selected tuning rule was applied to the actual typical process model.

➢ *Act Over (%)* is the actual closed loop percentage overshoot determined when the selected tuning rule was applied to the actual typical process model.

From Table F-2, it can be seen that the results for each of the specified objectives are divided into two sections, namely *Test Results* and *Applied Tuning Rules*. The results under the heading *Test Results* refer to those generated by the PI database, those under the heading *Applied Tuning Rules* refer to the results obtained due to additional simulations on the typical process models with the selected tuning rules applied. The *Applied Tuning Rules* were generated in order to verify the results displayed in the *Test Results* table.

F.1.1 PI tuning Rule Suggestions: Typical Process Model 2, Method 4

The results displayed in Table F-2 were obtained during the testing of typical process model 2 and its associated PI controller, illustrated in Table F-1, using PRBS pulse selection method 4 and a PRBS sequence length of 63 bits.

Table F-2 - Results obtained from application of suggested PI controller using model 2

PI Tuning Rules

Pulse period: Method 4 = 6.28sec
Sequence = 63 bits
Model 2 evaluated
Model 2= 1/(2s+1)*5.e-2s

Objective: Minimise Settling time

		Suggested PI Tuning rules	Kc	Ti	Est Ts (secs)	Est Tr (secs)	Est over (%)	Km	Tm	td
Test Results ====>	1.)	Rovira et al. pg31	0.55	7.87	24.4	12.56	1.6	1	6.7	7.25
	2.)	Smith & Corripio pg29	0.56	6.7	42.38	10.38	12.43	1	6.7	7.25
	3.)	Schneider pg34	0.37	6.7	39.4	20.5	0	1	6.7	7.25
		Current Rule (Estimated chars)			88.21	5.88	14.39			

			Kc	Ti	Act Ts (secs)	Act Tr (secs)	Act Over (%)
		Current: Edgar et al. pg52	1.1817	17.986	84.5	6.58	18
Applied Tuning Rules ====>	1.)	Rovira et al. pg31	0.55	7.87	49.6	11.5	0.24
	2.)	Smith & Corripio pg29	0.56	6.7	48.9	9.83	11.3
	3.)	Schneider pg34	0.37	6.7	55.9	20.9	0

Objective: Minimise Rise time

		Suggested PI Tuning rules	Kc	Ti	Est Ts	Est Tr	Est over	Km	Tm	td
Test Results ====>	1.)	Hang et al. pg36	0.68	5.69	72.98	7.34	37.55	1	6.7	7.25
	2.)	Cluett & Wang pg42	0.69	8.56	48.57	9.32	7.84	1	6.7	7.25
	3.)	Smith & Corripio pg29	0.56	6.7	42.38	10.38	12.43	1	6.7	7.25
		Current Rule (Estimated chars)			88.21	5.88	14.39			

			Kc	Ti	Act Ts (secs)	Act Tr (secs)	Act Over (%)
		Current: Edgar et al. pg52	1.1817	17.986	84.5	6.58	18
Applied Tuning Rules ====>	1.)	Hang et al. pg36	0.68	5.69	79.8	7.46	37.6
	2.)	Cluett & Wang pg42	0.69	8.56	46.3	9	9.62
	3.)	Smith & Corripio pg29	0.56	6.7	48.9	9.83	11.3

Objective: Minimise Overshoot

		Suggested PI Tuning rules	Kc	Ti	Est Ts	Est Tr	Est over	Km	Tm	td
Test Results ====>	1.)	Schneider pg34	0.34	6.7	36.81	20.81	0	1	6.7	7.25
	2.)	St Clair pg21	0.31	6.7	54.38	28.5	0	1	6.7	7.25
	3.)	Bi et al pg37	0.47	13.21	110	54.36	0	1	6.7	7.25
		Current Rule (Estimated chars)			88.21	5.88	14.39			

			Kc	Ti	Act Ts (secs)	Act Tr (secs)	Act Over (%)
		Current: Edgar et al. pg52	1.1817	17.986	84.5	6.58	18
Applied Tuning Rules ====>	1.)	Schneider pg34	0.34	6.7	61.4	26.8	0
	2.)	St Clair pg21	0.31	6.7	68.7	32.5	0
	3.)	Bi et al pg37	0.47	13.21	118	58.3	0

In Table F-2, the PI database estimates of the closed loop system characteristics for the specified objective are highlighted in blue and the actual characteristics obtained are highlighted in yellow. The table entries highlighted in red indicate instances where the estimate of the closed loop system's time domain characteristic made by the PI(D) database differs significantly from the actual characteristics value. The specified objectives are highlighted in light blue. The PI controller structure Implemented had the following form:

$$Kc\left(1+\frac{1}{T_i s}\right) \tag{F.1.1}$$

From an initial glance at Table F-2, it can be seen that there are a number of entries, associated with the *minimising settling time* objective, highlighted in red. On further investigation into these results, it can be seen that the inaccuracies in the PI database estimation of the closed loop settling times (*Est Ts (secs)*) are due to the fact that the high order typical process model (i.e. typical process model 2) is being modelled as a FOLPD model. Figure F-1 to Figure F-4 illustrate how the closed loop system response obtained using the FOLPD process model estimate and the actual closed loop system response differ, for two of the tuning rules selected to minimise closed loop settling time.

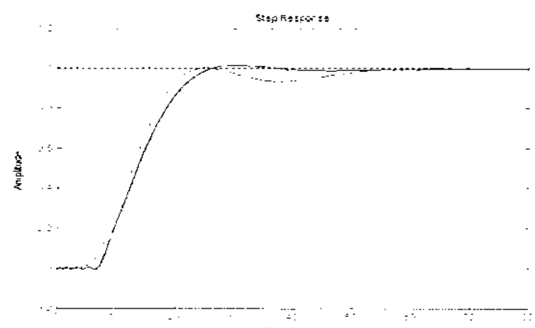

Figure F-1 - Actual step response of closed loop system using Rovira *et al.* tuning rule (green) and estimated response (blue) using estimated process model

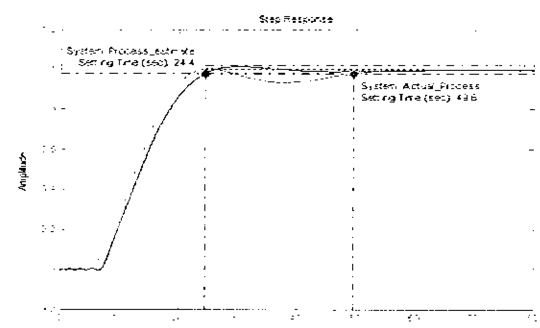

Figure F-2 - Step response of Figure F-1, with settling time characteristics labelled.

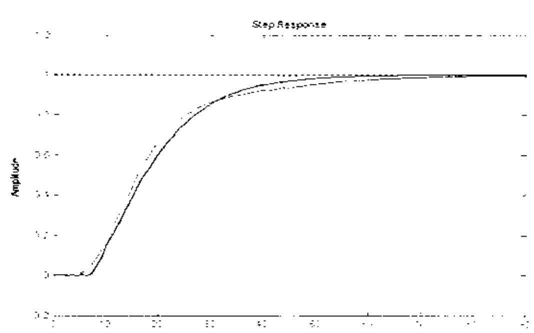

Figure F-3 - Actual step response of closed loop system using Schneider tuning rule (green) and estimated response (blue) using estimated process model

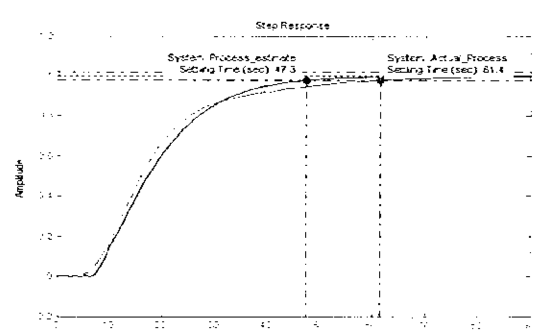

Figure F-4 - Step response of Figure F-3, with settling time characteristics labelled.

From Figure F-1 and Figure F-3, it can be seen that the estimated closed loop system step response (blue) and actual step response are quite similar. However, because a lower order model is being used to approximate the high order process, some inaccuracies in modelling the dynamic response of the closed loop system occur. This leads to the inaccurate estimates of the closed loop settling time highlighted in red in Table F-2.

Ignoring the fact that the estimates of the closed loop settling times calculated by the PI database are slightly inaccurate, each of the tuning rules suggested by the PI database asserted that, upon application of these new tuning rules, the required objective of minimising settling time could be achieved. When the suggested tuning rules were applied to the actual process, it can be seen that, in each case, the new settling times attained were shorter than the current controller being implemented. Thus, the controller parameters suggested by the PI database resulted in improved performance with respect to minimising closed loop settling time.

Considering the case where the objective was to minimise rise time, it can be seen from Table F-2 that no tuning rule within the database could be suggested that would reduce the closed loop rise time below its current values. To confirm this result, three tuning rules suggested that could achieve rise times closest to the current value were applied to the actual process in closed loop. From the results of Table F-2, it can be seen that, as indicated by the database, none of the tuning rules selected could improve upon the current rise time value. It can also be seen that, unlike the results obtained for the closed loop settling time, the estimates of the closed loop rise time (*Est Tr*) were quite accurate when compared to the actual results (*Act Tr (secs)*). Therefore, although the database was unable to provide any improvement with respect to reduced rise time values, the estimates of the closed loop system's characteristics generated accurately reflected this conclusion.

With respect to the objective of minimising overshoot, the results of Table F-2 illustrate how, for each of the tuning rules suggested, the estimated closed loop system percentage overshoot (*Est over*) with the selected tuning rules, and the actual closed loop percentage overshoot (*Act over (%)*), obtained upon application of the suggested tuning rules, was exactly the same. In this case, the each of the tuning rules suggested by the PI database results in improved performance with respect to reducing closed loop overshoot (i.e. overshoot was reduced form 18% to 0% upon application of each of the suggested tuning rules).It should also be noted that, the estimates generated by the PI database with respect to the current closed loop characteristics, labelled *Current Rule (Estimated Chars)* were quite accurate when compared to the actual results. For example, the estimated current settling time was determined to be 88.21 secs while the actual value was 84.5 secs.

F.1.2 PID Tuning Rule Suggestions: Typical Process Model 2, Method 4

The results of Table F-3 were generated using the PID tuning rule database. As discussed in section F.1.1, typical process model 2 and its associated controller were tested in closed loop, using PRBS pulse period selection method 4 and a PRBS sequence length of 63 bits.

Table F-3 - Results obtained from application of suggested PID controller using model 2

PID Tuning Rules

Pulse period: Method 4
sequence = 63 bits
Model 2 evaluated
Model 2= 1/(2s+1)^5 e-2s

Objective: Minimise Settling time

Test Results ====>

	Suggested PID Tuning rules	Kc	Ti	Td	1/N	Est Ts (secs)	Est Tr (secs)	Est over (%)	Km	Tm
1.)	Tsang et al. pg147	0.5	6.7	1.81	0.4	25.17	12.66	0.78	1	6.7
2.)	Tsang et al. pg147	0.42	6.7	1.81	0.4	27.91	15.31	0	1	6.7
3.)	Tsang et al. pg147	0.46	6.7	1.81	0.4	29.65	14.99	0	1	6.7
	Current Rule (Estimated chars)					88.21	5.88	14.39		

Applied Tuning Rules ====>

	Rule	Kc	Ti	Td	1/N	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current: Edgar et al. pg52	1.1817	17.986	-	-	84.5	6.58	18
1.)	Tsang et al. pg147	0.5	6.7	1.81	0.4	45.6	11.5	0
2.)	Tsang et al. pg147	0.42	6.7	1.81	0.4	51.5	17.8	0
3.)	Tsang et al. pg147	0.46	6.7	1.81	0.4	47.7	13.6	0

Objective: Minimise Rise time

Test Results ====>

	Suggested PID Tuning rules	Kc	Ti	Td	1/N	Est Ts	Est Tr	Est over	Km	Tm
1.)	Kaya & Scheib pg145	0.67	7.2	3.91	0.1	32.45	5.89	2.04	1	6.7
2.)	Tsang et al. pg147	0.797	6.7	1.81	0.4	55.37	6	27.49	1	6.7
3.)	Kaya & Scheib pg144	0.6	6.14	3.71	0.1	29.9	6.28	0.7	1	6.7
	Current Rule (Estimated chars)					88.21	5.88	14.39		

Applied Tuning Rules ====>

	Rule	Kc	Ti	Td	1/N	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current: Edgar et al. pg52	1.1817	17.986	-	-	84.5	6.58	18
1.)	Kaya & Scheib pg145	0.67	7.2	3.91	0.1	51.1	6.73	4.31
2.)	Tsang et al. pg147	0.797	6.7	1.81	0.4	65.8	6.63	29.3
3.)	Kaya & Scheib pg144	0.6	6.14	3.71	0.1	35.5	7.2	5.27

Objective: Minimise Overshoot

Test Results ====>

	Suggested PID Tuning rules	Kc	Ti	Td	1/N	Est Ts	Est Tr	Est over	Km	Tm
1.)	Tsang et al. pg147	0.42	6.7	1.81	0.4	27.91	15.31	0	1	6.7
2.)	Tsang et al. pg147	0.46	6.7	1.81	0.4	29.65	14.99	0	1	6.7
3.)	Kaya & Scheib pg144	0.6	6.14	3.71	0.1	29.91	6.28	0.7	1	6.7
	Current Rule (Estimated chars)					88.21	5.88	14.39		

Applied Tuning Rules ====>

	Rule	Kc	Ti	Td	1/N	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current: Edgar et al. pg52	1.1817	17.986	-	-	84.5	6.58	18
1.)	Tsang et al. pg147	0.42	6.7	1.81	0.4	51.5	17.8	0
2.)	Tsang et al. pg147	0.46	6.7	1.81	0.4	47.7	13.6	0
3.)	Kaya & Scheib pg144	0.6	6.14	3.71	0.1	35.5	7.2	5.27

Comparing Table F-3 to Table F-2, it can be seen that there are a number of additional table headings in Table F-3. These additional headings are associated with the additional parameters needed to implement a PID controller as opposed to a PI controller. These additional parameters needed are the derivative time constant labelled *Td* and the filter coefficient labelled *N*. The PID controller structure implemented had the following structure:

$$Kc\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right) \tag{F.1.2}$$

From Table F-3, it can be seen that there are a number of table entries highlighted in red. The results highlighted, illustrate results where the estimated closed loop time response characteristics are inaccurate when compared to the actual characteristics. This occurs as a result of modelling a high order process with a low order model. This issue has been discussed in section F.1.1.

Considering the results displayed in Table F-3, with respect to the results obtained in conjunction with the objective of minimising settling time, it can be seen that although the estimated settling times proved inaccurate, for each of the tuning rules suggested, a reduced settling time was achieved when each of the selected tuning rules were applied to the actual process. In some cases, an actual reduction in settling time of almost 50% was achieved upon implementation of the suggested tuning rule.

With respect to the results obtained for the objective of minimising rise time, it can be seen that the estimated closed loop rise time (*Est Tr*) for each of the selected tuning rules was reasonably accurate when compared to the actual closed loop rise time (*Act Tr (secs)*). From the results obtained, it can be seen that it was not possible to identify a tuning rule that could provide a closed loop rise time below the current rise time of the system.

Considering the results obtained for the objective of minimising percentage overshoot, it can be seen that for each of the tuning rules suggested, a significant reduction in overshoot was obtained upon application of the suggested rule.

An interesting result has been highlighted in orange in Table F-3. It can be seen that the actual settling time highlighted (*Act Ts (secs)*) is less than any of the results obtained under the objective of minimising settling time. In this instance the PID database failed to identify the tuning rule that could provide the shortest settling time when applied to the actual process. This erroneous result occurred because the PI(D) database calculates its estimates of settling time, rise time and overshoot using the estimate of the process model. Therefore, as highlighted in section F.1.1, inaccuracies resulting from the modelling of the high order process with a FOLPD model manifest itself as an inaccurate estimate of the closed loop time domain characteristics. This problem could be remedied by modelling the process with a higher order model.

F.1.3 PI Tuning Rule Suggestions: Typical Process Model 4, Method 4

The results of Table F-4 were generated using the PI tuning rule database. For this set of results, typical process model 4 and its associated controller were tested in closed loop, using PRBS pulse period selection method 4 and a PRBS sequence length of 63 bits.

Table F-4 - Results obtained from application of suggested PI controller using model 4

PI Tuning Rules

Pulse period: Method 4 = 5.21sec
Sequence = 63 bits
Model 4 evaluated
Model 4= 1/(s+1)(5s+1)^2.e-2.5s

Objective: Minimise Settling time

Test Results ====>

	Suggested PI Tuning rules	Kc	Ti	Est Ts (secs)	Est Tr (secs)	Est over (%)	Km	Tm	td
1.)	Zhuang & Atherton pg27	1.58	10.67	45.7	5.02	30.44	0.99	9.35	5.28
2.)	Murril pg26	1.51	9.41	47.74	4.96	33.96	0.99	9.35	5.28
3.)	McMillann pg45	1.92	15.41	53.4	4.23	32.56	0.99	9.35	5.28
	Current Rule (Estimated chars)			72.13	3.65	45.7			

Applied Tuning Rules ====>

		Kc	Ti	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current: Edgar et al. pg52	2.22	15.85	89.7	4.88	47.6
1.)	Zhuang & Atherton pg27	1.58	10.67	77.7	5.95	38.5
2.)	Murril pg26	1.51	9.41	80.6	5.98	42.4
3.)	McMillann pg45	1.92	15.41	80.6	5.46	36.4

Objective: Minimise Rise time

Test Results ====>

	Suggested PI Tuning rules	Kc	Ti	Est Ts	Est Tr	Est over	Km	Tm	td
1.)	Murill pg25	2.28	12.46	102.38	3.4	59.86	0.99	9.35	5.28
2.)	Murill pg21	1.61	6.22	103.3	4.13	70.73	0.99	9.35	5.28
3.)	Pemberton pg22	1.79	9.35	68.72	4.21	50.28	0.99	9.35	5.28
	Current Rule (Estimated chars)			72.13	3.65	45.7			

Applied Tuning Rules ====>

		Kc	Ti	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current: Edgar et al. pg52	2.22	15.85	89.7	4.88	47.6
1.)	Murill pg25	2.28	12.46	164	4.62	61.2
2.)	Murill pg21	1.61	6.22	240	5.12	78
3.)	Pemberton pg22	1.79	9.35	116	5.24	56.3

Objective: Minimise Overshoot

Test Results ====>

	Suggested PI Tuning rules	Kc	Ti	Est Ts	Est Tr	Est over	Km	Tm	td
1.)	Astrom & Hagglund pg51	0.28	3.36	90.12	15.34	18.89	0.99	9.35	5.28
2.)	Davydov pg36	0.53	4.19	88.32	10.11	30.33	0.99	9.35	5.28
3.)	Zhuang & Atherton pg27	1.58	10.67	45.68	5.02	30.44	0.99	9.35	5.28
	Current Rule (Estimated chars)			72.13	3.65	45.7			

Applied Tuning Rules ====>

		Kc	Ti	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current: Edgar et al. pg52	2.22	15.85	89.7	4.88	47.6
1.)	Astrom & Hagglund pg51	0.28	3.36	80.9	14.5	20.6
2.)	Davydov pg36	0.53	4.19	85.1	9.86	35.6
3.)	Zhuang & Atherton pg27	1.58	10.67	77.7	5.95	38.5

From Table F-4, it can be seen that a number of the results associated with the objective of minimising settling time have been highlighted in red. These highlighted results indicate a situation where the estimate of the closed loop system's time domain characteristics (in this case settling time *Est Ts (secs)*) was determined to be inaccurate when compared to the actual closed loop characteristic. This issue has been discussed in section F.1.1. For each of the suggested tuning rules, improved performance with respect to minimising settling time was achieved, over the current performance, upon application of the suggested rule.

For both the results obtained for the objective of minimising rise time and minimising overshoot, the tuning rules suggested by the PI database resulted in improved performance, when applied to the actual process, in each of the situations where it indicated it could. For the case of the *minimising rise time* objective, the database only suggested one tuning rule where improved performance could be achieved. The other two rules illustrated in Table F-4, resulted in closed loop rise times similar to, but greater than, the current closed loop system's rise time.

F.1.4 PID Tuning Rule Suggestions: Typical Process Model 4, Method 4

The results of Table F-5 were generated using the PID tuning rule database. Typical process model 4 and its associated controller were tested in closed loop, using PRBS pulse period selection method 4 and a PRBS sequence length of 63 bits.

Table F-5 - Results obtained from application of suggested PID controller using model 4

PID Tuning Rules

Pulse period: Method 4 = 5.21sec
Sequence = 63 bits
Model 4 evaluated
Model 4= 1/(s+1)(5s+1)^2.e-2.5s

Objective: Minimise Settling time

Test Results ====>

	Suggested PID Tuning rules	Kc	Ti	Td	1/N	Est Ts (secs)	Est Tr (secs)	Est over (%)	Km	Tm
1.)	Tsang et al. pg147	2.07	9.35	1.32	0.4	65.89	3.08	56.06	0.99	9.35
2.)	Kaya & Scheib pg142	1.88	6.78	3.1	0.1	65.88	1.54	75.38	0.99	9.35
3.)	Witt & Waggoner pg139	0.55	2.23	8.61	0.067	71.35	2.32	64.42	0.99	9.35
	Current Rule (Estimated chars)					72.13	3.65	45.7		

Applied Tuning Rules ====>

		Kc	Ti	Td	1/N	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current: Edgar et al. pg52	2.22	15.85	-	-	89.7	4.88	47.6
1.)	Tsang et al. pg147	2.07	9.35	1.32	0.4	101	4.45	57.1
2.)	Kaya & Scheib pg142	1.85	6.78	3.1	0.1	54.8	3.53	51.7
3.)	Witt & Waggoner pg139	0.55	2.23	8.61	0.067	80.3	3.86	56.6

Objective: Minimise Rise time

Test Results ====>

	Suggested PID Tuning rules	Kc	Ti	Td	1/N	Est Ts	Est Tr	Est over	Km	Tm
1.)	Kaya & Scheib pg142	1.88	6.78	3.1	0.1	65.88	1.54	75.38	0.99	9.35
2.)	Witt & Waggoner pg139	0.55	2.23	8.61	0.067	71.35	2.32	64.42	0.99	9.35
3.)	Tsang et al. pg147	2.47	9.35	1.32	0.4	120.32	2.56	78.15	0.99	9.35
	Current Rule (Estimated chars)					72.13	3.65	45.7		

Applied Tuning Rules ====>

		Kc	Ti	Td	1/N	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current: Edgar et al. pg52	2.22	15.85	-	-	89.7	4.88	47.6
1.)	Kaya & Scheib pg142	1.85	6.78	3.1	0.1	54.8	3.53	51.7
2.)	Witt & Waggoner pg139	0.55	2.23	8.61	0.067	80.3	3.86	56.6
3.)	Tsang et al. pg147	2.47	9.35	1.32	0.4	184	3.9	73

Objective: Minimise Overshoot

Test Results ====>

	Suggested PID Tuning rules	Kc	Ti	Td	1/N	Est Ts	Est Tr	Est over	Km	Tm
1.)	Tsang et al. pg147	2.07	9.35	1.32	0.4	65.89	3.08	56.06	0.99	9.35
2.)	Witt & Waggoner pg139	0.55	2.23	8.61	0.067	71.35	2.32	64.42	0.99	9.35
3.)	Kaya & Scheib pg142	1.88	6.78	3.1	0.1	65.88	1.54	75.38	0.99	9.35
	Current Rule (Estimated chars)					72.13	3.65	45.7		

Applied Tuning Rules ====>

		Kc	Ti	Td	1/N	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current: Edgar et al. pg52	2.22	15.85	-	-	89.7	4.88	47.6
1.)	Tsang et al. pg147	2.07	9.35	1.32	0.4	101	4.45	57.1
2.)	Witt & Waggoner pg139	0.55	2.23	8.61	0.067	80.3	3.86	56.6
3.)	Kaya & Scheib pg142	1.88	6.78	3.1	0.1	54.8	3.53	51.7

From Table F-5, the results highlighted in red indicate instances where the estimated closed loop system characteristic was determined to be inaccurate when compare to the actual characteristic value (refer to section F.1.1 for a detail explanation of these inaccuracies).

For the case of the minimising settling time objective, the suggested tuning rules resulted in improved performance in two out of the three cases where the PID database indicated improved performance could be achieved. For the case of the Tsang *et al.* tuning rule result, inaccuracies due to the modelling of the process as a FOLPD model resulted in an inaccurate estimate of the settling time, which, in turn, resulted in the PID database incorrectly identifying this rule as a means of improved performance. Using a higher order model of the process would eliminate erroneous results such as this one.

For the case of the minimising rise time objective, reduced closed loop rise times, and as a result, improved performance resulted in three out of the three cases where indicated by the PID database.

For the case of the minimising overshoot objective, the PID database indicated that no rule in the database could reduce the overshoot below the current value. To confirm this result, three tuning rules suggested that could generate a percentage overshoot closest to the current value were applied to the actual process and the resulting closed loop characteristics were evaluated. From the results of Table F-5, it can be seen that, as indicated by the database none of the tuning rules selected could improve upon the current percentage overshoot value.

F.2 Relay Approach Results:

The results of Table F-6 to Table F-9 were obtained using the relay based evaluation procedure developed in Chapter 3. As in section F.1, typical process models 2 and 4 and their associated PI controllers were evaluated. For each of the tests carried out, 250 output periods were recorded and processed in order to obtain the results of Table F-6 to Table F-9.

F.2.1 PI Tuning Rule Suggestions: Typical Process Model 2, M=250:

The results of Table F-6 were generated using the PI tuning rule database and the relay-based approach to system evaluation developed in Chapter 3.

Table F-6 - Results obtained from application of suggested PI controller using model 2

PI Tuning Rules

No of periods, M=250
Model 2 evaluated
Model 2= 1/(2s+1)*5.e-2s

Objective: Minimise Settling time

Test Results ====>

	Suggested PI Tuning rules	Kc	Ti	Est Ts (secs)	Est Tr (secs)	Est over (%)		Km	Tm
1.)	Rovira et al. pg31	0.54	7.86	**25.11**	12.63	1.45		1	6.66
2.)	Schneider pg34	0.37	6.66	**38.44**	19.87	0		1	6.66
3.)	Gorecki et al. pg38	0.53	7.21	**40.38**	11.73	5.9		1	6.66
	Current Rule (Estimated chars)			**88.96**	5.88	15.65			

Applied Tuning Rules ====>

		Kc	Ti	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current Edgar et al. pg52	1.1817	17.986	**84.5**	6.58	18
1.)	Rovira et al. pg31	0.54	7.86	**50**	11.8	0
2.)	Schneider pg34	0.37	6.66	**55.4**	20.4	0
3.)	Gorecki et al. pg38	0.53	7.21	**49**	11.2	3.35

Objective: Minimise Rise time

Test Results ====>

	Suggested PI Tuning rules	Kc	Ti	Est Ts	Est Tr	Est over		Km	Tm
1.)	Hang et al. pg36	0.69	5.67	73.43	**6.95**	40.4		1	6.66
2.)	Chien et al. pg20	0.54	6.66	41.75	**10.65**	11.62		1	6.66
3.)	Smith & Corripio pg 29	0.54	6.66	41.75	**10.65**	11.62		1	6.66
	Current Rule (Estimated chars)			88.96	**5.88**	15.65			

Applied Tuning Rules ====>

		Kc	Ti	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current Edgar et al. pg52	1.1817	17.986	84.5	**6.58**	18
1.)	Hang et al. pg36	0.69	5.67	80	**7.36**	39.1
2.)	Chien et al. pg20	0.54	6.66	49	**10.2**	9.46
3.)	Smith & Corripio pg 29	0.54	6.66	49	**10.2**	9.46

Objective: Minimise Overshoot

Test Results ====>

	Suggested PI Tuning rules	Kc	Ti	Est Ts	Est Tr	Est over		Km	Tm
1.)	St Clair pg21	0.299	6.66	55.13	28.91	**0**		1	6.66
2.)	Schneider pg 34	0.33	6.66	40.57	22.31	**0**		1	6.66
3.)	Cluett & Wang pg41	0.36	7.95	63.47	30.48	**0**		1	6.66
	Current Rule (Estimated chars)			88.96	5.88	**15.65**			

Applied Tuning Rules ====>

		Kc	Ti	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current Edgar et al. pg52	1.1817	17.986	84.5	6.58	**18**
1.)	St Clair pg21	0.299	6.66	71.1	34	**0**
2.)	Schneider pg 34	0.33	6.66	63	28.3	**0**
3.)	Cluett & Wang pg41	0.36	7.95	76.4	36.8	**0**

From Table F-6, it can be seen that there are two results, highlighted in red, for which the estimated settling times (*Est Ts (secs)*) generated by the PI database for the selected tuning rules turned out to be inaccurate when compared to the actual results (*Act Ts (secs)*). This issue has been discussed in section F.1.1.

From the result obtained for the objective of minimising settling time, it can be seen that for each of the tuning rules suggested by the PI database, improved performance, in the form of reduced settling time, was achieved upon implementation of the suggested rule. This was also the case for the results obtained for the objective of minimising overshoot. It can be seen that, when each of the suggested tuning rule were applied to the actual process, a reduction in overshoot was achieved. For the objective of minimising rise time, the PI database was unable to suggest any tuning rules that could reduce the rise time below the current value. This was confirmed by applying the three tuning rules, suggested by the PI database, that would generate rise times closest to the current closed loop system rise time. As the results indicate, none of the tuning rules selected and tested, generated rise times below the current closed loop rise time of the system.

F.2.2 PID Tuning Rule Suggestions: Typical Process Model 2, M=250:

The results of Table F-7 were generated using the PID tuning rule database and the relay based evaluation procedure. 250 output oscillations were recorded and processed in the generation of the results shown.

Table F-7 - Results obtained from application of suggested PID controller using model 2

No of periods, M=250
Model 2 evaluated
Model 2= 1/(2s+1)*5.e-2s

Objective: Minimise Settling time

	Suggested PID Tuning rules	Kc	Ti	Td	1/N	Est Ts (secs)	Est Tr (secs)	Est over (%)	Km	Tm
1.)	Tsang et al pg147	0.49	6.66	1.84	0.4	**25.59**	12.9	0.69	1	6.66
2.)	Tsang et al pg147	0.45	6.66	1.84	0.4	**29.73**	15.2	0	1	6.66
3.)	Kaya & Scheib pg144	0.59	6.09	3.76	0.1	**30.43**	6.38	0.79	1	6.66
	Current Rule (Estimated chars)					**88.96**	5.88	15.65		

Applied Tuning Rules ====>		Kc	Ti	Td	1/N	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current Edgar et al. pg52	1.1817	17.986	-	-	**84.5**	6.58	18
1.)	Tsang et al pg147	0.49	6.66	1.84	0.4	**27.6**	16.4	1.5
2.)	Tsang et al pg147	0.45	6.66	1.84	0.4	**32**	18.5	0.34
3.)	Kaya & Scheib pg144	0.59	6.09	3.76	0.1	**22.3**	12.4	0.21

Objective: Minimise Rise time

	Suggested PID Tuning rules	Kc	Ti	Td	1/N	Est Ts	Est Tr	Est over	Km	Tm
1.)	Tsang & Rad pg147	0.73	6.66	3.67	0.2	42.65	**5.08**	17.21	1	6.66
2.)	Kaya & Scheib pg145	0.65	7.19	3.96	0.1	32.95	**6.06**	1.65	1	6.66
3.)	Tsang et al pg147	0.78	6.66	1.84	0.4	56.12	**6.1**	27.5	1	6.66
	Current Rule (Estimated chars)					88.96	**5.88**	15.65		

Applied Tuning Rules ====>		Kc	Ti	Td	1/N	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current Edgar et al. pg52	1.1817	17.986	-	-	84.5	**6.58**	18
1.)	Tsang & Rad pg147	0.73	6.66	3.67	0.2	24.5	**9.39**	26.1
2.)	Kaya & Scheib pg145	0.65	7.19	3.96	0.1	36.7	**14**	0
3.)	Tsang et al pg147	0.78	6.66	1.84	0.4	47.8	**9.24**	15.3

Objective: Minimise Overshoot

	Suggested PID Tuning rules	Kc	Ti	Td	1/N	Est Ts	Est Tr	Est over	Km	Tm
1.)	Tsang et al pg147	0.45	6.66	1.84	0.4	29.73	15.2	**0**	1	6.66
2.)	Tsang et al pg147	0.41	6.66	1.84	0.4	31.65	16.74	**0**	1	6.66
3.)	Kaya & Scheib pg144	0.59	6.09	3.76	0.1	30.43	6.38	**0.79**	1	6.66
	Current Rule (Estimated chars)					88.96	5.88	**15.65**		

Applied Tuning Rules ====>		Kc	Ti	Td	1/N	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current Edgar et al. pg52	1.1817	17.986	-	-	84.5	6.58	**18**
1.)	Tsang et al pg147	0.45	6.66	1.84	0.4	32	18.5	**0.34**
2.)	Tsang et al pg147	0.41	6.66	1.84	0.4	38.5	21.3	**0**
3.)	Kaya & Scheib pg144	0.59	6.09	3.76	0.1	22.3	12.4	**0.21**

The results of Table F-7, highlighted in red, indicate situations were the estimate of the closed loop time domain characteristics of the system were determined to be inaccurate when compared to the actual results. As can be seen, these inaccurate estimates are associated with the estimates of the rise time of the closed loop systems. This issue has been discussed in more detail in section F.1.1.

From the results obtained with respect to the objectives of minimising settling time and minimising overshoot, it can be seen that for each of the tuning rules suggested by the PID database, improved performance, with respect to the specified objectives, was achieved when the suggested rule was applied to the actual process model. With respect to the results obtained under the objective of minimising rise time, the PID database was unable to identify any tuning rules that would result in reduced rise times when applied to the process model.

F.2.3 PI Tuning Rule Suggestions: Typical Process Model 4, M=250:

The results of Table F-8 were generated using the PI tuning rule database and the relay based evaluation procedure.

Table F-8 - Results obtained from application of suggested PI controller using model 4

PI Tuning Rules

No of periods, M=250
Model 4 evaluated
Model 4= 1/(s+1)(5s+1)^2.e-2.5s

Objective: Minimise Settling time

		Suggested PI Tuning rules	Kc	Ti	Est Ts (secs)	Est Tr (secs)	Est over (%)		Km	Tm	td
Test Results ====>	1.)	Znuang & Atherton pg27	1.54	10.91	47.26	5.13	30.32		1	9.4	5.51
	2.)	McMillan pg45	1.86	16.02	55.76	4.54	30.87		1	9.4	5.51
	3.)	Murril pg22	1.68	10.6	59.25	4.63	40.32		1	9.4	5.51
		Current Rule (Estimated chars)			94.27	3.68	49.67				

			Kc	Ti	Act Ts (secs)	Act Tr (secs)	Act Over (%)
		Current Edgar et al. pg52	2.22	15.85	89.7	4.88	47.6
Applied Tuning Rules ====>	1.)	Znuang & Atherton pg27	1.54	10.91	76.6	6.11	35.4
	2.)	McMillan pg45	1.86	16.02	83.2	5.65	32.3
	3.)	Murril pg22	1.68	10.6	89.2	5.65	43.6

Objective: Minimise Rise time

		Suggested PI Tuning rules	Kc	Ti	Est Ts	Est Tr	Est over		Km	Tm	td
Test Results ====>	1.)	Huang et al. pg24	2.06	10.92	90.02	3.72	60.08		1	9.4	5.51
	2.)	Murril pg25	2.2	12.88	97.74	3.76	57.88		1	9.4	5.51
	3.)	Pemberton pg 22	1.72	9.43	70.56	4.4	49.73		1	9.4	5.51
		Current Rule (Estimated chars)			94.27	3.68	49.67				

			Kc	Ti	Act Ts (secs)	Act Tr (secs)	Act Over (%)
		Current Edgar et al. pg52	2.22	15.85	89.7	4.88	47.6
Applied Tuning Rules ====>	1.)	Huang et al. pg24	2.06	10.92	144	4.96	59.1
	2.)	Murril pg25	2.2	12.88	141	4.9	56.2
	3.)	Pemberton pg 22	1.72	9.43	104	5.4	52.4

Objective: Minimise Overshoot

		Suggested PI Tuning rules	Kc	Ti	Est Ts	Est Tr	Est over		Km	Tm	td
Test Results ====>	1.)	Znuang & Atherton pg27	1.54	10.91	47.26	5.13	30.32		1	9.4	5.51
	2.)	Davydov pg36	0.52	4.26	91.96	10.22	30.8		1	9.4	5.51
	3.)	McMillan pg45	1.86	16.02	55.78	4.54	30.87		1	9.4	5.51
		Current Rule (Estimated chars)			94.27	3.68	49.67				

			Kc	Ti	Act Ts (secs)	Act Tr (secs)	Act Over (%)
		Current Edgar et al. pg52	2.22	15.85	89.7	4.88	47.6
Applied Tuning Rules ====>	1.)	Znuang & Atherton pg27	1.54	10.91	76.6	6.11	35.4
	2.)	Davydov pg36	0.52	4.26	85.5	10.1	33.5
	3.)	McMillan pg45	1.86	16.02	83.2	5.65	32.3

The results of Table F-8, highlighted in red, indicate instances where the PI database estimates of closed loop system time domain characteristics differed significantly from the actual characteristics obtained when the suggested tuning rules were applied to the actual process model. This issue has been discussed in detail in section F.1.1.

From Table F-8, it can be seen that for each of the tuning rules suggested with respect to the objectives of minimising settling time and overshoot, improved performance was achieved when the suggested rule was applied to the actual process model.

With respect to the objective of minimising rise time, the PI database was unable to suggest tuning rules that could improve upon the current closed loop system rise time. The results displayed in Table F-8 were generated using three tuning rules suggested that would give rise times approximately equal to the current closed loop system rise time.

F.2.4 PID Tuning Rule Suggestions: Typical Process Model 4, M=250:

The results of Table F-9 were generated using the PI tuning rule database and the relay based evaluation procedure.

Table F-9 - Results obtained from application of suggested PID controller using model 2

PID Tuning Rules

No of periods, M=250
Model 4 evaluated
Model 4= 1/(s+1)(5s+1)^2.e-2.5s

Objective: Minimise Settling time

	Suggested PID Tuning rules	Kc	Ti	Td	1/N	Est Ts (secs)	Est Tr (secs)	Est over (%)	Km	Tm
1.)	Tsang et al pg147	1.71	9.43	1.38	0.4	42.77	3.68	39.77	1	9.4
2.)	Tsang et al pg147	2	9.4	1.38	0.4	70.14	3.22	55.33	1	9.4
3.)	Kaya & Scheib pg142	1.82	7.07	3.22	0.1	73.99	1.68	74.44	1	9.4
	Current Rule (Estimated chars)					94.27	3.68	49.67		

Applied Tuning Rules ====>

		Kc	Ti	Td	1/N	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current Edgar et al. pg52	2.22	15.85	-	-	89.7	4.88	47.6
1.)	Tsang et al pg147	1.71	9.43	1.38	0.4	70.4	5.18	41.2
2.)	Tsang et al pg147	2	9.4	1.38	0.4	90.6	4.56	53.6
3.)	Kaya & Scheib pg142	1.82	7.07	3.22	0.1	52.8	3.61	46.5

Objective: Minimise Rise time

	Suggested PID Tuning rules	Kc	Ti	Td	1/N	Est Ts	Est Tr	Est over	Km	Tm
1.)	Kaya & Scheib pg142	1.82	7.07	3.22	0.1	73.99	1.68	74.44	1	9.4
2.)	Witt & Waggoner pg139	0.53	2.32	8.93	0.067	75.78	2.42	63.31	1	9.4
3.)	Tsang et al pg147	2.38	9.43	1.38	0.4	126.01	2.67	77.66	1	9.4
	Current Rule (Estimated chars)					94.27	3.68	49.67		

Applied Tuning Rules ====>

		Kc	Ti	Td	1/N	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current Edgar et al. pg52	2.22	15.85	-	-	89.7	4.88	47.6
1.)	Kaya & Scheib pg142	1.82	7.07	3.22	0.1	52.8	3.61	46.5
2.)	Witt & Waggoner pg139	0.53	2.32	8.93	0.067	69.9	3.97	50.9
3.)	Tsang et al pg147	2.38	9.43	1.38	0.4	152	3.99	69.1

Objective: Minimise Overshoot

	Suggested PID Tuning rules	Kc	Ti	Td	1/N	Est Ts	Est Tr	Est over	Km	Tm
1.)	Tsang et al pg147	1.71	9.43	1.38	0.4	42.77	3.68	39.77	1	9.4
2.)	Tsang et al pg147	2	9.43	1.38	0.4	55.33	3.22	55.33	1	9.4
3.)	Witt & Waggoner pg139	0.53	2.32	8.93	0.067	75.78	2.42	63.31	1	9.4
	Current Rule (Estimated chars)					94.27	3.68	49.67		

Applied Tuning Rules ====>

		Kc	Ti	Td	1/N	Act Ts (secs)	Act Tr (secs)	Act Over (%)
	Current Edgar et al. pg52	2.22	15.85	-	-	89.7	4.88	47.6
1.)	Tsang et al pg147	1.71	9.43	1.38	0.4	70.4	5.18	41.2
2.)	Tsang et al pg147	2	9.43	1.38	0.4	90.6	4.56	53.6
3.)	Witt & Waggoner pg139	0.53	2.32	8.93	0.067	69.9	3.97	50.9

The results of Table F-9, highlighted in red, illustrate results obtained where the PID database estimates of closed loop system time domain characteristics differed significantly from the actual characteristics obtained when the suggested tuning rules were applied to the actual process model. This issue has been discussed previously in section F.1.1.

From the results obtained with respect to the objective of minimising rise time, it can be seen that, for each of the tuning rules suggested by the PID database, improved performance in the form of reduced rise times was achieved when the suggested tuning rules were applied to the actual processes.

With respect to the objective of minimising overshoot, it can be seen from Table F-9 that the PID database suggested one tuning rule, namely the first tuning rule by Tsang *et al.*, that would result in improved performance and two tuning rules that would results in similar performance to the current control system. When the suggested rules were applied to the actual process in closed loop, these results were confirmed; with the first tuning rule (Tsang *et al.*) providing improved performance and the other two rules providing marginally larger percentage overshoots, thus poorer performance, than the current control system.

From the results obtained with respect to the objective of minimising settling time, it can be seen from Table F-9 that the PID database suggested three tuning rules that would results in improved performance. When applied to the actual process, it was determined that improved performance was only achieved for two of these suggested tuning rules, namely the first rule designed by Tsang *et al.* and the rule designed by Kaya & Scheib. The second rule designed by Tsang *et al.* actually resulted in marginally poorer performance when applied to the actual process. This erroneous results occurred due to an inaccurate estimate of the closed loop settling time caused by the model mismatching problem discussed in section F.1.1.

G: P, PI and PID control explained:

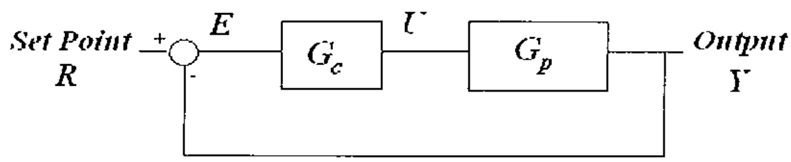

Figure G-1 - Simple feedback system

Proportional controllers work by simply multiplying the error between the set point and the process output by some constant value [73]. Simple proportional control proves to be effective in situations where the static gain of the process is approximately one. Proportional controllers are quite effective in controlling applications such as liquid level control where no particular economic penalty applies to small deviations in level [73]. Traditionally, instead of using integral action, it was common practice for operators to apply their own correction by applying a bias to the output of the controller. This practice is known as manual reset. The following formula may be used to describe a proportional controller with bias for the set up shown in Figure G-1.

$$G_p(s) = K_c\left(r - c\right) + bias \qquad \text{(G.1.1)}$$

The Proportional plus Integral (PI) controller combines the stability of proportional control with the offset elimination of integral action. PI controllers frequently have the following structure:

$$G_{PI}(s) = K_c\left(1 + \frac{1}{sT_i}\right)E(s) \qquad \text{(G.1.2),}$$

where T_i is known as the integral time. In introducing the integral term to a control system, the control engineer is also introducing a phase lag into the system. In effect the system designer is in fact moving the system's closed loop poles closer to the imaginary axis or, to put it another way, reducing the phase and gain margins of the system thus reducing the robustness of the overall system [73]. While this integral term may eliminate offset error and improve the response times of the system (i.e. rise time, settling time, etc.) it also has the effect of making the response of the system more oscillatory by its nature, thus increasing the risks of overshoot or, worse, system instability.

Derivative action is not used alone, but always in conjunction with proportional action. This type of corrective action has the effect of adding phase lead to a system. It may therefore be used to cancel out any phase lag introduced by integral action. In a PID controller derivative action may be useful in shortening the period of the loop, thereby hastening its recovery from disturbances. One major disadvantage in applying this type of control action to a loop is that, if any disturbances affecting the loop come in the form of step disturbances, the derivative action will produce a large pulse output therefore overriding the controller output and causing a large overshoot in the controlled variable. Derivative action is therefore always accompanied by some kind of filter; hence the lead introduced is always accompanied by a lag. The classical PID structure is of the following form:

$$G_{PID}(s) = K_c\left(1+\frac{1}{sT_i}\right)\left(\frac{1+sT_d}{1+s\frac{T_d}{N}}\right) \qquad \text{(G.1.3)},$$

where T_d is the derivative time constant; N is the filter coefficient and typically has a value of 10. In process control industries, more than 90% of the control loops are of the PID type [100].

H: Publications:

[1.] O'Connor, N. and O'Dwyer, A. (2005). 'Comparative study of two techniques for determining critical system response characteristics', Proceedings of the 4th Wismar Automatisierungssymposium, Wismar, Germany, September

Niall O'Connor, A O'Dwyer, School of Control Systems and Electrical Engineering, Dublin Institute of Technology, Dublin 8, Ireland

Abstract: The paper presents a review of a comparative study of two separate techniques for obtaining important frequency and time domain characteristics of a system consisting of a process in series with a PI/PID controller in closed loop. The first technique involves the use of a Pseudo Random Binary Sequence (PRBS) to obtain the closed loop frequency response of the system. This closed loop frequency information may then be manipulated to obtain both open loop frequency response information and process model parameters. A further mathematical simulation using Matlab may then be performed independent of the actual process in order to obtain important time domain characteristics of the system such as rise time, settling time and overshoot. The second technique is a relay-based method described in [1], which may be used to obtain the open loop frequency response of the system. In this case, it is the open loop frequency response information that is manipulated in order to obtain closed loop frequency response information and process model parameters.

Introduction:

According to [2], in the testing of thousands of control loops in hundreds of operating plants, Techmation Inc. and others have found that more than 30% of the automatic control loops actually increase variability over manual control due to poor controller tuning. One reason why so many control loops perform poorly is that there are often numerous (more than a thousand) loops in a large process plant and not enough control engineers to maintain every loop. Jamsa-Jounela *et al.* [3] make the point that in order to ensure highest product quality it is essential to maintain the control system in an adequate manner. Vishnubhotla *et al.* [4] discuss how the current standard practice for industrial process control is to install DCS (Distributed Control Systems) and PLC control system platforms. These system platforms accumulate large volumes of process data, but there are very few data mining tools. As stated in [7], only about 20% of all control loops surveyed in mill audits have been found to actually reduce process variability in automatic mode. The remaining 80% of loops were found to increase variability. Of these, about 30% were found to oscillate due to control valve nonlinearities, another 30% performed poorly due to poor controller tuning and controller equipment design limitations, approximately 15% performed poorly due to deficiencies in control strategy design and about 5% performed poorly due to poor process design.

It is obvious, therefore, that there is a strong need for automatic assessment and monitoring of control loop performance. The goal of monitoring should be to provide information that can be used to assess the current status of the existing controller and to assist control engineers in deciding whether redesign is necessary [5]. When the controller performance is determined to be inadequate, it is important to ascertain whether an acceptable level of performance can be achieved with the existing control structure [6].

The paper presents two separate schemes for evaluating a control system's performance. The first method presented is a PRBS based technique, which basically involves the application of a binary sequence test signal to the system under investigation and the subsequent employment of the Fast Fourier Transform (FFT) to the resulting system response. The second testing scheme, again involves the use of the FFT, however the system is excited through the use of a relay placed in series with the controller and process. For each of the techniques discussed, a graphical user interface (GUI) was developed using Matlab 6.5 in order to provide a user-friendly environment in which the testing procedure could be carried out.

The paper is divided into the following sections. Section 0 provides a brief outline of the steps involved in each of the assessment schemes developed. The evaluation measures used to quantify control performance are defined in section 0. Results from a variety of simulations and experiments involving a process simulator are presented in section 0. Finally, a summary of the concepts discussed throughout the paper is provided in section 0.

Assessment Techniques:

The following section is designed to highlight the testing procedures used for both the PRBS based method and the relay based approach. The basic advantages and theory associated with each of the techniques will be presented in an effort to clarify their operational capacities. Also provided is theory behind the manipulation of frequency response information in order to obtain process model parameters.

PRBS Approach:

Pseudo random binary sequences (PRBS's), also known as pseudo noise (PN), linear feedback shift register (LFSR) sequences or maximal length binary sequences (m-sequences), are widely used in the field of system identification [8]. A pseudo random binary sequence, as its name suggests, is a semi-random binary sequence in the sense that it appears random within the sequence length, but the entire sequence repeats indefinitely. A PRBS sequence is an ideal test signal, as it simulates the random characteristics of a digital signal and can be easily generated. Pseudorandom binary sequences (PRBS's) are very effective as persistent excitation stimuli in dynamic testing [9]. Because the PRBS testing method is based on the cross-correlation techniques it is highly immune to extraneous noise of all kinds and as a result, its amplitude can be controlled to within safe limits without the risk of driving the plant outside the bounds of linear operation. The PRBS signal can also be easily coupled with the command input signal (set point) for normal plant operation. PRBS signal energy can be controlled over a range of frequencies with low amplitude by appropriate choice of the PRBS test signal parameters.

Correlation may be defined as a measure of similarity between two sequences. If the two sequences compared are different, 'crosscorrelation' is the term used and if they are the same, 'autocorrelation' is the term used. Mathematically, the autocorrelation of a sequence x(k) of length L may expressed as follows:

$$R_{xx}(m) = \frac{\sum_{k=1}^{L-m}\left(x(k)-\bar{x}\right)\left(x(k+m)-\bar{x}\right)}{\sum_{k=1}^{L}\left(x(k)-\bar{x}\right)^2}$$

$$m = 1, 2, 3 \ldots, L \qquad (1.2.1)$$

For the case of a PRBS sequence, its 'cyclic' autocorrelation function has the following values:

$$R_{xx}(k\Delta t) = \begin{cases} V^2 & k = 0 \\ -\dfrac{V^2}{L} & k \neq 0 \end{cases} \qquad (1.2.2)$$

where V is the bit interval voltage level, k is an integer and Δt is the pulse period (duration of each bit) of the PRBS. From (3.2.2) it can be seen that the autocorrelation function is a periodic triangular pulse train similar to that of the autocorrelation function of a truly random binary waveform. As the pulse period Δt vanishes and L becomes large the autocorrelation function tends closer to that of a periodic white noise source.

It has been well documented that the cross-correlation between the input x(k) and the output y(k) of a linear system, is related to the auto-correlation of the input by a convolution with the impulse response [10]:

$$\begin{aligned} & y(k) = h(k) \bullet x(k) \\ \Rightarrow\ & x(k) * y(k) = h(k) \bullet x(k) * x(k) \qquad (1.2.3) \\ \Rightarrow\ & R_{xy}(k) = h(k) \bullet R_{xx}(k) \end{aligned}$$

where $\bullet$ symbolises convolution and $*$ represents correlation. As already discussed, an important property of any PRBS is that its auto-correlation function is essentially an impulse. This impulse is represented by the Dirac delta function:

$$R_{xx}(k) \approx \delta(k) \qquad (1.2.4)$$

The result of convolving a sequence with a Dirac delta function is the sequence itself. Thus the impulse response h(k) can be found by cross-correlating the PRBS input x(k) with the output y(k):

$$R_{xy}(k) = \delta(k) \bullet h(k) = h(k) \qquad (1.2.5)$$

Hence it is possible to measure the impulse response of linear systems by calculating the cross-correlation between the PRBS and the system output signal. The system's frequency response may then be determined by applying the FFT to the system's impulse response.

Relay Based Approach:

There are a number of interesting features associated with relay based system testing. Two main advantages with this form of testing are 1) it is a closed loop test, which keeps the process variable under control and is usually preferred to open loop tests, and 2) a linear stable process with relay feedback is likely to automatically reach a sustained stationary oscillation known as a 'limit cycle'. Using the amplitude and period of this oscillation, information about the process critical point (point on the frequency response curve when input/output phase ratio is at -180 degrees) can easily be determined.

There are a number of different structures in which a relay feedback system (RFS) may be set up in order to obtain a control system's critical response information. The most efficient of these relay-based methods appears to be the 'Weighting' method as discussed in [1]. This technique involves applying a decay weighting on the signals such that the weighted signals die off as time passes. This technique is also known as windowing. If y(t) and u(t) are the process output and relay output in a relay feedback system, a decaying exponential $e^{-\alpha t}$ ($\alpha>0$) may be introduced to moderate these signals. Figure 2 illustrates how a typical relay output u(t) is affected when the decaying exponential is applied to it, thus producing weighted output ũ(t).

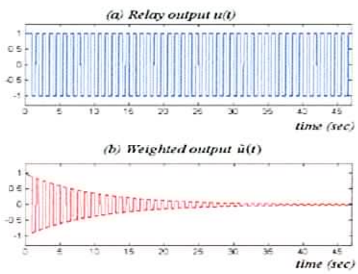

Figure 2 - Relay output and weighted version of relay output

The shifted process frequency response $G(j\omega_l + \alpha)$ may be determined using the following formula:

$$G(j\omega_l+\alpha)=\frac{Y(j\omega_l+\alpha)}{U(j\omega_l+\alpha)}=\frac{FFT\{\tilde{y}(kT)\}}{FFT\{\tilde{u}(kT)\}}, \quad l=1,\ 2,\ 3...,\frac{\mathrm{N}}{2} \qquad (1.2.6)$$

where $\omega_l = 2\pi l/(NT_s)$, $\tilde{y}(kT)$ and $\tilde{u}(kT)$ are the weighted system output and relay output respectively, N is the total number of samples taken of the output and T_s is the sampling period. According to [1], this weighting method yields the best results when compared with the other relay-based approaches.

Manipulating Frequency Response Information:

Consider the simple feedback system of Figure 3

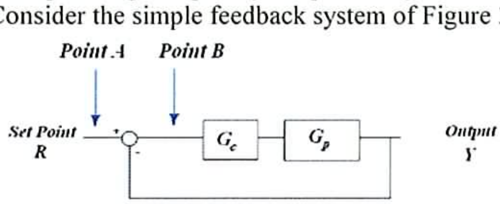

Figure 3 – Simple feedback system

For the case of the PRBS system-testing technique the binary testing sequence would be superimposed onto the set point and thus injected into the system at point A of Figure 3. The signals of interest in this testing procedure are therefore the PRBS signal and the system output, Y. After applying the FFT to these signals and determining the system's frequency response, as discussed in section 3.3.1.1, we are left with the closed loop frequency response, M(s), of the control system.

$$M(s)=\frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} \qquad (1.2.7)$$

While this is useful in itself, it is also possible to determine the open loop frequency response, $G_c(s)G_p(s)$, using this closed loop data. This may be done by applying the following formula:

$$G_c(s)G_p(s)=\frac{M(s)}{1-M(s)} \qquad (1.2.8)$$

Now considering the case of the relay-based approach, the relay is inserted at point B of Figure 3 and it is the open loop frequency response of the system that is obtained. In a similar fashion to that of the PRBS case, it is possible to obtain the closed loop frequency response of the system by simply applying equation (3.2.8). Using both the open loop and closed loop frequency response, important characteristics of the system may be determined. These frequency domain characteristics will be discussed further in the section entitled 'Evaluation Measures'.

For both the PRBS based approach and the relay based technique, a least squares based method known as the Gradient approach [13] is used, in conjunction with the control system's open loop frequency response, to obtain the process model parameters for a First Order Lag Plus Dead-time (FOLPD) model. This FOLPD model takes the form

$$G_p(s)=\frac{K_m}{sT_c+1}e^{-sT_d} \qquad (1.2.9)$$

where the static gain K_m, model time constant T_c and model time delay T_d are determined through the use of the Gradient approach. A further mathematical simulation using Matlab may then performed independent of the actual process (using the process model determined through the gradient approach) in order to obtain important time domain characteristics of the control system.

Evaluation Measures:

In order to quantify a control system's performance, a number of assessment measures were identified. These measures were considered appropriate as they provided a straightforward and easily interoperable measure of performance, and in most cases, these measures were industrial standard means of quantifying performance. These evaluation measures may be divided into two categories, namely time domain measures and frequency domain measures. While the time domain measures tend to provide a straightforward indication as to the response times of a system, the frequency domain measures tend to focus on system stability.

The time domain measures considered include the following: rise time, settling time, offset and overshoot. The rise time of a system can be defined as the time taken for the step response of a system to change from 10% to 90% of it final steady state value. A short rise time is usually desired. The settling time (T_s) is defined as the time the system takes to attain a 'nearly constant' value, usually +/- 5 percent of its final value [11]. Again, a short settling time is usually desired. Offset can be defined as the difference between the final, steady state value of the set point and that of the system output. In most cases, a zero steady state offset is desired [11]. The overshoot is the maximum amount that the system output exceeds its final steady state value and is usually expressed as a percentage.

The frequency domain measures decided upon include gain margin (Gm), phase margin (Pm), closed loop log modulus (L_{cmax}) and closed loop bandwidth (Bw). Both Pm and Gm are a direct measure of how much the phase and gain of the open loop system may vary before the closed loop system becomes unstable. While the phase and gain margin specifications can sometimes give poor results when the shape of the frequency response curve is unusual, the maximum closed loop log modulus does not have this problem. It is related to the closeness of the open loop frequency domain transfer function to the (-1,0) point at all frequencies [12]. The maximum closed loop log modulus is basically the peak value of the closed loop frequency response of the control system. A system's closed loop bandwidth is a direct measure of the response time capabilities of a system.

Results:

Table H-1 provides an illustration of the type of results obtained using both the PRBS and relay based evaluation methods. The results of Table H-1 were obtained when testing a process of three consecutive 1-second lags with a process gain of 1:

$$Process = \frac{1}{(s+1)(s+1)(s+1)} \qquad (1.3.1)$$

This process was implemented using a Process Control Simulator PCS327 Mk2. For this example the controller was considered to be a proportional controller with a gain of 1. The percentage error was calculated as 100% times the relative error:

$$\partial x = \frac{x_0 - x}{x} * 100\% = \left(\frac{x_0}{x} - 1\right) * 100\% \qquad (1.3.2)$$

where x was the sum of the absolute 'actual' results and x_0 was the sum of the absolute measured results. As can be seen from the table the PRBS based evaluation method proved to be consistently accurate when estimating frequency domain measures whereas the relay based approach appeared to be profoundly affected by noise levels. It should also be noted that the majority of the error associated with the PRBS based method came as a result of modeling the actual 3rd order process as a FOLPD model and then estimating the time domain characteristics based on this model. If a higher order model was used this error could be greatly reduced.

Summary:

This paper presented a review of the concepts associated with two independent methods for evaluating important time and frequency domain characteristics of a control system. Following a section outlining the theory and properties associated with each technique, the results of a comparative process test were provided. From these results it was obvious that the PRBS based testing scheme proved more reliable and more accurate, in the majority of instances, especially with increased noise levels

Table H-1 – Results obtained from a Matlab based Experiment

	Ts (secs)	Tr (secs)	Overshoot (%)	Offset (%)	Time error (%)	Gm (dBs)	Pm	Lcmax (dBs)	Bw (rads)	Frequency Error (%)	Noise (%)	Km	Tc (secs)	Td (secs)
Actual	5 79	1 87	13.9	50	-	18.1	0	-5 24	1.17	-	-	-	-	-
PRBS	3.1	1.76	0	53.64	-18.25%	18.68	0	-5.83	1	4.08%	0	0.88	2.49	0.84
Relay	3.78	1.2	5.79	52 37	-11.77%	17.61	0	-5.75	1.2	0 20%		0.91	1 99	0 96
PRBS	3.07	1.74	0	54.43	-17.22%	18.29	0	-5.87	1	2.65%	5	0.85	2.4	0.81
Relay	2.59	1.43	0.87	40.9	-36.01%	20.97	122	-4.77	1.07	507.14%		1.41	2.86	0.79
PRBS	2.9	1.64	0	55.4	-16.24%	18.15	0	-5.86	1	2.04%	10	0.85	2.38	0.79
Relay	2.99	1.72	0	38.37	-39 80%	20 33	93.83	-3.14	0.13	379.11%		1.62	3.6	0.77
PRBS	2.9	1.64	0	55.26	-16.43%	18.07	0	-5.77	1	1.35%	15	0.85	2.34	0.79
Relay	2 6	1 45	1.07	30.26	-50.56%	21 43	87 59	-2 21	0 78	357.00%		2 22	4 09	0.78
PRBS	2.9	1.66	0	55.25	-16.42%	17.92	0	-5.89	1	1.22%	20	0.85	2.37	0.8
Relay	3.41	2.18	0	62.9	-4.29%	1.06	0	-5.35	1 4	-68.14%		0.59	1.92	0.46
PRBS	2.88	1.65	0	55.36	-16.31%	17.86	0	-5.89	1	0.98%	25	0.85	2.35	0.79
Relay	3.53	2.19	0	66.66	1.15%	26.59	0	-5 7	1.4	37.45%		0.5	1.87	0.58
PRBS	2.87	1.63	0	55.28	-16.46%	17.95	0	-5.74	1	0.73%	30	0.84	2.31	0.79
Relay	2.28	1.15	0.13	72 14	5 79%	17.04	0	-5.99	1.54	0.24%		0.38	1.09	0.8
PRBS	2.9	1.63	0	55.58	-16.00%	17.87	0	-5.87	1	0.94%	35	0.84	2.34	0.79
Relay	2 08	0.46	12.7	71.8	21.63%	27	0	-5.4	4.7	51.37%		0 39	0.48	0.63
PRBS	2.9	1.64	0	55.33	-16.34%	17.74	0	-5.9	1	0.53%	40	0.85	2.35	0.79
Relay	0.58	0.42	0	684.12	857.41%	4.5	44.8	6.55	33.79	265.73%		3.07	0.07	0.13

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Abstract – **This paper presents an overview of a number of controller performance assessment techniques. The techniques discussed are divided into five categories, namely, time domain assessment, frequency domain assessment, minimum variance control (MVC) as a benchmark, statistical analysis, and other more 'problem specific' assessment techniques. Recent work, by various authors, in each of the five categories is outlined.**

I INTRODUCTION

According to [1] monitoring of process variables is useful, not only for assessing the status of the process, but also for controlling product quality. According to [2], in the testing of thousands of control loops in hundreds of operating plants, Techmation Inc. and others have found that more than 30% of the automatic control loops actually increase variability over manual control due to poor controller tuning. One reason why so many control loops perform poorly is that there is often numerous (more than a thousand) loops in a large process plant and not enough control engineers to maintain every loop.

In [3] Jamsa-Jounela et al. make the point that in order to ensure highest product quality it is essential to maintain the control system in an adequate manner. In [4] Vishnubhotla et al. discuss how the current standard practice for industrial process control is to install DCS (Distributed Control Systems) and PLC control system platforms. These system platforms accumulate large volumes of process data, but there are very few data mining tools.

It should be obvious, therefore, that there is a strong need for automatic assessment and monitoring of control loop performance. The goal of monitoring should be to provide information that can be used to assess the current status of the existing controller and to assist control engineers in deciding whether redesign is necessary [5]. When the controller performance is determined to be inadequate, it is important to ascertain whether an acceptable level of performance can be achieved with the existing control structure [6].

With these goals in mind, the next step is to review some of the existing loop performance assessment techniques. It was decided to divide the assessment techniques into the following categories:

1. Time domain assessment,
2. Frequency domain assessment,
3. Minimum variance control (MVC) as a benchmark,
4. Statistical analysis techniques, and
5. Other more 'problem specific' assessment techniques

II TIME DOMAIN ASSESSMENT

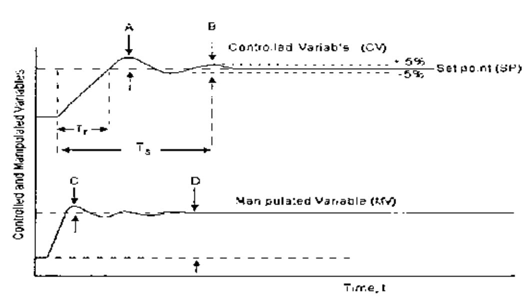

Figure (1). Typical transient response of a feedback control system to a step set point change

The dynamic response characteristics of a system may be accurately assessed using a number of useful time domain measures. These measures include rise time, settling time and integral error measures, see Figure (1). The rise time (T_r) is defined as the time from the step change in the set point until the controlled variable first reaches the new set point [7]. A short rise time is usually desired. The settling time (T_s) is defined as the time the system takes to attain a 'nearly constant' value, usually + or − 5 percent of its final value [7]. This measure is related to the rise time and decay ratio. A short settling time is usually desired.

The integral error measures indicate the cumulative deviation of the controlled variable from its set point during the transient response. The Integral of Absolute Error (IAE) criterion is determined from the sum of areas above and below the set point. It is an appropriate measure of control performance when the effect on control performance is linear with the deviation magnitude. The Integral of Squared Error (ISE) criterion is appropriate when large deviations cause greater performance degradation than small deviations. The Integral of Time multiplied by Absolute Error (ITAE) criterion penalizes deviations that endure for a long time. The Integrated Error (IE) criterion is not normally used because positive and negative errors cancel in the integral, resulting in the possibility of large positive and negative errors giving a small IE [7]. The formulae for calculating the integral error measures are given below:

$$IAE = \int_0^{\infty} |SP(t) - CV(t)|.dt \qquad (1)$$

$$ISE = \int_0^{\infty} [SP(t) - CV(t)]^2.dt \qquad (2)$$

$$ITAE = \int_0^{\infty} t.|SP(t) - CV(t)|.dt \qquad (3)$$

$$IE = \int_0^{\infty} [SP(t) - CV(t)]dt \qquad (4)$$

In [3] Jamasa-Jounela et al. present a set of performance indices appropriate to process monitoring and assessment. These indices include IAE, ITAE, rise time and settling time. In [8] Swanda and Seborg have developed a new methodology to assess the performance of PI controllers from closed loop response data for a set point step change. This method is based on two new dimensionless performance indices, the dimensionless settling time and the dimensionless IAE. This methodology is also applicable to PID controllers. In [9] Horch and Stattin extend this method to analyse the settling time-normalised by the apparent process time delay-of a set point step response. In [10] Ruel discusses a number of metrics used to assess loop performance. These include IAE, set point crossing, and average error. In [11] Huang and Jeng assess a simple feedback system by analysing IAE and rise time observed from the response of the system to a step set point change. Optimal IAE's and associated rise times are computed. Comparing its current IAE to the optimal IAE allows an assessment of the performance of the system.

Explained in more detail below, there are a variety of other time domain measures that may be used to assess a system's performance. These include offset, decay ratio, manipulated variable overshoot, maximum deviation of the controlled variable, and magnitude of the controlled variable in response to a sine disturbance. Offset is defined as the difference between the final, steady state value of the set point and of the controlled variable. In most cases, a zero steady state offset is desired [7]. The decay ratio (B/A), see Figure (1), is the ratio of neighbouring peaks in an underdamped controlled-variable response. Usually, periodic behaviour with large amplitudes is avoided in process variables; therefore, a small decay ratio is usually desired, and an overdamped response is sometimes desired [7]. The manipulated variable overshoot (C/D), see Figure (1), is of concern because the manipulated variable is also a process variable that influences performance. Some large variations can cause long-term degradation in equipment performance. The overshoot is the maximum amount that the manipulated variable exceeds its final steady state value and is usually expressed as a percentage of the change in manipulated variable from its initial to its final value. Some overshoot is acceptable in some cases [7]. The maximum deviation of the controlled variable from the set point is an important measure of the process degradation experienced due to disturbances. Usually a small value is desirable so that the process variable remains close to its set point [7]. In many cases the disturbance is composed predominantly of one or a few sine waves. Therefore, the behaviour of the control system in response to sine inputs is of great practical importance, because through this analysis the relationship between the frequency of the disturbances and the control performance is deduced. Control performance is assessed by measuring the amplitude of the output sine wave; the metric is often expressed as the ratio of the output to input sine wave amplitudes [7].

In [12] Stanfelji et al. present a method for monitoring and diagnosing the performance of single loop-control systems based primarily on normal operating data. This method involves analysing the autocorrelation and cross correlations of a time series of control loop variables. In [13] Hagglund describes a procedure for the automatic detection of sluggish control loops obtained from conservatively tuned controllers. The 'idle index' describes the relation between the times of positive and negative correlation between the control and measurement signal increments. From this index the sluggishness of the control loop can be determined.

III FREQUENCY DOMAIN ASSESSMENT

According to [14] traditional measures such as overshoot, rise time, decay ratio, settling time and the ISE are difficult to translate into an economic measure so as to justify process or control system redesign. They state, however, that frequency domain measures can be used to provide a measure of performance that can be translated into an economic measure. This section will review some of the more common frequency domain assessment methods.

Three different types of plots are commonly used to graphically illustrate the frequency response of a controlled system, see Figure (2). These three plots are the Nyquist, Bode and Nichols plots. **Nyquist plots**, also called polar plots, may be obtained by either plotting the real versus the imaginary part of the frequency domain transfer function, G(jw) (using rectangular coordinates), or by plotting the magnitude at a particular phase angle of G(jw) (using polar coordinates). **Bode plots** require two curves to be plotted; these plots show how the magnitude ratio and phase angle vary with frequency. The **Nichols plot** is a single curve in a coordinate system with phase angle as the abscissa and log modulus as the ordinate. Frequency is a parameter along the curve [15].

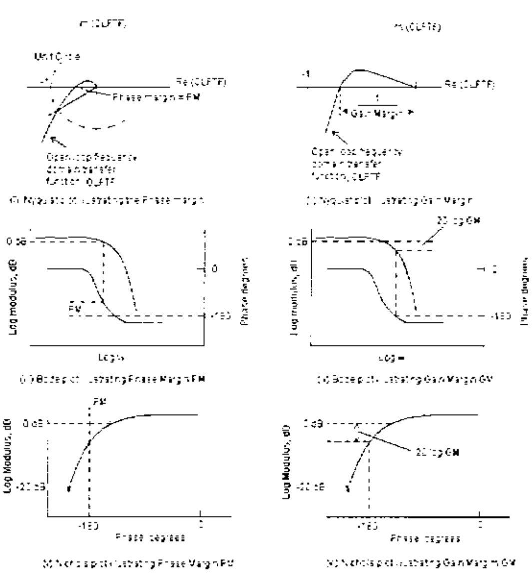

Figure (2) Nyquist, Bode and Nichols plots illustrating Gain and Phase margin.

Phase margin and **gain margin** are two commonly used assessment measures. Phase margin (PM) is defined as the angle between the negative real axis and a radial line drawn from the origin to the point where the open loop frequency domain transfer function intersects the unit circle. The bigger the phase margin the more stable the closed loop system. Phase margins of 45^0 are often considered appropriate [15]. The gain margin (GM) is defined as the reciprocal of the intersection of the open loop frequency domain transfer function polar plot on the negative real axis. The bigger the gain margin, the more stable the system. Typically gain margin values of about 2 are recommended [15]. In [16] Astrom and Hagglund discuss a simple method for estimating the critical gain of a controlled system, from which the gain margin may be deduced.

The **maximum closed loop log modulus**, L_{cmax}, is another quantity used to assess performance in the frequency domain, see Figure (3). While the phase and gain margin specifications can sometimes give poor results when the shape of the frequency response curve is unusual, the maximum closed loop log modulus does not have this problem since it directly measures the closeness of the open loop frequency domain transfer function to the (-1,0) point at all frequencies [15]. In [17] Chiou and Yu propose a monitoring procedure that identifies the maximum closed loop log modulus in two to three relay feedback experiments. In [5] Ju and Chiu present a monitoring procedure incorporating the FFT (Fast Fourier Transform) technique to identify L_{cmax} on line. This proposed method addresses some of the problems in the method presented in [17] i.e. too many relay tests are required, the frequency search range is confined to the third quadrant, and the identified value of L_{cmax} cannot be used on-line to redesign the controller. In [14] Belanger and Luyben propose a new test to locate the peak regulator log modulus. The test involves the insertion of a relay between the controlled variable and a given load disturbance model, with the feedback controller on automatic. This causes the plant to exhibit a sustained oscillation at the frequency where the L_{cmax} curve exhibits a peak. This test can be applied to both simulated models as well as existing plants.

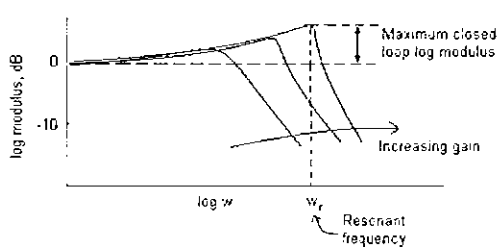

Figure (3). Plot illustrating the maximum closed loop log modulus L_{cmax}.

The **capacity based method for quantifying controllability** is a method used to quantitatively incorporate the economics of control into conventional steady-state design methods [15]. In [18] Elliot and Luyben outline a generic methodology called the capacity based economic approach that can be used to compare or screen preliminary plant designs by quantifying both steady-state economics and dynamic controllability. In [19] Elliot et al. demonstrate that the capacity based economic approach can be successfully applied to a large industrial scale process. In [20] Elliot and Luyben analyse the effectiveness of the capacity based economic approach when controlling a complex recycle system consisting of a reactor and two distillation columns.

In [21] Kendra and Cinar discuss a method used to estimate the closed loop transfer function of a system by exciting the reference input with a zero mean, pseudo random binary sequence and observing the process output and error response. Performance assessment is based on the comparison between the observed frequency response characteristics and the design specifications.

IV MINIMUM VARIANCE CONTROL (MVC) AS A BENCHMARK

According to [22] and [23], minimum variance control is considered the optimal feedback control provided that the process can be described by a linear transfer function with additive disturbance. In [24] Spring states that minimum variance is a better benchmark than zero variance for evaluating controller performance. Control systems cannot reduce the variance in product quality below the variance inherent in the process. On the basis of minimum variance, an investment in controller maintenance can be evaluated realistically.

According to [4], this benchmark control may or may not be achievable in practice depending on process invertibilty and other process physical constraints. Also, it is worth noting that this technique requires knowledge of the process time delay, which may not always be available. However, as a benchmark, it provides useful information such as how much 'potential' there is to improve controller performance. In [25] Thornhill et al. make the point that minimum variance control may require excessively vigorous action of the manipulated variable and, as a result, can lead to maintenance problems for the actuators. This section presents a review of some of the papers available that discuss some of these details.

A number of papers are recommended that give an overview of the MVC method. In [23] Harris discusses how an estimate of the best possible control can be obtained by fitting a univariate time series to process data collected under routine control. In [26] Harris et al. discuss some of the concepts associated with assessing the effectiveness of a control system. Also discussed in this paper is how these concepts were initially developed using a performance benchmark of minimum variance control for SISO systems. In [25] Thornhill et al. examine some of the factors that influence the minimum variance performance measure of a SISO control loop. The authors show that, for an arbitrary controller, the calculated minimum variance benchmark is different for servo and regulator operation. In [27] Grimble discusses the use of the generalised minimum variance control law for control loop performance assessment and benchmarking. In [28] Huang and Shah discuss, in detail, some of the theory behind the MVC method.

Based on MVC theory, a performance index (the Harris index) was first introduced by Harris [23]. This index compares the actual variance in the process variable to that of a minimum variance controller. In [22] and [29] Desborough and Harris present a normalised performance index used to characterise the performance of control systems. This index provides a measure of the proximity of control to minimum variance control. Time domain and spectral interpretations of the index are discussed and a fast, simple on-line method for estimating the index is given. In [30] Bezergianni and Georgakis introduce a modified version of the Harris index in which the closed loop performance is compared with that obtained with the best theoretical control action (minimum variance control) and no control action. In [4] Vishnubhotla et al. discuss a method of performance assessment based on the Harris index. The resulting index, gives an indication of the level of performance of the controller, and an indication of the action required to improve performance. In [24] Spring discusses a performance index based on minimum variance control. In [31] Ko and Edgar outline a scheme for the estimation of achievable PI control performance, measured by output variance, in linear processes with dead time when stochastic load disturbances are affecting the process.

A number of papers have been written in which modifications to the MVC benchmark have been made. In [32] Eriksson and Isaksson discuss how this technique provides an inadequate measure of performance if the aim is not control of statistically random disturbances. Some modifications to the Harris index are suggested. In [33] Horch and Isaksson discuss a modification to the index introduced by Harris

[23]. The modified index and the original index are then evaluated and compared using data from industrial processes. In [34] Isaksson discusses the MVC benchmarking technique and suggests a set of alternative indices. In [35] and [36] Huang discusses some of the aspects associated with the minimum variance control law for linear time variant processes. Alternative benchmarks that are more suitable for time variant processes are suggested. In [37] Venkatesan introduces a minimum variance feedback control algorithm (MVFCA) that can be used to calculate a series of adjustments required at the input that minimises the variance of the output variable. In [38] Kucera presents a tutorial paper emphasising the contribution of V. Peterka to the steady state minimum variance control problem. In [39] Qin presents an overview of the current status of control performance monitoring using minimum variance principles.

V STATISTICAL ANALYSIS

According to [1], the goal of statistical process monitoring (SPM) is to detect the existence, magnitude and time of occurrence of changes that cause a process to deviate from its desired operation. A number of useful techniques for the monitoring of process variables are discussed in this paper. These methods include Shewhart control charts, moving average control charts, cumulative sum charts and partial least squares methods.

The likelihood method is a useful technique for assessing performance. According to [1], this method may be used to determine if the error response characteristics are acceptable based on specified dynamic performance bounds. Dynamic response characteristics such as overshoot or settling time can be extracted from the pulse response of a fitted time series model of the output error. The pulse response of the estimated output error can be compared to the pulse response of the desired response specification to determine if the output error characteristics are acceptable. In [40] Tyler and Morari propose a framework in which acceptable performance is expressed by constraints on the closed loop transfer function impulse response coefficients. Using likelihood methods, a hypothesis test is outlined to determine if control deterioration has occurred. In [41] Zhang and Ho propose the use of the likelihood ratio method as a means of sensitivity analysis of stochastic system performance.

In [42] Li et al. develop a monitor to automatically detect poor control performance. The monitor provides a measure (Relative Performance Index – RPI) of a control loop performance relative to a reference model of acceptable control. The reference model simulates the controlled variable output of a user defined, acceptably tuned, control loop. In [43] Zhong demonstrates how to improve the effectiveness of equipment monitoring and process induced defect control through properly selecting, validating and using the hypothetical distribution models. In [44] Mosca and Agnoloni study the early detection problem of stability losses or close-to-instability conditions in feedback control systems, where the plant dynamics are uncertain and possibly time-varying.

VI OTHER ASSESSMENT TECHNIQUES

This section contains a number of more 'problem specific' assessment techniques, as opposed to the more general methods discussed in previous sections. The focus of this section is on methods to both detect and diagnose oscillations in control loops. The techniques discussed here may well be considered special cases of the methods discussed in previous sections.

The first step in dealing with an under performing control loop with suspected oscillation disturbances is the detection stage. In [45] Hagglund presents a closed loop performance monitor (CLPM) to detect oscillations in the control loop. The procedure presented is automatic in the sense that no additional parameters, other than the normal controller parameters, have to be specified. In [46] Huang et al. discuss a method of determining the presence of oscillations in selected frequency ranges, based on the regularity of the zero crossings of filtered auto-covariance functions. In [47] Chang et al. present a system-wide dynamic performance monitoring system (DPMS), which includes special features such as oscillation detection. In [48] Stenman et al. propose a model-based method for detecting static friction (stiction) in control valves. In contrast to existing methods, only limited process knowledge is needed and it is not required that the loop has oscillating behaviour. In [49] Wallen proposes an integrated system for valve diagnostics and automatic PID tuning. The purpose of the method is to detect non-linearities such as friction and hysteresis since these may drastically decrease the control performance.

Once an oscillation has been detected, the next step is to determine its cause. In [50] Thornhill and Hagglund present a set of 'operational signatures' that indicate the cause of an oscillation. This method involves the offline analysis of ensembles of data from control loops. In [51] Horch proposes a simple method for the diagnosis of oscillations in process control loops based on the cross correlation between control variable and loop output. This method is shown to correctly identify the two most important reasons for oscillations in control loops in the process industry, namely, external oscillating disturbances and stiction in control valves. In [52] Taha et al. present an on line automatic procedure for the diagnosis of oscillations in control loops. This method works without disturbing normal plant operation.

VII CONCLUSIONS

According to [53], minimum variance control (MVC) as a benchmark (as discussed in [22]) or variants of it, is used in virtually all industrial controller assessment packages due to its theoretical and practical advantages. In [53] Hugo lists some of these software packages as follows: Performance assessment tool-kit [54]; loop scout [55]; Process Doc [56]; and Aspen Watch [57]. Software packages such as Probe [58] and Plant Triage [59] also offer a number of useful routines and algorithms related to MVC and some of the other assessment techniques mentioned previously. In [26] Harris et al. state that a comprehensive approach for assessing the effectiveness of control systems requires determination of the capability of the control system, development of suitable statistics for monitoring the performance of the existing system, development of methods for diagnosing the underlying causes for changes in the performance of the control system, and incorporation of these methods in an industrial setting.

The main advantage that MVC as a benchmark has over the other four categories discussed in the paper is that it not only gives an indication as to the current level of performance of the controlled system under investigation, but it can also determine whether or not current performance can be improved by retuning the controller. In [4] Vishnubhotla et al. highlight this point by stating that 'as a benchmark (MVC) … provides useful information such as how well the current controller was tuned compared to the minimum variance controller and how much 'potential' there is to improve controller performance'. For example, an index (ratio of minimum achievable output variance to actual variance) value of 1 indicates that current performance cannot be improved by retuning the existing controller. However, an index value below 1 indicates retuning the controller will have an impact on improving system performance.

While time domain, frequency domain or statistical analysis techniques may give an accurate indication as to the current level of performance of the controller, no indication is given as to whether or not retuning will lead to improved performance. Simulations must be run and re-run with differently tuned parameters in order to determine if improved control is possible. This could prove to be an inefficient use of time if it was discovered, after numerous simulations had been run and analysed, that it is not possible to improve on the current control performance using the current controller structure.

Therefore, these findings would suggest that whatever assessment techniques are used, benchmarks specific to the controller under assessment must be used in order to determine whether retuning or controller redesign is necessary. According to [10], continuous performance monitoring requires benchmarking so that it may be observed how performance has changed with time. Also, this benchmark must be specific to the plant under investigation. Future work will focus on the development of a method to calculate controller specific benchmarks, in one of the assessment categories outlined in this paper, in order to provide a more efficient monitoring and assessment tool.

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